Lecture 1: A taste of experimental quantum information techniques

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Asher Peres International School, Chowder Bay @ Sydney, Nov 2008



#### Quantum physics does not happen in Hilbert space -

# quantum physics happens in the lab.

Misquoted? Don't know the exact reference but certainly a view of Asher Peres I subscribe to

# Overview



- Photons as qubits
  - obvious things one can do in quantum information
  - what is commonly understood by a photon?
  - manipulate qubits
  - what really works well: detectors, pair sources
  - what also works: pair (sometimes multi) photon sources
  - Hamilton operators are useful !!
- Quantum key distribution schemes as the usual quantum info suspects
  - basic ideas: BB84
  - Bell inequalities to evaluate knowledge of eavesdropper
  - a practical implementation
  - some dreams and problems

# Photons - quantum information



 want distinguishable things representing qubits with possible entanglement in system state:



What is....a photon?





Highly recommended read: W.E. Lamb: Anti-photon, Appl. Phys. B **60**, 77 (1995)



- Definition via detection: photoelectric effect localizable in space/time, no energy eigenstates
- Definition via cavity mode excitation clean energy eigenstate labelling, very hard to observe
- Definition via spontaneous emission localizable in time/origin, well-defined preparation scheme "single photon sources" rely on that
- Definition via total energy in ~monochromatic field over hbar



....in quantum information?

- transport light
- encode qubits....without loosing the superposition states



Easy: single qubit operations



• 1-qubit rotation



measurement



arbitrary 1-qubit operation



state preparation







 Inherit all the phase and amplitude manipulation abilities from classical optics / electrodynamics



Birefringent materials, polarization states of light

# POVMs in the lab....





Bloch / Stokes vectors corresponding to an optimal measurement scheme for polarization qubits optical implementation by embedding in larger Hilbert space



A. Ling et al., J. Mod. Opt. 53, 1523 (2006)

# Convergence in an experiment







universal 2-qubit operations, require large optical nonlinearity



- hopeless with typical bulk nonlinearities
- possible with atoms close to resonance:

work by S. Harris & friends in Stanford: M. Lukin, Harvard: atomic clouds atoms in fibers





Use photoelectric effect: Convert electromagnetic field into a charge



detect an electron

becomes an irreversible process in a very short time

# Single photon detectors 1



- Photomultiplier low-medium quantum efficiency @IR...red, well understood can be fast
- Avalanche photodiodes above breakdown QE about 20-50% (manufacturer quotes 70%), repetition rate ~1MHz
- superconducting detectors very high QE (>95% for some), photon number resolving, currently slow response rate
- (PIN photodiodes for continuous variables) not suitable for single photoelectron counting, high noise at low frequencies, up to veryhigh quantum efficiency (>99%)

# Single photon detectors 2



#### properties:

- quantum efficiency at operational wavelength (0.1..99%)
- timing jitter (30...500 ps)
- dark counts (0...20...100 000 cps)
- dead time (10ns...1us)
- ugly artefacts: afterpulses, selective blindability
- (intensity-dependent) delays
- gating needs
- temperature requirements (50mK...room temp)

# Avalanche photodetector



National

of Singapore

University

• passive quenching topology:



### Detector dark counts



#### Lower temperature to limit dark count rate...



# 'Making' photons



spontaneous emission from atom-like systems



cascade decays for making pairs



A. Aspect, P. Grangier, P. Roger, Phys. Rev. Lett. **47,** 460 (1981)

parametric conversion in optical nonlinearities





making single photons at random times:





spontaneous emission from atoms / color centers / quantum dots / single molecules

making single photons deterministically (and sequences)

use pulsed excitation,

cavities to enforce emission in one mode via Purcell effect

Cascade Decays 2





indistinguishable decay paths

$$(0,0) - (1,1) - (0',0)$$
 and  
 $(0,0) - (1,-1) - (0',0)$ 

leads to polarization correlation of the photons:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\sigma^+ \sigma^-\rangle - |\sigma^- \sigma^+\rangle)$$

singlet Bell state  $|\Psi^angle$ 

excitation - decay

A. Aspect, P. Grangier, P. Roger, Phys. Rev. Lett. 47, 460 (1981)

Atomic Cascade Decays





A. Aspect, P. Grangier, P. Roger, Phys. Rev. Lett. 47, 460 (1981)

# Nonlinear Optical Processes



 Nonlinear optical processes: 3-wave mixing, 4-wave mixing: response of a medium:

$$P = \epsilon_0 (\chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + ...)$$
refractive index,
birefringence
Kerr nonlinearity

second harmonic generation parametric conversion

# Nonlinear Optical Processes 2



making sometimes pairs of photons at random/fixed times



high fidelity maximally entangled photon pair states

• making combinations of 2/4/6....photon events at fixed times:







D.C. Burnham, D.L. Weinberg, Phys. Rev. Lett. 25, 84 (1970)



correlation between detection times – makes us talk about pairs



D.C. Burnham, D.L. Weinberg, Phys. Rev. Lett. 25, 84 (1970)

# Photon pairs form PDC 3



• 'spatial' correlations:

coincidences show up for particular detector positions

 indicates momentum conservation



D.C. Burnham, D.L. Weinberg, Phys. Rev. Lett. 25, 84 (1970)











# Entangled photon sources



#### • Use non-collinear type-II parametric down conversion



two indistinguishable decay paths lead to  $|\Psi^{-}\rangle = \frac{1}{\sqrt{2}} (|HV\rangle - |VH\rangle)$  $= \frac{1}{\sqrt{2}} (|+-\rangle - |-+\rangle)$ 

P.G. Kwiat et al., PRL 75, 4337 (1995)

*Type-II SPDC fluorescence* 



#### Look at a patricular wavelength range camera or $\chi^{(2)}$ pump detector interference filter -8 -6 -4 -2 0 2 4 6 8 -8 -6 -4 -2 0 2 4 6 8 -8 -6 -4 -2 0 2 4 6 8 $\Delta \Theta = -0.73^{\circ}$ $\Delta\Theta = +0.44^{\circ}$ 8 8 8 $\Delta \Theta = -0^{\circ}$ 8 8 8 6 6 6 6 6 6 -H 4 4 4 4 4 2 2 2 2 2 2 $\theta[\circ]_0$ 0 0 0 o 0 -2 -2 -2 -2 -2 -2 -4 -4 -4 -4 -4 -4 -6 -6 -6 -6 -6 -6 -8 -8 -8 -8 -8 -8 -8 -6 -4 -2 0 2 6 8 -8 -6 -4 -2 0 2 6 8 -8 - 6 - 4 - 2 0 24 4 4 6 8 φ[°] φ[°] φ[°] 20 k 40 k 60 k 80 k 100 k 120 k

SPDC, quantitatively



• Free electromagnetic field:

$$\hat{H}_F = \frac{\epsilon_0}{2} \int \hat{\vec{E}}^2(\vec{r}) + c^2 \hat{\vec{B}}^2(\vec{r}) d^3r$$

Linear susceptible material

$$\hat{H}_0 = \frac{\epsilon_0}{2} \int \chi \,\hat{\vec{E}}^2(\vec{r}) + c^2 \,\hat{\vec{B}}^2(\vec{r}) \,d^3r$$

• With nonlinear material:

$$\hat{H} = \frac{1}{2} \int \hat{\vec{E}} \cdot \hat{\vec{P}}(\vec{r}) + \epsilon_0 c^2 \hat{\vec{B}}^2(\vec{r}) d^3 r$$

$$= \hat{H}_{0} + \frac{\epsilon_{0}}{2} \int \hat{\vec{E}} \chi^{(2)} : \hat{\vec{E}}^{2}(\vec{r}) d^{3}r = : \hat{H}_{0} + \hat{H}_{I}$$

Choose fields





• Pump is treated as classical field (non-depleted):

$$\vec{E}_{p}(\vec{r},t) = E_{0}\vec{\epsilon} \left[g(\vec{r})e^{i\omega_{p}t} + g^{*}(\vec{r})e^{-i\omega_{p}t}\right]$$
amplitude
mode function







• Electrical field operator









$$g(\vec{r})=e^{i\vec{k}\cdot\vec{r}}$$

conceptually nice, not really implemented in experiments

Plane waves with a Gaussian envelope

$$g(\rho, z) = e^{ikz} e^{-\rho^2/w^2}$$

typical laser beams and light that propagates in optical fibers (approx)

# Normalize operator...The Works





quantization volume V

Get the magnetic field operator

$$\hat{\vec{B}}_{s,i}(\vec{r},t) = \frac{1}{i\omega} E_{s,i} \left[ \nabla \times (\vec{\epsilon} g(\vec{r})) \hat{a}(t) + \nabla \times (\vec{\epsilon} g^*(\vec{r})) \hat{a}^+(t) \right]$$

Free field should look like harmonic oscillators:

$$\hat{H}_{0} = \frac{\epsilon_{0}}{2} \int_{V} \left[ \hat{\vec{E}}^{2}(\vec{r}) + c^{2} \hat{\vec{B}}^{2}(\vec{r}) \right] dV = \sum_{modes j} \hbar \omega_{j} (\hat{a}_{j}^{+} \hat{a}_{j} + \frac{1}{2})$$
you need dispersion here:
$$c^{2} \vec{k}^{2} = \omega^{2} \text{ for plane waves}$$

you need dispersion here:

you get 
$$E_{s,i} = \sqrt{\frac{\hbar \omega_j}{2\epsilon_0 V}}$$

for plane waves

Simplify H<sub>1</sub>



$$\hat{H}_{I} = \frac{\epsilon_{0}}{2} \int \vec{E}_{p}(\vec{r}, t) \chi^{(2)} \hat{\vec{E}}_{s}(\vec{r}, t) \hat{\vec{E}}_{i}(\vec{r}, t) d^{3}r$$

• eight terms, integral only over mode functions:

$$\hat{H}_{I} = \sum_{n,m} \frac{\epsilon_{0}}{2} E_{0} E_{s_{n}} E_{i_{m}} (\vec{\epsilon}_{P} \chi^{(2)} \vec{\epsilon}_{s_{n}} \vec{\epsilon}_{i_{m}}) \times \text{ constants}$$

$$\text{mode indices} \begin{bmatrix} \left( \int_{V} g_{P}(\vec{r}) g_{s_{n}}(\vec{r}) g_{i_{m}}(\vec{r}) d^{3} r \times \text{ spatial aspect} \right) \\ \hat{a}_{s_{n}}(t) \hat{a}_{i_{m}}(t) e^{i\omega_{P}t} \end{bmatrix} \xrightarrow{\text{operator character} + \text{time dependencies}} +7 \, more \, terms \end{bmatrix}$$





$$S := \int_{V} g_{P}(\vec{r}) g_{s_{n}}(\vec{r}) g_{i_{m}}(\vec{r}) d^{3}r \quad \text{for terms} \quad \hat{a}_{s} \hat{a}_{i}$$

• plane waves:

$$S = (2\pi)^3 \delta_{xyz} (k_P + k_s + k_i)$$

finite size of interaction thickness:

$$S = (2\pi)^2 \delta_{xy} (k_P + k_s + k_i) \times a \operatorname{sinc} \left[ (k_{z,P} + k_{z,s} + k_{z,i}) a/2 \right]$$


*Phase matching 2* 



a

$$S := \int_{V} g_{P}(\vec{r}) g_{s_{n}}^{*}(\vec{r}) g_{i_{m}}^{*}(\vec{r}) d^{3}r \qquad \text{for terms} \quad \hat{a}_{s}^{+} \hat{a}_{i}^{+}$$

• plane waves:

$$S = (2\pi)^3 \delta_{xyz} (k_P - k_s - k_i)$$

finite size of interaction thickness:

$$S = (2\pi)^2 \delta_{xy} (k_P - k_s - k_i) \times a \operatorname{sinc} \left[ (k_{z,P} - k_{z,s} - k_{z,i}) a/2 \right]$$

approximate momentum conservation





$$\hat{H}_{I} = A \left( \hat{a}_{s}^{+} \hat{a}_{i}^{+} e^{-i\omega_{p}t} + \hat{a}_{s} \hat{a}_{i} e^{+i\omega_{p}t} \right)$$
.....and we know A quantitatively

## Where do we go from here on?

consider asymptotic states (vacuum fields -> pair states)

$$|\Psi_{f}
angle = \! \left| 1_{s}, 1_{i} 
ight
angle = \! \hat{a}_{s}^{+} \, \hat{a}_{i}^{+} \left| 0 
ight
angle$$

• do Fermi's golden rule for rates from the vacuum  $|\Psi_i
angle = |0
angle$ 

...brings energy conservation



• for fixed target wave index  $k_s$ 

mode density for ki

$$R(k_{s}) = \frac{2\pi}{\hbar} \left| \langle \Psi_{i} | \hat{H}_{I} | \Psi_{f} \rangle \right|^{2} \rho(\Delta E)$$

 total rate in all possible ks: (Gaussian beams, collinear modes of waist w<sub>s</sub>=w<sub>i</sub>=w<sub>p</sub>):

$$R_T = \frac{4d^2 a P \omega_P}{9n_s n_i n_p \epsilon_0 \pi w_P^2 (n_i - n_s) c^2}$$

d: effective nonlinearitya: crystal thicknessn: refractive indicesP: pump power

typically ~5000 pairs/sec/mW for 2mm thick BBO in type-II

### An efficient pair source





- Choose a target wavelengh which is suitable for detectors
- couple target modes in single mode optical fibers
- Remove residual distinguishability between photons due to birefringence

### Match angular dispersion





- Choose an optical bandwidth  $\Delta\lambda$
- Choose collection angle of fiber modes to  $\Delta \theta = \Delta \lambda \, d\theta / d\lambda$
- Restrict pump mode to collection region

C. K., M. Oberparleiter, and H. Weinfurter, Phys. Rev. A 64, 010102(R) (2001)





spectral brightness









 high brightness by mode matching, observed pair/single ratio: 28%

### Entanglement Quality





- Visibility of polarization correlations: HV: 98.2%, ±45deg: 96.3%
- Violation of a CHSH-type Bell inequality: S=2.6989±0034 (204  $\sigma$  in 1sec/point)

### Other experimental tests



- Visibility of Polarization correlation >99% in all bases
- Leggett-type inequalities for (nonlocal) hidden variable models





C. Branciard, A. Ling, N. Gisin, C.K., A. Lamas-Linares, V. Scarani, PRL 99, 210407 (2007)

### Practical pair source



#### Blue diode-laser as pump source, BBO as nonlinear crystal





- 24,000 s<sup>-1</sup> detected pairs from 40 mW pump @ 407nm in single mode fibers at 810/818 nm, 2mm BBO crystal
- polarization correlation visibility in 45° basis: 92%

## Much better implementations



#### Colinear down conversion, periodically poled materials



- Up to 1000 times brighter than non-colinear sources
- Polarization correlation visibility in 45° basis > 99%





- Make single qubits and qubit pairs
- Manipulate and transport photonc qubits
- Fundamental tests of quantum mechanics

test Bell-type inequalities & friends

Quantum communication

quantum cryptography

*Time for Coffee....* 





# Thank you !

#### http://qoptics.quantumlah.org/lah/

CQT Graduate program: http://cqtphd.quantumlah.org

## Lecture 2: Aspects of Quantum Cryptography

**Christian Kurtsiefer** 



Asher Peres International School, Chowder Bay @ Sydney, Nov 2008

Quantum 'cryptography'



- better: quantum key distributoion
- even better: quantum key growing

- BB84 protocol
- Ekert protocol
- Device independent key distribution

### BB84 protocol



#### Prepare & measure protocols (BB84 & friends/derivatives):



uses error fraction to estimate eavesdropper's knowledge

### Encoding information....



....works also with other perpendicular polarizations.....



• ....but you need correct measurement basis:





#### ALICE: 0111 0101 0101 0110 1010 0111 0101 ....

#### BOB: 0110 0101 0111 1110 1010 0111 0101 ....

- Some errors are due to imperfect devices, detectors, background light etc.
- Some errors indicate an eavesdropping attempt
- Correct errors by discussing parity bits over blocks openly:

p=0 **p=0 p=0**  $A \rightarrow B$ : p=1 **p=0 p=0** p=1 OK **B->A:** ERR OK ERR ERR OK OK

### Other encoding techniques



• Encoding qubit in relative phase between two packets



Replace fiber pair by time structure (early / late)







• same basis: always same outcome

$$\longrightarrow \ \ \bigcirc \ \ \overset{}{\longrightarrow} \ \ \bigcirc \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow} \ \ \overset{}{\longrightarrow} \ \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow} \ \ \overset{}{\longrightarrow} \ \ \overset{}{\longrightarrow} \ \ \overset{}{\longrightarrow} \ \overset{}{\longrightarrow}$$

• different bases







- Raw key with errors:  $N_r$  bits
- Quantum bit error ratio (QBER): η
- Number of bits leaked to an eavesdropper  $N_e$

$$N_e = N_r (h(\eta) + h(\eta))$$

possible knowledge of an eavesdropper due to measurements revealed in (optimal) error correction

binary entropy:  $h(\eta) = -\eta \log_2 \eta - (1 - \eta) \log_2 (1 - \eta)$ 







\* depends on the attack model (individual attack); for *infinite* key length and (!) single photons

### Privacy amplification



compress raw key to the information advantage vs. Eve..



 All information leakaged to Eve (attacks + error correction) has to be considered

Tricky: finite key length may make privacy amplification more difficult –  $\sim 10^7$  to  $10^{10}$  bits

### **BB84** original implementation





C. Bennett, F. Bessette, G. Brassard, L. Savail, J. Smolin J. Cryptology 5, 3 (1992)

Imperfect 'single photons'



• use faint coherent pulses instead of single photons

$$p(n) = \frac{\lambda^n}{n!} e^{-\lambda}$$
 for  $\langle n \rangle = 0.1$   $p(0) = 90.48\%$   
 $p(1) = 9.05\%$   
 $p(n>1) = 0.47\%$ 

• much simpler to prepare than true single photons:



potentially insecure: photon number splitting attack
 ---> decoy state protocol

H.-K. Lo, X. Ma, K. Chen, Phys. Rev. Lett. **94** 230504 (2004) T. Schmitt-Manderbach et al., Phys. Rev. Lett. **98**, 010504 (2007)





### ...needs lots\* of trusted random numbers!



\*Mbit/sec for kbit/sec key





• use beam splitters and single (post-selected) photons



J.G. Rarity et al., J. Mod. Opt. 41, 2345 (1994)

T. Jennewein et al., Rev. Sci. Inst. 71, 1675 (2000)

need to remove bias, two detectors





#### extract Poissonian photon statistics





• Make use of good intrinsic polarization of laser diodes



**BB84:** Spectral attack



Don't measure polarization, but e.g. color: The Hilbert Space in your system is larger than it appears







Replace active basis choice by passive choice in a beam splitter



### Bridging distances





C. K., P. Zarda, M. Halder, H. Weinfurter, P. M. Gorman, P. R. Tapster, and J. G. Rarity, Nature **419**, 450 (2002)

Even further....









• Use satellites as trusted relays between distant locations



• .....but why should you trust it?







of Singapore

public discussion (sifting, key gen / state estimation)

error correction, privacy amplification

- no trusted random numbers for key
- direct use of quantum randomness for measurement basis

### Practical pair source



#### Blue diode-laser as pump source, BBO as nonlinear crystal





- 24,000 s<sup>-1</sup> detected pairs from 40 mW pump @ 407nm in single mode fibers at 810/818 nm, 2mm BBO crystal
- polarization correlation visibility in 45° basis: 92%
## NUS campus test range





### Receiver unit





polarization analyzer passively quenched Silicon APD - QE ~50% ~1000s<sup>-1</sup> dark cnt rate

### spatial filter (150 µrad)







(40 mm FWHM)







#### Identified raw coincidences between close and remote receiver



(with interference filter 5nm FWHM, 50% peak transmission)

....and after The Works:





- CASCADE error correction with ~6000 bit packets
- assume incoherent attack strategy for privacy amplification
- average efficiency of EC/PA: >57%
- average final key rate: 650 bits/sec
- residual error rate ~10<sup>-6</sup> due to a stupid error

### No interference filter





- use a RG780 long pass filter to suppress visible light
- average final key rate 850 bits/sec

```
(link loss 8.3 dB)
```

Atmospheric absorption



 representative vertical atmoshpere layer (corresponds to ~11 km air on ground)



**Optical fibers as 'channel'** 



- Use existing telecom infrastructure
- independent of environment
- high transmission:
  - 800nm:2dB/km(T=63% in 1km)1310nm:0.2dB/km(T=63% in 10km)1550nm:0.35dB/km(T=44% in 10km)
- stress birefringence and geometric phases are time dependent:



## Birefringence compensation



Probe fiber birefringence via two passes with Faraday mirror



- Basis of "Plug & Play" or autocompensation schemes in commercial QKD systems (id quantique, NEC)
- Bridging ~100 km

N. Gisin & team, GAP optique, Geneva D. Bethune / W. Risk, IBM Almaden A. Karlsson, KTH Stockolm NEC

Compare figures of merit







Find eavesdropper not via errors, but via testing entanglement: Ekert91 – type and tomographic protocols



### Bell inequality I





Correlation between setting *i*, *j*:

$$E(i,j) := \frac{n(i,j) + n(\overline{i},\overline{j}) - n(i,\overline{j}) - n(\overline{i},j)}{n(i,j) + n(\overline{i},\overline{j}) + n(\overline{i},\overline{j}) + n(\overline{i},\overline{j})}$$

combined correlation function:

$$S := E(1,1') + E(1,2') + E(2,1') - E(2,2')$$



If there is a local hidden parameter  $\lambda$  (= knowledge of **E** ) governing the measurement outcomes of **A** and **B**, then:

$$|S| \leq 2$$

### Bell inequality II





For proper settings 1, 2, 1', 2' and state  $|\Psi^-\rangle$ :  $S=\pm 2\sqrt{2}$ 

 Estimate quantitatively the knowledge of Eve of raw key between A and B from S:

$$I_{E}(S) = h \left( 1 + \frac{\sqrt{S^{2}/4 - 1}}{2} \right)$$

- Assume "fair sampling" between key measurement and Bell test
- No fingerprint problems of photons due to side channels A. Acin, N. Brunner, N. Gisin, S. Massar, S. Pironio, V. Scarani, PRL 98, 230501 (2007)

### E91 Implementation









Field results (1.4km range)



#### typical data run (with tropical rainfall inbetween)







of Singapore

 For non-lossy detectors and a measurement basis decision at "free will" of the observers:

No assumptions on devices and source is necessary to get an upper bound for eavesdropping!

A. Acin, N. Brunner, N. Gisin, S. Massar, S. Pironio, V. Scarani, PRL 98, 230501 (2007)

## Field usage...



### PDC pair source & sender



- Open source: http://code.google.com/p/qcrypto
- 24C3, Berlin 2007, Black Hat / DEFCON16, 2008

 System gets simpler and more robust

### receiving side



### Detector breakdown signature





### Timing channel attack I





## Timing channel attack II



Classical timing information carries fingerprint of detectors:



*Timing ch attack – The Cure* 



Make sure no detail timing information is revealed.....



- Alternative cures (costly for background):
  - coarser quantized timing information
  - add timing noise

Nastier attacks: V. Makarov Trondheim H.K. Lo, Toronto

*Time for Coffee....* 





# Thank you !

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