Lecture 1: A taste of experimental quantum information techniques

Christian Kurtsiefer



Asher Peres International School, Chowder Bay @ Sydney, Nov 2008



Quantum physics does not happen in Hilbert space -

quantum physics happens in the lab.

Misquoted? Don't know the exact reference but certainly a view of Asher Peres I subscribe to

Overview



- Photons as qubits
 - obvious things one can do in quantum information
 - what is commonly understood by a photon?
 - manipulate qubits
 - what really works well: detectors, pair sources
 - what also works: pair (sometimes multi) photon sources
 - Hamilton operators are useful !!
- Quantum key distribution schemes as the usual quantum info suspects
 - basic ideas: BB84
 - Bell inequalities to evaluate knowledge of eavesdropper
 - a practical implementation
 - some dreams and problems

Photons - quantum information



 want distinguishable things representing qubits with possible entanglement in system state:



What is....a photon?





Highly recommended read: W.E. Lamb: Anti-photon, Appl. Phys. B **60**, 77 (1995)



- Definition via detection: photoelectric effect localizable in space/time, no energy eigenstates
- Definition via cavity mode excitation clean energy eigenstate labelling, very hard to observe
- Definition via spontaneous emission localizable in time/origin, well-defined preparation scheme "single photon sources" rely on that
- Definition via total energy in ~monochromatic field over hbar



....in quantum information?

- transport light
- encode qubits....without loosing the superposition states



Easy: single qubit operations



• 1-qubit rotation



measurement



arbitrary 1-qubit operation



state preparation







 Inherit all the phase and amplitude manipulation abilities from classical optics / electrodynamics



Birefringent materials, polarization states of light

POVMs in the lab....





Bloch / Stokes vectors corresponding to an optimal measurement scheme for polarization qubits optical implementation by embedding in larger Hilbert space



A. Ling et al., J. Mod. Opt. 53, 1523 (2006)

Convergence in an experiment







universal 2-qubit operations, require large optical nonlinearity



- hopeless with typical bulk nonlinearities
- possible with atoms close to resonance:

work by S. Harris & friends in Stanford: M. Lukin, Harvard: atomic clouds atoms in fibers





Use photoelectric effect: Convert electromagnetic field into a charge



detect an electron

becomes an irreversible process in a very short time

Single photon detectors 1



- Photomultiplier low-medium quantum efficiency @IR...red, well understood can be fast
- Avalanche photodiodes above breakdown QE about 20-50% (manufacturer quotes 70%), repetition rate ~1MHz
- superconducting detectors very high QE (>95% for some), photon number resolving, currently slow response rate
- (PIN photodiodes for continuous variables) not suitable for single photoelectron counting, high noise at low frequencies, up to veryhigh quantum efficiency (>99%)

Single photon detectors 2



properties:

- quantum efficiency at operational wavelength (0.1..99%)
- timing jitter (30...500 ps)
- dark counts (0...20...100 000 cps)
- dead time (10ns...1us)
- ugly artefacts: afterpulses, selective blindability
- (intensity-dependent) delays
- gating needs
- temperature requirements (50mK...room temp)

Avalanche photodetector



National

of Singapore

University

• passive quenching topology:



Detector dark counts



Lower temperature to limit dark count rate...



'Making' photons



spontaneous emission from atom-like systems



cascade decays for making pairs



A. Aspect, P. Grangier, P. Roger, Phys. Rev. Lett. **47,** 460 (1981)

parametric conversion in optical nonlinearities





making single photons at random times:





spontaneous emission from atoms / color centers / quantum dots / single molecules

making single photons deterministically (and sequences)

use pulsed excitation,

cavities to enforce emission in one mode via Purcell effect

Cascade Decays 2





indistinguishable decay paths

$$(0,0) - (1,1) - (0',0)$$
 and
 $(0,0) - (1,-1) - (0',0)$

leads to polarization correlation of the photons:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\sigma^+ \sigma^-\rangle - |\sigma^- \sigma^+\rangle)$$

singlet Bell state $|\Psi^angle$

excitation - decay

A. Aspect, P. Grangier, P. Roger, Phys. Rev. Lett. 47, 460 (1981)

Atomic Cascade Decays





A. Aspect, P. Grangier, P. Roger, Phys. Rev. Lett. 47, 460 (1981)

Nonlinear Optical Processes



 Nonlinear optical processes: 3-wave mixing, 4-wave mixing: response of a medium:

$$P = \epsilon_0 (\chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + ...)$$
refractive index,
birefringence
Kerr nonlinearity

second harmonic generation parametric conversion

Nonlinear Optical Processes 2



making sometimes pairs of photons at random/fixed times



high fidelity maximally entangled photon pair states

• making combinations of 2/4/6....photon events at fixed times:







D.C. Burnham, D.L. Weinberg, Phys. Rev. Lett. 25, 84 (1970)



correlation between detection times – makes us talk about pairs



D.C. Burnham, D.L. Weinberg, Phys. Rev. Lett. 25, 84 (1970)

Photon pairs form PDC 3



• 'spatial' correlations:

coincidences show up for particular detector positions

 indicates momentum conservation



D.C. Burnham, D.L. Weinberg, Phys. Rev. Lett. 25, 84 (1970)











Entangled photon sources



• Use non-collinear type-II parametric down conversion



two indistinguishable decay paths lead to $|\Psi^{-}\rangle = \frac{1}{\sqrt{2}} (|HV\rangle - |VH\rangle)$ $= \frac{1}{\sqrt{2}} (|+-\rangle - |-+\rangle)$

P.G. Kwiat et al., PRL 75, 4337 (1995)

Type-II SPDC fluorescence



Look at a patricular wavelength range camera or $\chi^{(2)}$ pump detector interference filter -8 -6 -4 -2 0 2 4 6 8 -8 -6 -4 -2 0 2 4 6 8 -8 -6 -4 -2 0 2 4 6 8 $\Delta \Theta = -0.73^{\circ}$ $\Delta\Theta = +0.44^{\circ}$ 8 8 8 $\Delta \Theta = -0^{\circ}$ 8 8 8 6 6 6 6 6 6 -H 4 4 4 4 4 2 2 2 2 2 2 $\theta[\circ]_0$ 0 0 0 o 0 -2 -2 -2 -2 -2 -2 -4 -4 -4 -4 -4 -4 -6 -6 -6 -6 -6 -6 -8 -8 -8 -8 -8 -8 -8 -6 -4 -2 0 2 6 8 -8 -6 -4 -2 0 2 6 8 -8 - 6 - 4 - 2 0 24 4 4 6 8 φ[°] φ[°] φ[°] 20 k 40 k 60 k 80 k 100 k 120 k

SPDC, quantitatively



• Free electromagnetic field:

$$\hat{H}_F = \frac{\epsilon_0}{2} \int \hat{\vec{E}}^2(\vec{r}) + c^2 \hat{\vec{B}}^2(\vec{r}) d^3r$$

Linear susceptible material

$$\hat{H}_0 = \frac{\epsilon_0}{2} \int \chi \,\hat{\vec{E}}^2(\vec{r}) + c^2 \,\hat{\vec{B}}^2(\vec{r}) \,d^3r$$

• With nonlinear material:

$$\hat{H} = \frac{1}{2} \int \hat{\vec{E}} \cdot \hat{\vec{P}}(\vec{r}) + \epsilon_0 c^2 \hat{\vec{B}}^2(\vec{r}) d^3 r$$

$$= \hat{H}_{0} + \frac{\epsilon_{0}}{2} \int \hat{\vec{E}} \chi^{(2)} : \hat{\vec{E}}^{2}(\vec{r}) d^{3}r = : \hat{H}_{0} + \hat{H}_{I}$$

Choose fields





• Pump is treated as classical field (non-depleted):

$$\vec{E}_{p}(\vec{r},t) = E_{0}\vec{\epsilon} \left[g(\vec{r})e^{i\omega_{p}t} + g^{*}(\vec{r})e^{-i\omega_{p}t}\right]$$
amplitude
mode function







• Electrical field operator









$$g(\vec{r})=e^{i\vec{k}\cdot\vec{r}}$$

conceptually nice, not really implemented in experiments

Plane waves with a Gaussian envelope

$$g(\rho, z) = e^{ikz} e^{-\rho^2/w^2}$$

typical laser beams and light that propagates in optical fibers (approx)

Normalize operator...The Works





quantization volume V

Get the magnetic field operator

$$\hat{\vec{B}}_{s,i}(\vec{r},t) = \frac{1}{i\omega} E_{s,i} \left[\nabla \times (\vec{\epsilon} g(\vec{r})) \hat{a}(t) + \nabla \times (\vec{\epsilon} g^*(\vec{r})) \hat{a}^+(t) \right]$$

Free field should look like harmonic oscillators:

$$\hat{H}_{0} = \frac{\epsilon_{0}}{2} \int_{V} \left[\hat{\vec{E}}^{2}(\vec{r}) + c^{2} \hat{\vec{B}}^{2}(\vec{r}) \right] dV = \sum_{modes j} \hbar \omega_{j} (\hat{a}_{j}^{+} \hat{a}_{j} + \frac{1}{2})$$
you need dispersion here:
$$c^{2} \vec{k}^{2} = \omega^{2} \text{ for plane waves}$$

you need dispersion here:

you get
$$E_{s,i} = \sqrt{\frac{\hbar \omega_j}{2\epsilon_0 V}}$$

for plane waves

Simplify H₁



$$\hat{H}_{I} = \frac{\epsilon_{0}}{2} \int \vec{E}_{p}(\vec{r}, t) \chi^{(2)} \hat{\vec{E}}_{s}(\vec{r}, t) \hat{\vec{E}}_{i}(\vec{r}, t) d^{3}r$$

• eight terms, integral only over mode functions:

$$\hat{H}_{I} = \sum_{n,m} \frac{\epsilon_{0}}{2} E_{0} E_{s_{n}} E_{i_{m}} (\vec{\epsilon}_{P} \chi^{(2)} \vec{\epsilon}_{s_{n}} \vec{\epsilon}_{i_{m}}) \times \text{ constants}$$

$$\text{mode indices} \begin{bmatrix} \left(\int_{V} g_{P}(\vec{r}) g_{s_{n}}(\vec{r}) g_{i_{m}}(\vec{r}) d^{3} r \times \text{ spatial aspect} \right) \\ \hat{a}_{s_{n}}(t) \hat{a}_{i_{m}}(t) e^{i\omega_{P}t} \end{bmatrix} \xrightarrow{\text{operator character} + \text{time dependencies}} +7 \, more \, terms \end{bmatrix}$$





$$S := \int_{V} g_{P}(\vec{r}) g_{s_{n}}(\vec{r}) g_{i_{m}}(\vec{r}) d^{3}r \quad \text{for terms} \quad \hat{a}_{s} \hat{a}_{i}$$

• plane waves:

$$S = (2\pi)^3 \delta_{xyz} (k_P + k_s + k_i)$$

finite size of interaction thickness:

$$S = (2\pi)^2 \delta_{xy} (k_P + k_s + k_i) \times a \operatorname{sinc} \left[(k_{z,P} + k_{z,s} + k_{z,i}) a/2 \right]$$


Phase matching 2



a

$$S := \int_{V} g_{P}(\vec{r}) g_{s_{n}}^{*}(\vec{r}) g_{i_{m}}^{*}(\vec{r}) d^{3}r \qquad \text{for terms} \quad \hat{a}_{s}^{+} \hat{a}_{i}^{+}$$

• plane waves:

$$S = (2\pi)^3 \delta_{xyz} (k_P - k_s - k_i)$$

finite size of interaction thickness:

$$S = (2\pi)^2 \delta_{xy} (k_P - k_s - k_i) \times a \operatorname{sinc} \left[(k_{z,P} - k_{z,s} - k_{z,i}) a/2 \right]$$

approximate momentum conservation





$$\hat{H}_{I} = A \left(\hat{a}_{s}^{+} \hat{a}_{i}^{+} e^{-i\omega_{p}t} + \hat{a}_{s} \hat{a}_{i} e^{+i\omega_{p}t} \right)$$
.....and we know A quantitatively

Where do we go from here on?

consider asymptotic states (vacuum fields -> pair states)

$$|\Psi_{f}
angle = \! \left| 1_{s}, 1_{i}
ight
angle = \! \hat{a}_{s}^{+} \, \hat{a}_{i}^{+} \left| 0
ight
angle$$

• do Fermi's golden rule for rates from the vacuum $|\Psi_i
angle = |0
angle$

...brings energy conservation



• for fixed target wave index k_s

mode density for ki

$$R(k_{s}) = \frac{2\pi}{\hbar} \left| \langle \Psi_{i} | \hat{H}_{I} | \Psi_{f} \rangle \right|^{2} \rho(\Delta E)$$

 total rate in all possible ks: (Gaussian beams, collinear modes of waist w_s=w_i=w_p):

$$R_T = \frac{4d^2 a P \omega_P}{9n_s n_i n_p \epsilon_0 \pi w_P^2 (n_i - n_s) c^2}$$

d: effective nonlinearitya: crystal thicknessn: refractive indicesP: pump power

typically ~5000 pairs/sec/mW for 2mm thick BBO in type-II

An efficient pair source





- Choose a target wavelengh which is suitable for detectors
- couple target modes in single mode optical fibers
- Remove residual distinguishability between photons due to birefringence

Match angular dispersion





- Choose an optical bandwidth $\Delta\lambda$
- Choose collection angle of fiber modes to $\Delta \theta = \Delta \lambda \, d\theta / d\lambda$
- Restrict pump mode to collection region

C. K., M. Oberparleiter, and H. Weinfurter, Phys. Rev. A 64, 010102(R) (2001)





spectral brightness









 high brightness by mode matching, observed pair/single ratio: 28%

Entanglement Quality





- Visibility of polarization correlations: HV: 98.2%, ±45deg: 96.3%
- Violation of a CHSH-type Bell inequality: S=2.6989±0034 (204 σ in 1sec/point)

Other experimental tests



- Visibility of Polarization correlation >99% in all bases
- Leggett-type inequalities for (nonlocal) hidden variable models





C. Branciard, A. Ling, N. Gisin, C.K., A. Lamas-Linares, V. Scarani, PRL 99, 210407 (2007)

Practical pair source



Blue diode-laser as pump source, BBO as nonlinear crystal





- 24,000 s⁻¹ detected pairs from 40 mW pump @ 407nm in single mode fibers at 810/818 nm, 2mm BBO crystal
- polarization correlation visibility in 45° basis: 92%

Much better implementations



Colinear down conversion, periodically poled materials



- Up to 1000 times brighter than non-colinear sources
- Polarization correlation visibility in 45° basis > 99%





- Make single qubits and qubit pairs
- Manipulate and transport photonc qubits
- Fundamental tests of quantum mechanics

test Bell-type inequalities & friends

Quantum communication

quantum cryptography

Time for Coffee....





Thank you !

http://qoptics.quantumlah.org/lah/

CQT Graduate program: http://cqtphd.quantumlah.org

Lecture 2: Aspects of Quantum Cryptography

Christian Kurtsiefer



Asher Peres International School, Chowder Bay @ Sydney, Nov 2008

Quantum 'cryptography'



- better: quantum key distributoion
- even better: quantum key growing

- BB84 protocol
- Ekert protocol
- Device independent key distribution

BB84 protocol



Prepare & measure protocols (BB84 & friends/derivatives):



uses error fraction to estimate eavesdropper's knowledge

Encoding information....



....works also with other perpendicular polarizations.....



•but you need correct measurement basis:





ALICE: 0111 0101 0101 0110 1010 0111 0101

BOB: 0110 0101 0111 1110 1010 0111 0101

- Some errors are due to imperfect devices, detectors, background light etc.
- Some errors indicate an eavesdropping attempt
- Correct errors by discussing parity bits over blocks openly:

p=0 **p=0 p=0** $A \rightarrow B$: p=1 **p=0 p=0** p=1 OK **B->A:** ERR OK ERR ERR OK OK

Other encoding techniques



• Encoding qubit in relative phase between two packets



Replace fiber pair by time structure (early / late)







• same basis: always same outcome

$$\longrightarrow \ \ \bigcirc \ \ \overset{}{\longrightarrow} \ \ \bigcirc \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow} \ \ \overset{}{\longrightarrow} \ \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow} \ \ \overset{}{\longrightarrow} \ \ \overset{}{\longrightarrow} \ \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow}$$

• different bases







- Raw key with errors: N_r bits
- Quantum bit error ratio (QBER): η
- Number of bits leaked to an eavesdropper N_e

$$N_e = N_r (h(\eta) + h(\eta))$$

possible knowledge of an eavesdropper due to measurements revealed in (optimal) error correction

binary entropy: $h(\eta) = -\eta \log_2 \eta - (1 - \eta) \log_2 (1 - \eta)$







* depends on the attack model (individual attack); for *infinite* key length and (!) single photons

Privacy amplification



compress raw key to the information advantage vs. Eve..



 All information leakaged to Eve (attacks + error correction) has to be considered

Tricky: finite key length may make privacy amplification more difficult – $\sim 10^7$ to 10^{10} bits

BB84 original implementation





C. Bennett, F. Bessette, G. Brassard, L. Savail, J. Smolin J. Cryptology 5, 3 (1992)

Imperfect 'single photons'



• use faint coherent pulses instead of single photons

$$p(n) = \frac{\lambda^n}{n!} e^{-\lambda}$$
 for $\langle n \rangle = 0.1$ $p(0) = 90.48\%$
 $p(1) = 9.05\%$
 $p(n>1) = 0.47\%$

• much simpler to prepare than true single photons:



potentially insecure: photon number splitting attack
 ---> decoy state protocol

H.-K. Lo, X. Ma, K. Chen, Phys. Rev. Lett. **94** 230504 (2004) T. Schmitt-Manderbach et al., Phys. Rev. Lett. **98**, 010504 (2007)





...needs lots* of trusted random numbers!



*Mbit/sec for kbit/sec key





• use beam splitters and single (post-selected) photons



J.G. Rarity et al., J. Mod. Opt. 41, 2345 (1994)

T. Jennewein et al., Rev. Sci. Inst. 71, 1675 (2000)

need to remove bias, two detectors





extract Poissonian photon statistics





• Make use of good intrinsic polarization of laser diodes



BB84: Spectral attack



Don't measure polarization, but e.g. color: The Hilbert Space in your system is larger than it appears







Replace active basis choice by passive choice in a beam splitter



Bridging distances





C. K., P. Zarda, M. Halder, H. Weinfurter, P. M. Gorman, P. R. Tapster, and J. G. Rarity, Nature **419**, 450 (2002)

Even further....





• Use satellites as trusted relays between distant locations

•but why should you trust it?

of Singapore

public discussion (sifting, key gen / state estimation)

error correction, privacy amplification

- no trusted random numbers for key
- direct use of quantum randomness for measurement basis

Practical pair source

Blue diode-laser as pump source, BBO as nonlinear crystal

- 24,000 s⁻¹ detected pairs from 40 mW pump @ 407nm in single mode fibers at 810/818 nm, 2mm BBO crystal
- polarization correlation visibility in 45° basis: 92%
NUS campus test range





Receiver unit





polarization analyzer passively quenched Silicon APD - QE ~50% ~1000s⁻¹ dark cnt rate

spatial filter (150 µrad)







(40 mm FWHM)







Identified raw coincidences between close and remote receiver



(with interference filter 5nm FWHM, 50% peak transmission)

....and after The Works:





- CASCADE error correction with ~6000 bit packets
- assume incoherent attack strategy for privacy amplification
- average efficiency of EC/PA: >57%
- average final key rate: 650 bits/sec
- residual error rate ~10⁻⁶ due to a stupid error

No interference filter





- use a RG780 long pass filter to suppress visible light
- average final key rate 850 bits/sec

```
(link loss 8.3 dB)
```

Atmospheric absorption



 representative vertical atmoshpere layer (corresponds to ~11 km air on ground)



Optical fibers as 'channel'



- Use existing telecom infrastructure
- independent of environment
- high transmission:
 - 800nm:2dB/km(T=63% in 1km)1310nm:0.2dB/km(T=63% in 10km)1550nm:0.35dB/km(T=44% in 10km)
- stress birefringence and geometric phases are time dependent:



Birefringence compensation



Probe fiber birefringence via two passes with Faraday mirror



- Basis of "Plug & Play" or autocompensation schemes in commercial QKD systems (id quantique, NEC)
- Bridging ~100 km

N. Gisin & team, GAP optique, Geneva D. Bethune / W. Risk, IBM Almaden A. Karlsson, KTH Stockolm NEC

Compare figures of merit







Find eavesdropper not via errors, but via testing entanglement: Ekert91 – type and tomographic protocols



Bell inequality I





Correlation between setting *i*, *j*:

$$E(i,j) := \frac{n(i,j) + n(\overline{i},\overline{j}) - n(i,\overline{j}) - n(\overline{i},j)}{n(i,j) + n(\overline{i},\overline{j}) + n(\overline{i},\overline{j}) + n(\overline{i},\overline{j})}$$

combined correlation function:

$$S := E(1,1') + E(1,2') + E(2,1') - E(2,2')$$



If there is a local hidden parameter λ (= knowledge of **E**) governing the measurement outcomes of **A** and **B**, then:

$$|S| \leq 2$$

Bell inequality II





For proper settings 1, 2, 1', 2' and state $|\Psi^-\rangle$: $S=\pm 2\sqrt{2}$

 Estimate quantitatively the knowledge of Eve of raw key between A and B from S:

$$I_{E}(S) = h \left(1 + \frac{\sqrt{S^{2}/4 - 1}}{2} \right)$$

- Assume "fair sampling" between key measurement and Bell test
- No fingerprint problems of photons due to side channels A. Acin, N. Brunner, N. Gisin, S. Massar, S. Pironio, V. Scarani, PRL 98, 230501 (2007)

E91 Implementation









Field results (1.4km range)



typical data run (with tropical rainfall inbetween)







of Singapore

 For non-lossy detectors and a measurement basis decision at "free will" of the observers:

No assumptions on devices and source is necessary to get an upper bound for eavesdropping!

A. Acin, N. Brunner, N. Gisin, S. Massar, S. Pironio, V. Scarani, PRL 98, 230501 (2007)

Field usage...



PDC pair source & sender



- Open source: http://code.google.com/p/qcrypto
- 24C3, Berlin 2007, Black Hat / DEFCON16, 2008

 System gets simpler and more robust

receiving side



Detector breakdown signature





Timing channel attack I





Timing channel attack II



Classical timing information carries fingerprint of detectors:



Timing ch attack – The Cure



Make sure no detail timing information is revealed.....



- Alternative cures (costly for background):
 - coarser quantized timing information
 - add timing noise

Nastier attacks: V. Makarov Trondheim H.K. Lo, Toronto

Time for Coffee....





Thank you !

http://qoptics.quantumlah.org/lah/

CQT Graduate program: http://cqtphd.quantumlah.org