Centre for Quantum Technologies

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Absolute rate of SPDC into single transverse Gaussian modes

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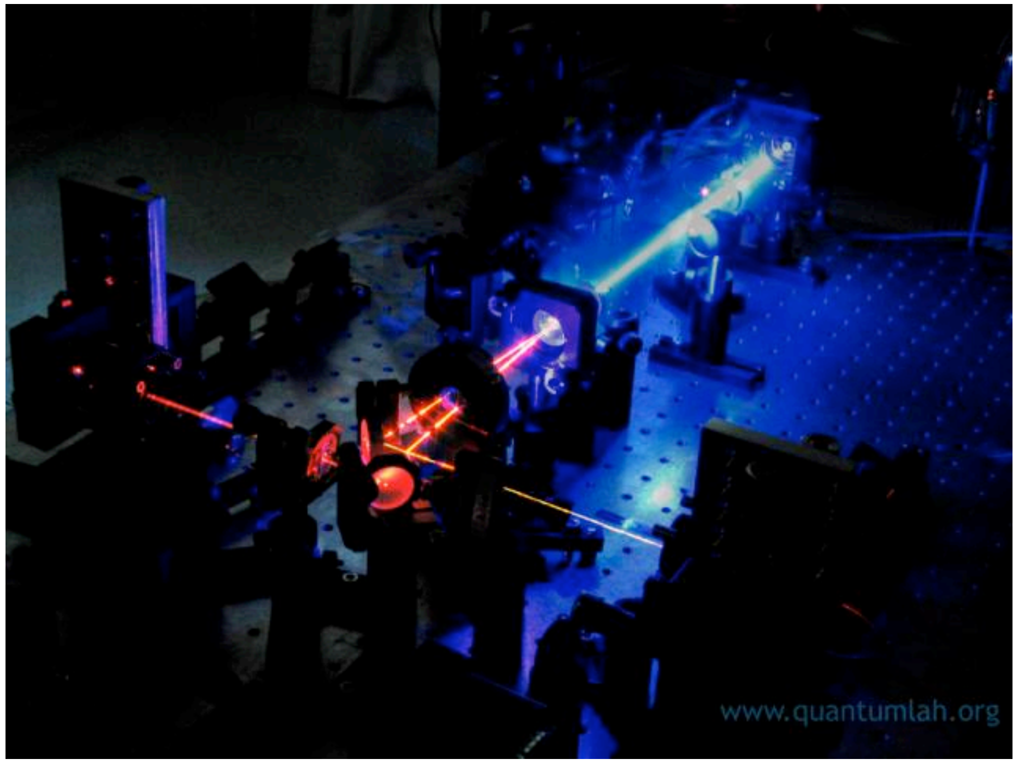
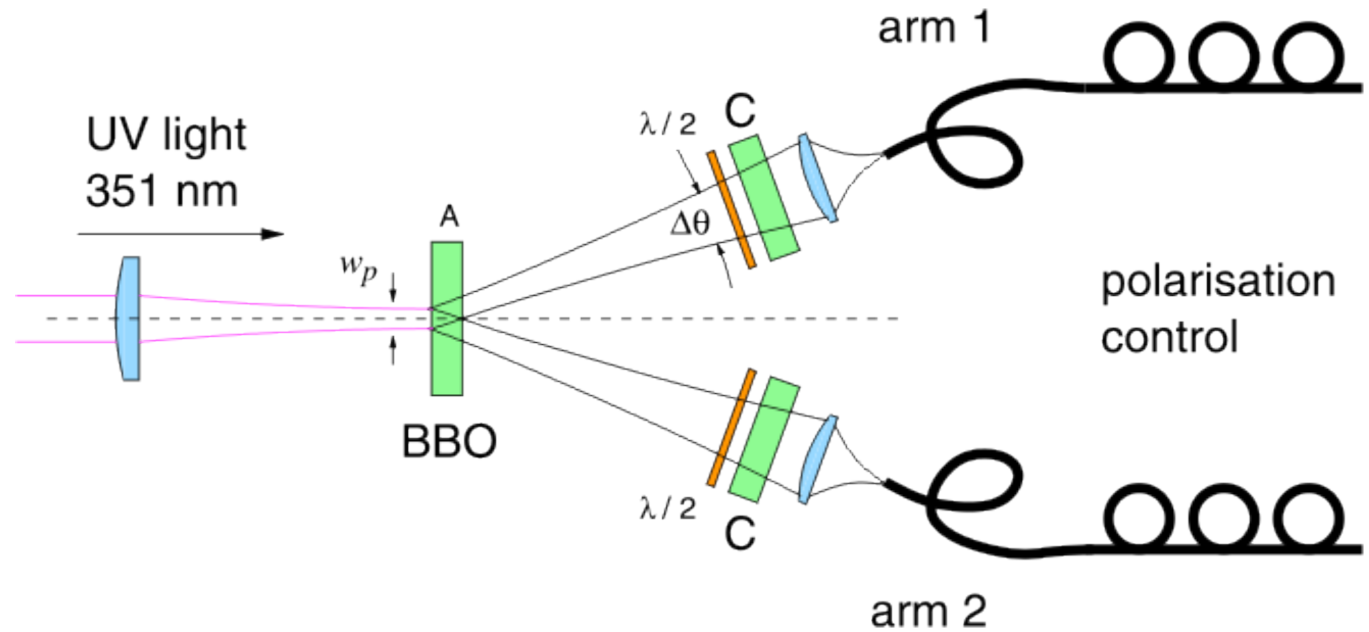
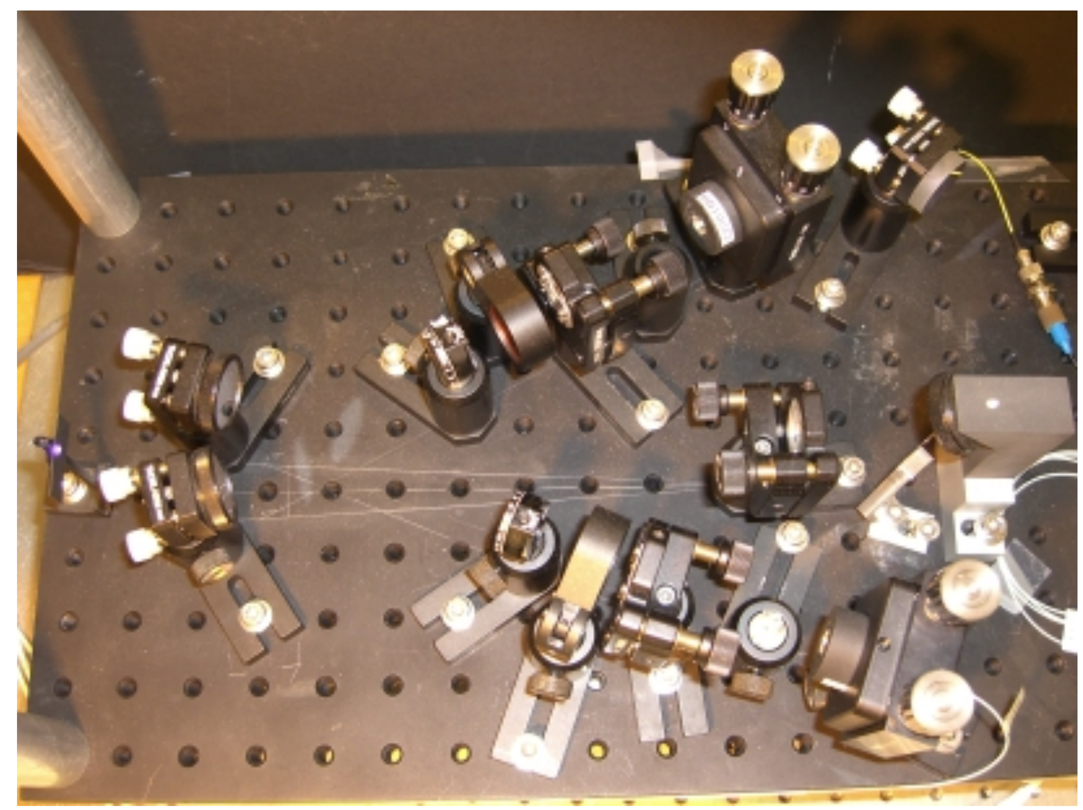
SPDC typical setups





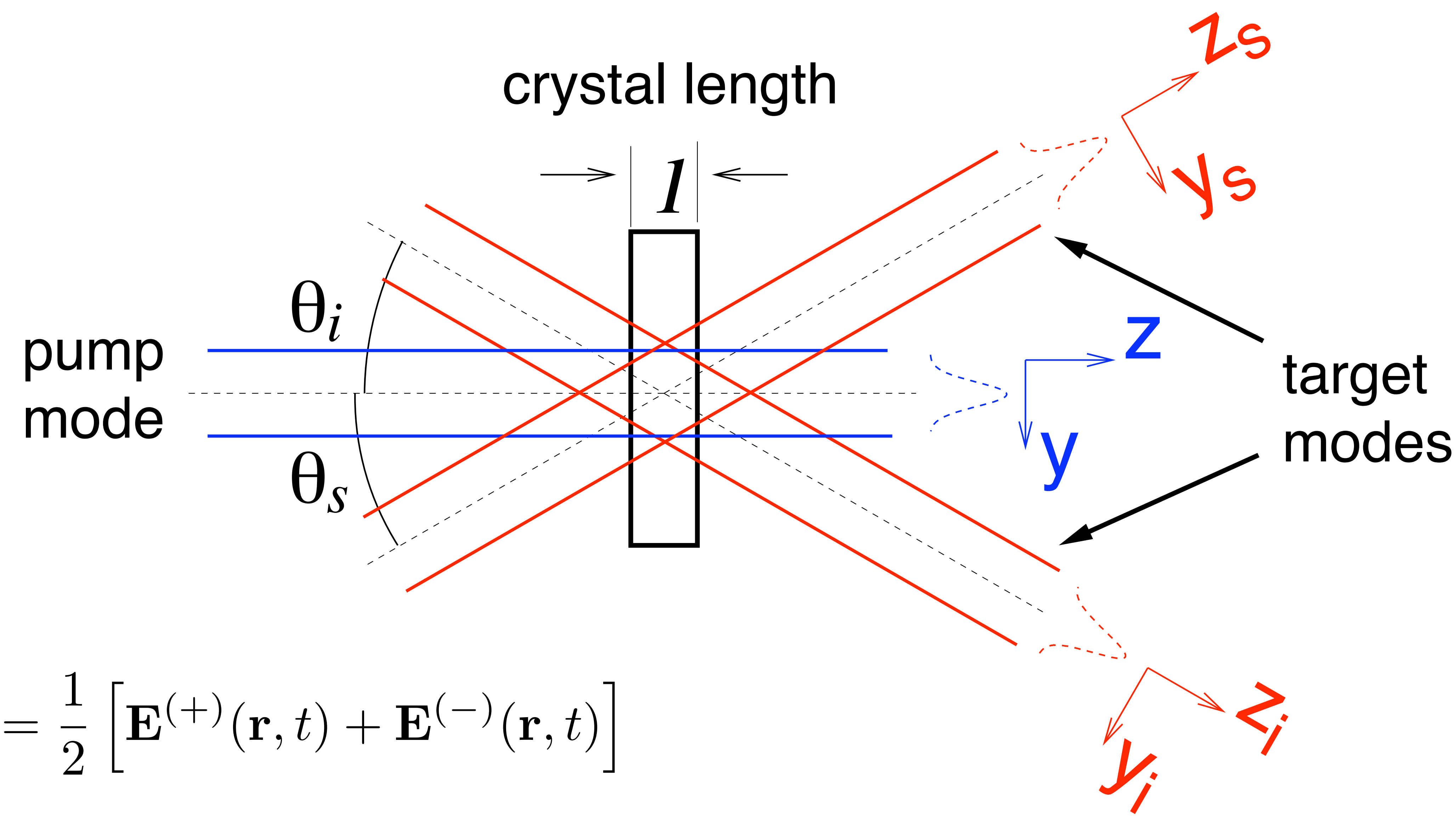
- Bulk crystal
- Type-II (or Type-I)
- Non-collinear
- Coupled in to single mode fibers for further manipulation
- Experimentally reported performance: ~1000 detected pairs/s/mW
- Efficiency (coincidences/singles): ~30%



Early theory of SPDC: Klyshko, Kleinman, Mandel, Hong, Yariv, Siegman, Burnham, … (mostly 70s and 80s)

Optimization of particular figures of merit (mostly efficiency) and spectral aspects: Castellato et al. (04, 05), Bovino et al. (03), Ljunggren et al (05), Kurtsiefer et al (01).

Physical model



$$\mathbf{E}(\mathbf{r},t) = \frac{1}{2}\left[\mathbf{E}^{(+)}(\mathbf{r},t) + \mathbf{E}^{(-)}(\mathbf{r},t)\right]$$

$$g(\mathbf{r}) = e^{ikz} \cdot U(x,y) = e^{ikz} \cdot e^{-\frac{x^2+y^2}{W^2}}$$ geometrical factor

$$\mathbf{E}_p(\mathbf{r},t) = \frac{1}{2}\left[E_p^0 \mathbf{e}_p g_p(\mathbf{r}) e^{-i\omega_p t} + c.c\right]$$ classical field, non-depleted

$$\hat{\mathbf{E}}_{s,i} = \frac{i}{2}\sum_{k_{s,i}} \sqrt{\frac{2\hbar\omega_{s,i}}{n_{s,i}^2 \epsilon_0}} \frac{\alpha_{s,i}}{\sqrt{L}} \mathbf{e}_{s,i} g_{s,i}(\mathbf{r})\, e^{-i\omega_{s,i} t}\, \hat{a}_{k_{s,i}} + h.c.$$ quantized field

Interaction Hamiltonian

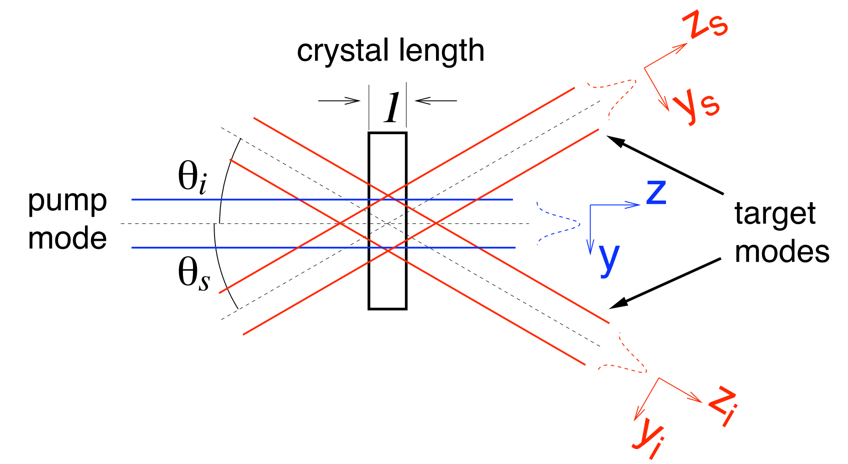
$$\hat{H}_I = -\frac{2\epsilon_0\chi^{(2)}}{8}\int_{-\infty}^{\infty} dxdy \int_{-l/2}^{l/2} dz\, \mathbf{E}_p^{(+)}\hat{\mathbf{E}}_s^{(-)}\hat{\mathbf{E}}_i^{(-)} + h.c.$$

$$= d\int_{-\infty}^{\infty} dxdy \int_{-l/2}^{l/2} dz \sum_{k_s,k_i} \frac{\hbar\sqrt{\omega_i\omega_s}}{n_s n_i}\frac{\alpha_s\alpha_i E_p^0}{L} \times$$

$$\times\, e^{-i\Delta\omega t} g_p(\mathbf{r}) g_s^*(\mathbf{r}) g_i^*(\mathbf{r}) \hat{a}_{k_s}^\dagger(t)\hat{a}_{k_i}^\dagger(t) + h.c.$$

- (yellow) crystal is taken to be infinitely large in transverse direction
- (blue) d is the effective non-linearity $2d = \mathbf{e}_p\chi^{(2)} : \mathbf{e}_s\mathbf{e}_i$
- (green) geometrical overlap

Overlap integral



$$\Phi(\Delta\mathbf{k}) = \int dz \int dy\, dx\, g_p(\mathbf{r}) g_s^*(\mathbf{r}) g_i^*(\mathbf{r})$$
$$= \int dz \int dy\, dx\, e^{i\Delta\mathbf{k}\cdot\mathbf{r}} U_p(\mathbf{r}) U_s(\mathbf{r}) U_i(\mathbf{r}).$$

phase mismatch

$$\Phi(\Delta\mathbf{k}) = \frac{\pi}{\sqrt{A\cdot C}}\, e^{-\frac{\Delta k_y^2}{4C}} \int dz\, e^{-Hz^2 + izK}$$

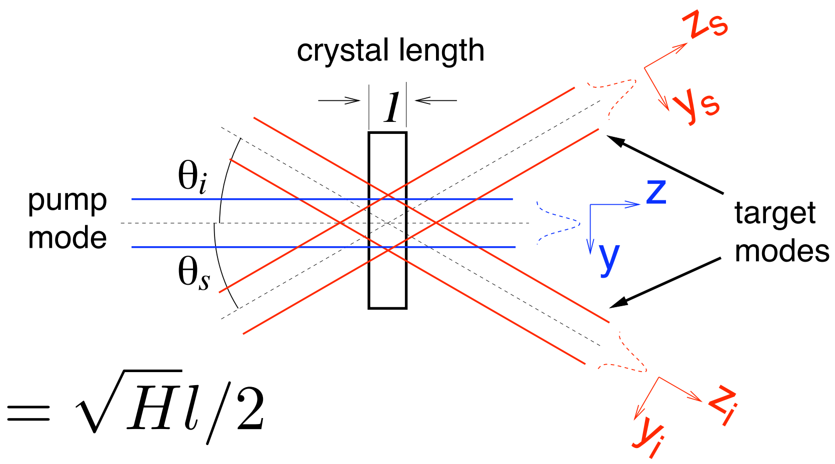
$= \Phi_z$

$$A = \frac{1}{W_p^2} + \frac{1}{W_s^2} + \frac{1}{W_i^2}$$
$$C = \frac{1}{W_p^2} + \frac{\cos^2\theta_s}{W_s^2} + \frac{\cos^2\theta_i}{W_i^2}$$
$$D = \frac{\sin 2\theta_s}{W_s^2} - \frac{\sin 2\theta_i}{W_i^2}$$
$$F = \frac{\sin^2\theta_s}{W_s^2} + \frac{\sin^2\theta_i}{W_i^2}$$
$$H = F - \frac{D^2}{4C}$$
$$K = \Delta k_y \frac{D}{2C} + \Delta k_z$$

$\Delta k_x = 0$ since all waves in y–z plane

$\Delta k_y = ?$ can be neglected only when perfect transverse phase matching,

Physical interpretation



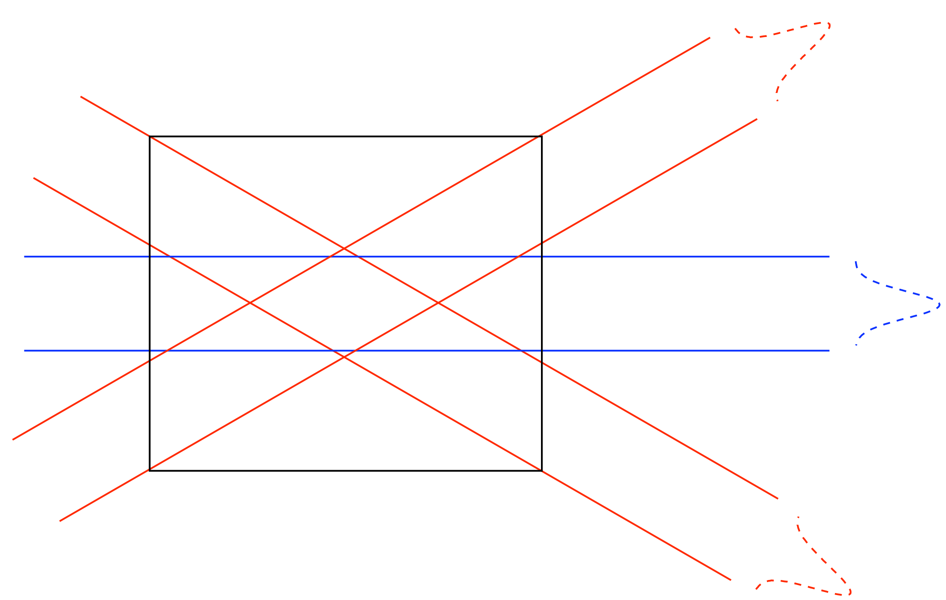
$$\Phi_z = l \cdot \int_0^1 du\, e^{-\Xi^2 u^2} \cos(\Delta\varphi u)$$

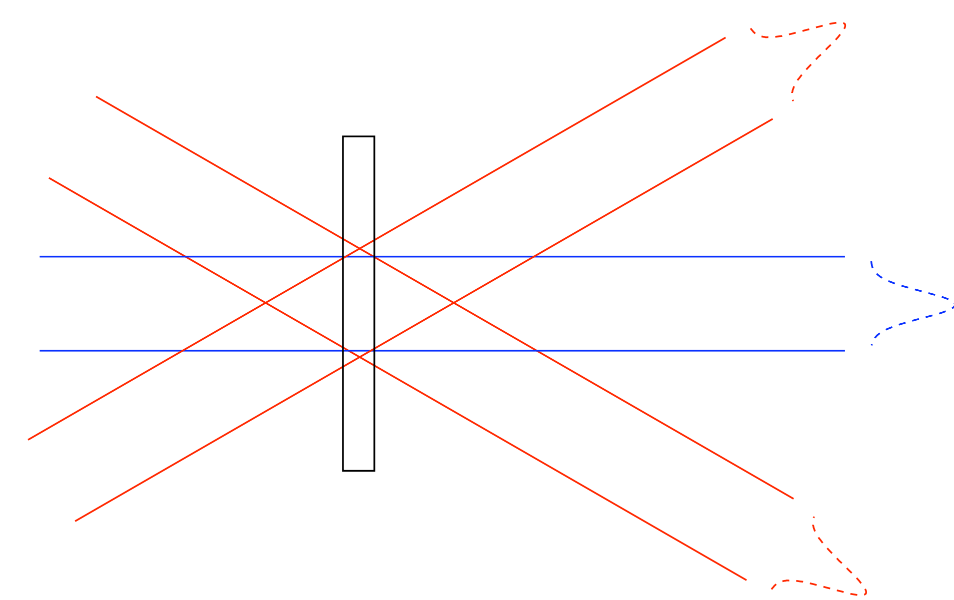
$$\Xi := \sqrt{H} l/2$$
$$\Delta\varphi := K l/2$$

Ξ can be interpreted as a “walk–off” parameter

$\Xi > 1$ thick crystal

$\Xi < 1$ thin crystal





Thick and thin crystal limits

$\Xi > 1$ thick crystal

$$\Phi_z \approx l \frac{\sqrt{\pi}}{2\Xi}\,\mathrm{Erf}(\Xi)$$

$$\stackrel{\Xi \to \infty}{=} \sqrt{\frac{\pi}{H}}\,\mathrm{Erf}(\Xi)$$

The result does not depend strongly on the length of the crystal. In the limit of really long crystals there is no dependence.

$\Xi < 1$ thin crystal

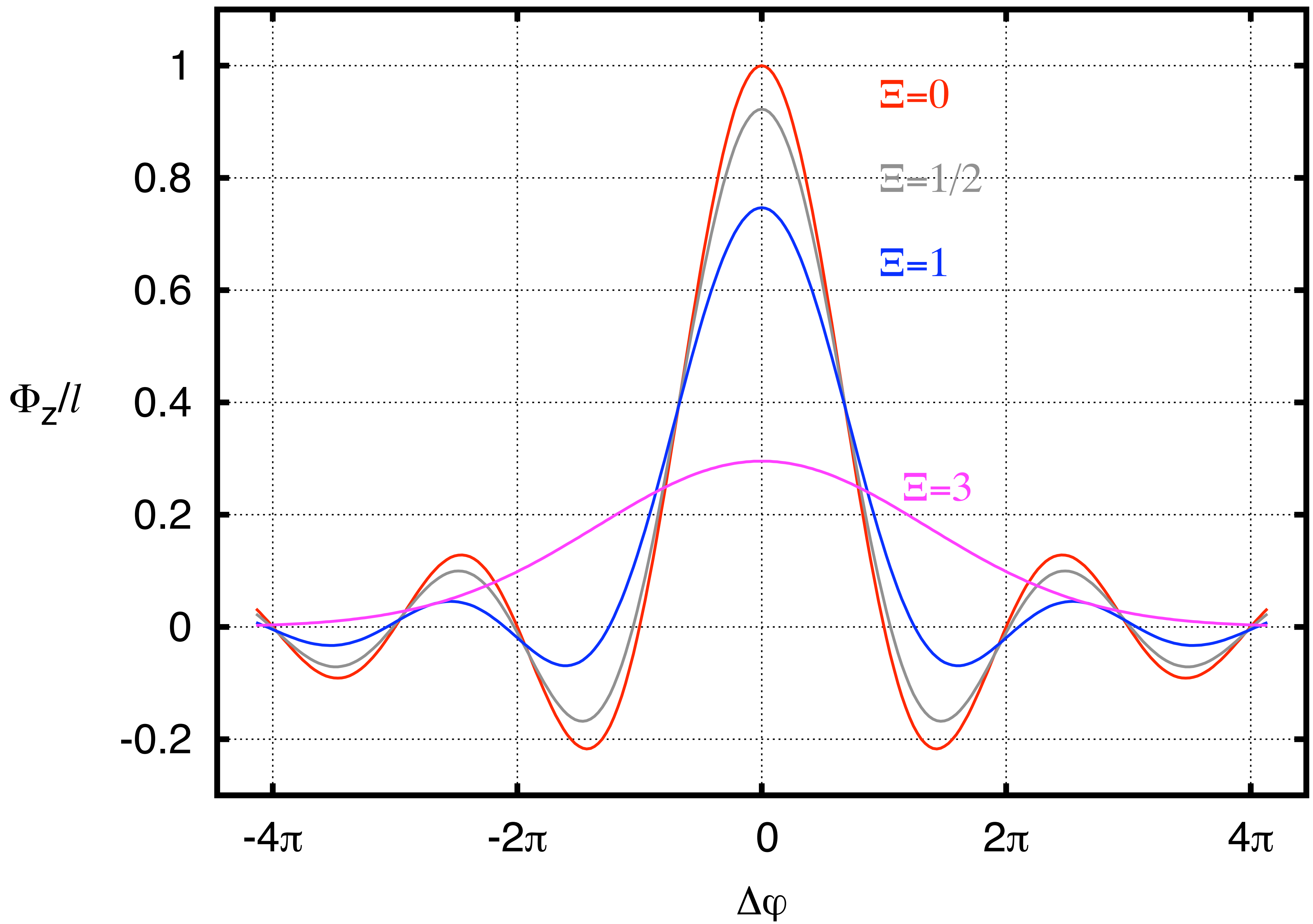
In particular collinear beams will satisfy:

$$\theta_i = \theta_s = 0$$

$$K = \Delta k_z$$

$$\Phi_z = l\,\mathrm{sinc}(\Delta\varphi)$$

Thick and thin crystal limits



Emission rates

Emission into each spectral mode

$$\rho(\Delta E) = \frac{\Delta m}{\Delta k_i} \frac{\partial k_i}{\partial(\hbar \Delta\omega)} = \frac{L}{2\pi} \frac{n_i}{\hbar c}$$

mode density

$$\Delta m / \Delta k_i = L/2\pi$$

$$R(k_s) = \frac{2\pi}{\hbar} \left| \langle f | \hat{H}_I | i \rangle \right|^2 \rho(\Delta E) = \left| d \ \alpha_s \alpha_i E_p^0 \Phi(\Delta \mathbf{k}) \right|^2 \frac{\omega_s \omega_i}{n_s^2 n_i c L}$$

Emission per unit of angular frequency

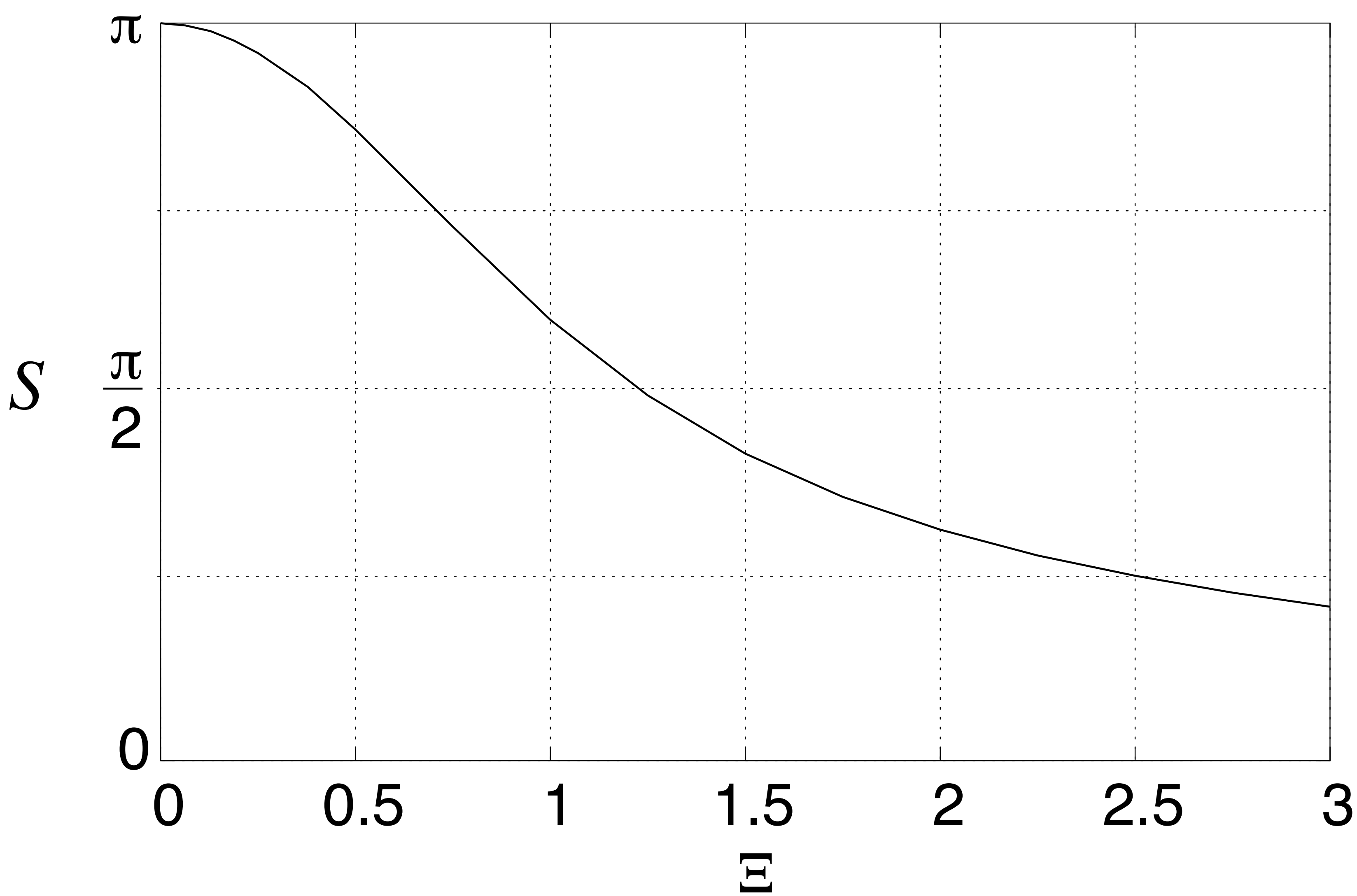
$$\frac{dR(\omega_s)}{d\omega_s} = \left[\frac{d\alpha_s \alpha_i E_p^0 \Phi(\Delta \mathbf{k})}{c} \right]^2 \frac{\omega_s \omega_i}{2\pi n_s n_i}$$

quantization length L
dependence has vanished
as expected

Total rate

$$R_T = \left[\frac{d\alpha_s\alpha_i E_p^0}{c}\right]^2 \frac{\omega_s\omega_i}{2\pi n_s n_i} \boxed{\int d\omega_s \left|\Phi(\Delta\mathbf{k})\right|^2}$$

S



Largest absolute rate for thin crystal limit

$$\Xi = 0$$

Optimal waist matching

In the thin crystal limit and collinear emission we can find a closed analytical solution

$$\tilde{R}_T = \frac{4d^2 P l \omega_p^2}{9 n_s n_i n_p \epsilon_0 \pi W_p^2 (n_i - n_s) c^2}$$

Optimal waist matching

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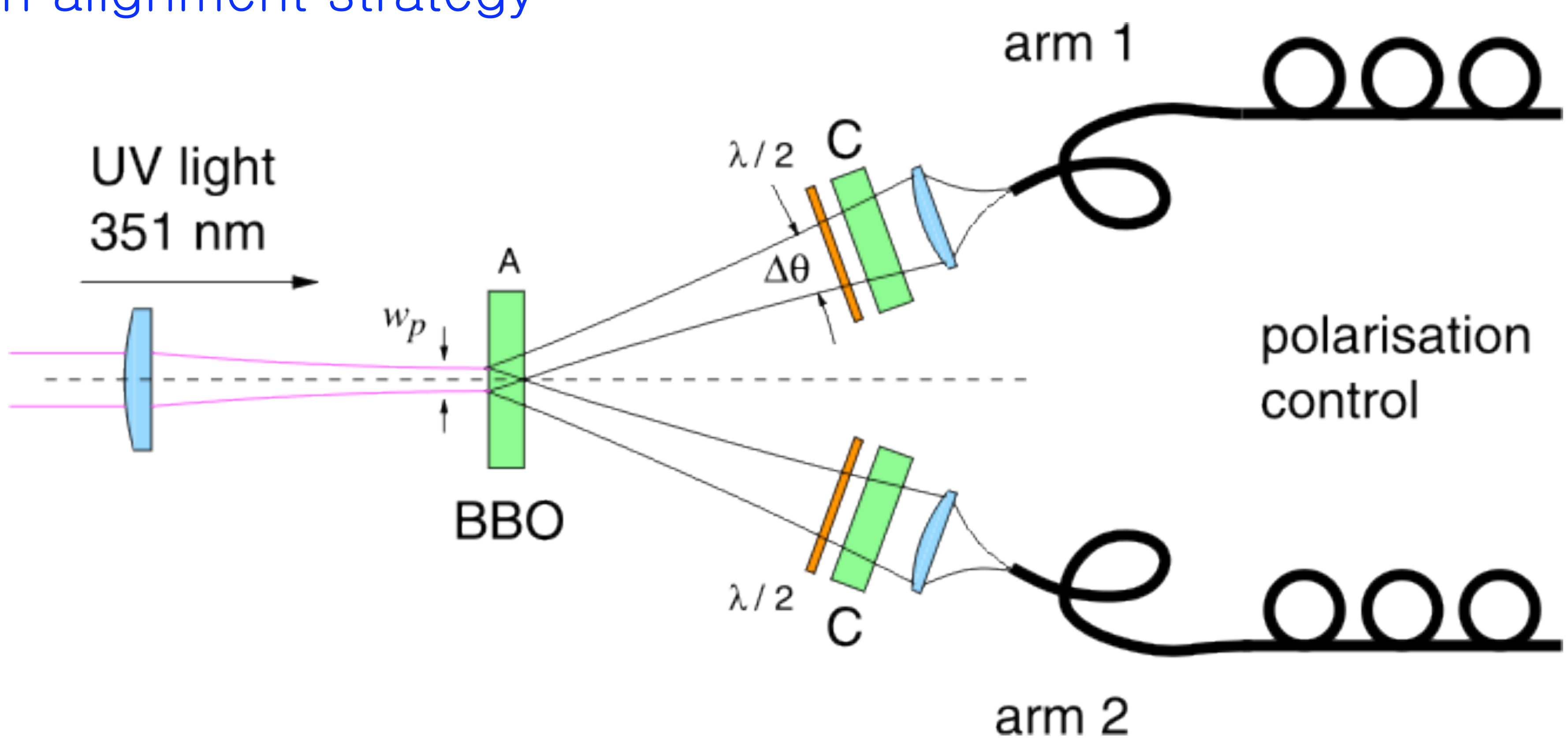
assumption $W_p = W_s = W_i$

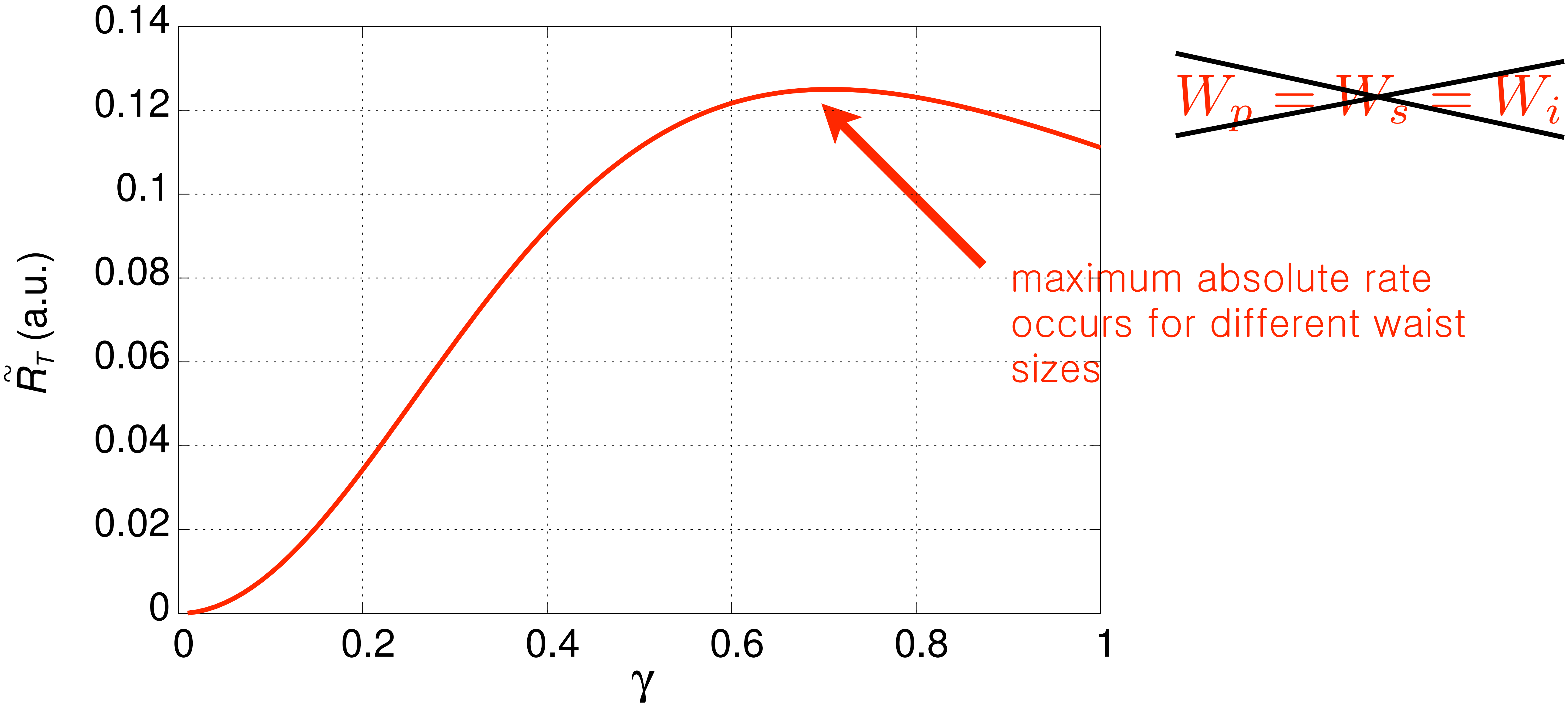
Common alignment strategy



Optimal waist matching

$$\tilde{R}_T \approx \frac{1}{W_p^2 W_s^2 W_i^2 (1/W_p^2 + 1/W_s^2 + 1/W_i^2)}$$

$$\tilde{R}_T \approx R_T \approx \frac{1}{W^2(1/ + 2\gamma)^2} \qquad W_p = \gamma W$$



Comparisons

Type-II non-collinear SPDC for entanglement production

$W_p = W_s = W_i = 82\mu\text{m}$

$\Xi = 0.93$

Experimental: 800 pairs/s/mW
Predicted maximum: 1100 pairs/s/mW

} Absolute count rate

Total conversion efficiency from our model:

$3 \times 10^{-12}\text{mm}^{-1}$ $\quad$ $7 \times 10^{-8}\text{mm}^{-1}\text{sr}^{-1}$

Estimate of conversion efficiency from general arguments (Klyshko):

$3 \times 10^{-8}\text{mm}^{-1}\text{sr}^{-1}$

} Total efficiency

Conclusions

- Simple closed expression for the absolute rate in “classic” SPDC setups

- Good agreement with existing experimental values

- Current configurations are nearly optimal

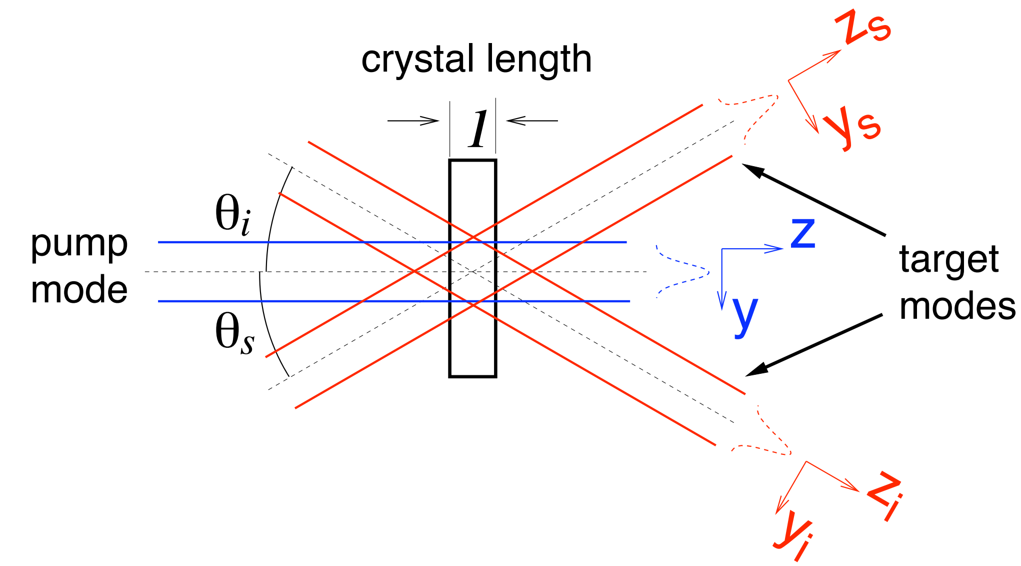
- There might still be something to gain from adjustment in the ratio of collection to pump mode beyond the $W_p = W_s = W_i$

- Relates to recent work done for waveguides (Fiorentino et al. Opt. Exp. 15, 2007)

- More details to be found in PRA 77, 043834 (2008); arXiv:0801.2220v2

Thanks!

imposing ...



normalization:

$$\alpha^2 \int dxdy \left|U(x,y)\right|^2 = 1$$

$$\alpha_{p,s,i} = \sqrt{\frac{2}{\pi W_{p,s,i}^2}}$$

optical power/electrical field amplitude:

$$\left|E_p^0\right|^2 = \alpha_p^2 \frac{2P}{\epsilon_0 n_p c}$$

relevant fields

$$\mathbf{E}_p(\mathbf{r},t) = \frac{1}{2}\left[\mathbf{E}_p^{(+)}(\mathbf{r},t) + \mathbf{E}_p^{(-)}(\mathbf{r},t)\right]$$

$$= \frac{1}{2}\left[E_p^0 \mathbf{e}_p g_p(\mathbf{r}) e^{-i\omega_p t} + c.c\right]$$

$$\hat{\mathbf{E}}_{s,i} = \frac{1}{2}[\hat{\mathbf{E}}_{s,i}^{(+)}(\mathbf{r},t) + \hat{\mathbf{E}}_{s,i}^{(-)}(\mathbf{r},t)]$$

$$= \frac{i}{2}\sum_{k_{s,i}} \sqrt{\frac{2\hbar\omega_{s,i}}{n_{s,i}^2 \epsilon_0}}\ \frac{\alpha_{s,i}}{\sqrt{L}}\ \mathbf{e}_{s,i} g_{s,i}(\mathbf{r})\ e^{-i\omega_{s,i} t}\ \hat{a}_{k_{s,i}} + \ h.c.$$