Atom-Light interaction in the strong focusing regime

28-29 May 2012 - IAS summer school physics

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Why strong focusing?

Motivation:

- Alternative way into the strong coupling regime compared to resonant structures like cavities & waveguides
- Intrinsically compatible with “flying qubits”, but this needs a slightly different treatment of “the photon”
- Explore possibilities of controlled phase gates & friends for photonic qubits

M.K. Tey, G. Maslennikov, T.C.H. Liew, et al., New J. Phys. ***11****, 040311 (2009)*
S.A. Aljunid et al, , New J. Phys. ***11****, 040311 (2009)*

Intended structure of lecture

- Part 1:

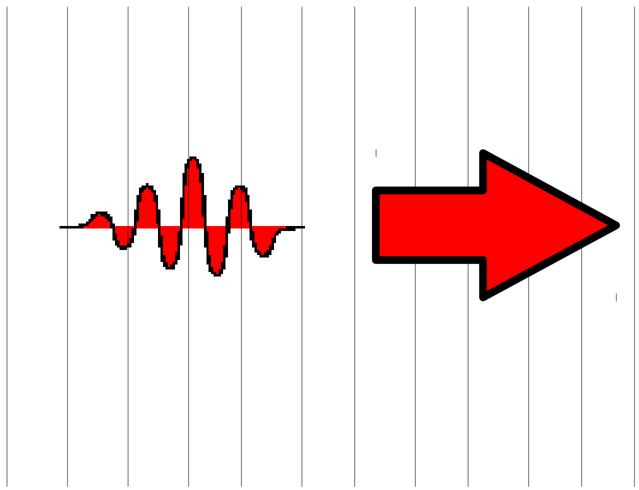
  Spatial aspects of strongly focused optical modes:
  Field geometries, stationary experiments

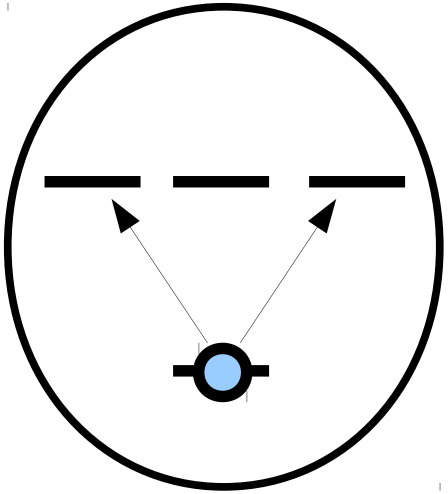
- Part 2:

  Temporal aspects:
  Flying photons, pulsed experiments

Atom-Photon interface

- e.g. transfer of information from flying qubits into a quantum memory





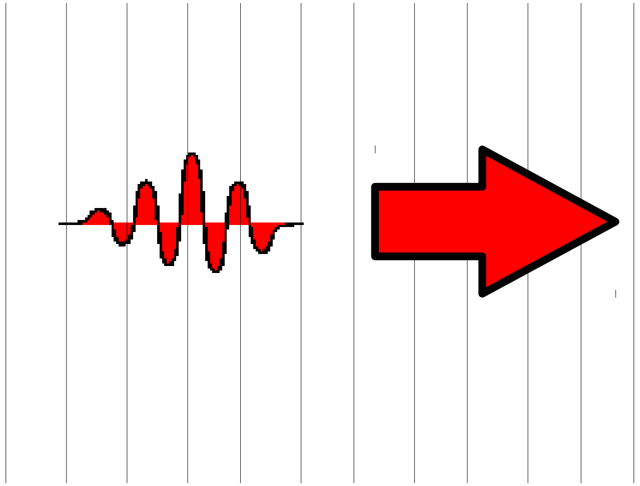
$$|\Psi_L\rangle = \alpha|L\rangle + \beta|R\rangle$$

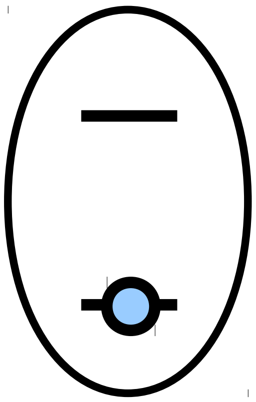
$$|\Psi_A\rangle = \alpha|m=-1\rangle + \beta|m=+1\rangle$$

- requires internal states of atom and an **absorption process**

The basic problem

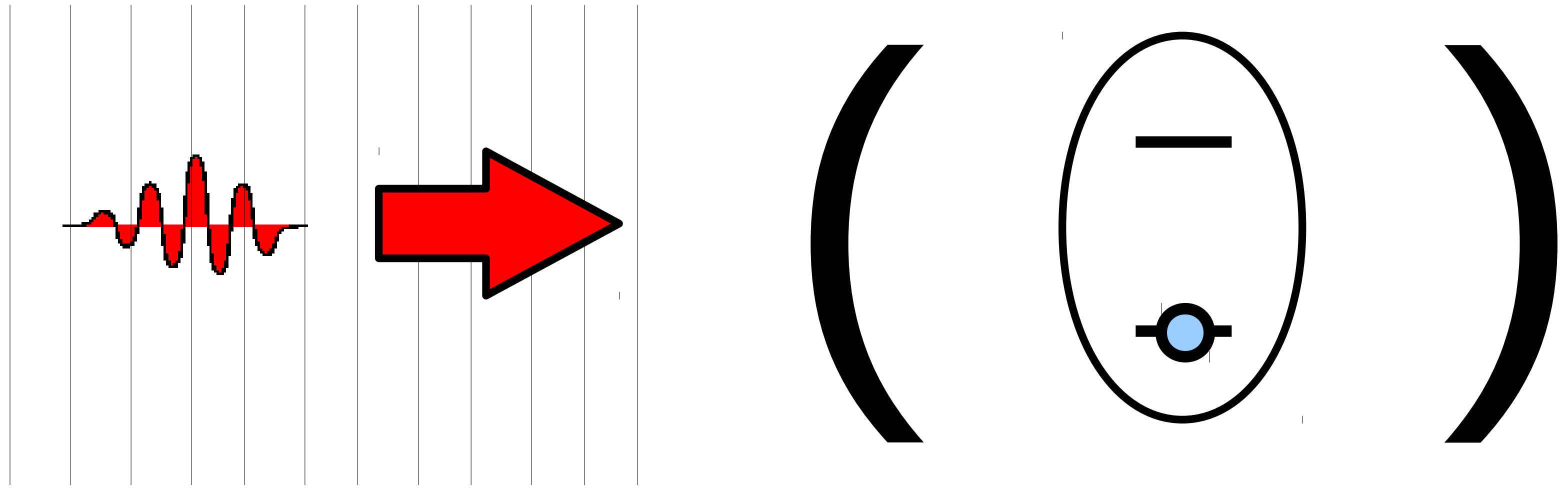
- Get strong coupling between an atom and a light field on the single photon level





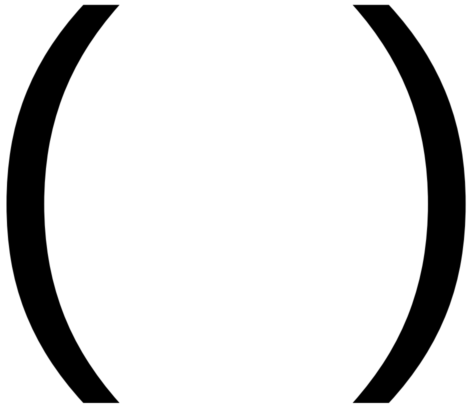
electromagnetic field / photon **2-level atom**

One solution: Use a cavity



- High electrical field strength even for a single photon
- Preferred spontaneous emission into the cavity mode
- A cavity can enhance the interaction between a propagating external mode and an atom

Why cavities are nice



- discrete mode spectrum
- 'textbook' field energy eigenstates

$$\hat{H}_{field} = \frac{\epsilon_0}{2} \int \left(\hat{\boldsymbol{E}}^2 + c^2 \hat{\boldsymbol{B}}^2 \right) dV = \hbar \omega \left(\hat{n} + \frac{1}{2} \right)$$

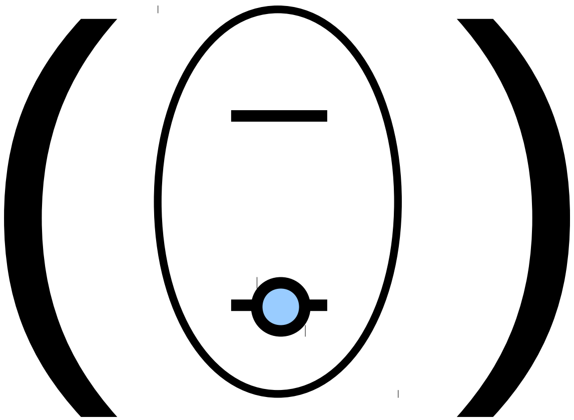
Electrical field operator (single freq):

$$\hat{\boldsymbol{E}}(x, y, z) = i \sqrt{\frac{\hbar \omega}{2 \pi \epsilon_0 V}} \left(\boldsymbol{g}(x, y, z) \hat{a}^+ - \boldsymbol{g}^*(x, y, z) \hat{a} \right)$$

mode function, e.g.

$$\boldsymbol{g}(x, y, z) = \boldsymbol{e} \sin kz \, e^{-\frac{x^2 + y^2}{w^2}}$$

Atom in a cavity



- atom Hamiltonian

$$\hat{H}_{atom} = E_g |g\rangle\langle g| + E_e |e\rangle\langle e|$$

- electric dipole interaction

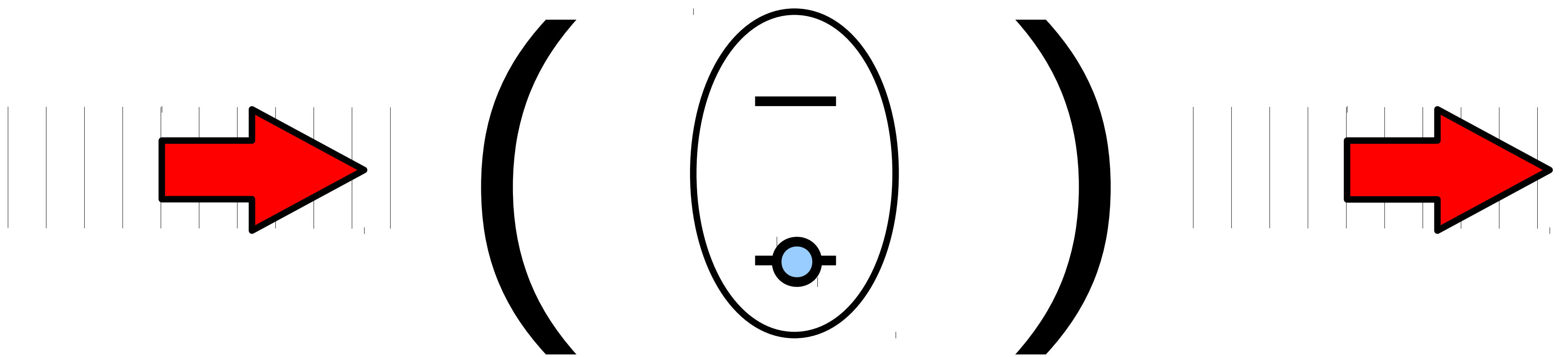
$$\hat{H}_I = \hat{\boldsymbol{E}} \cdot \hat{\boldsymbol{d}} \quad \text{with} \quad \hat{\boldsymbol{d}} = \boldsymbol{e}\, d_{eff} \left(|e\rangle\langle g| + |g\rangle\langle e|\right)$$

- (treat other field mode as losses)...

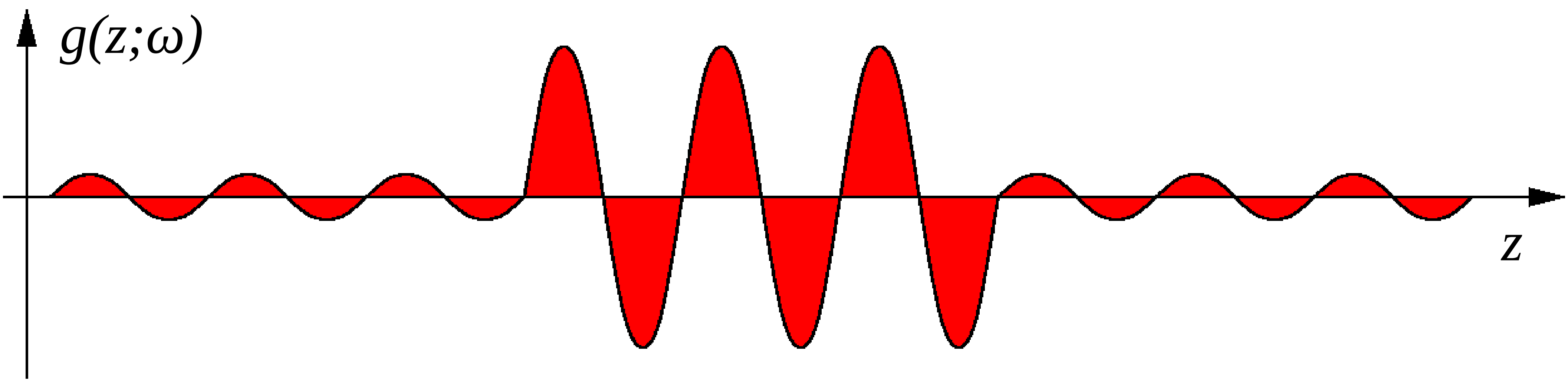
.....Jaynes-Cummings model with all its aspects

- treat external fields as perturbation/spectator of internal field

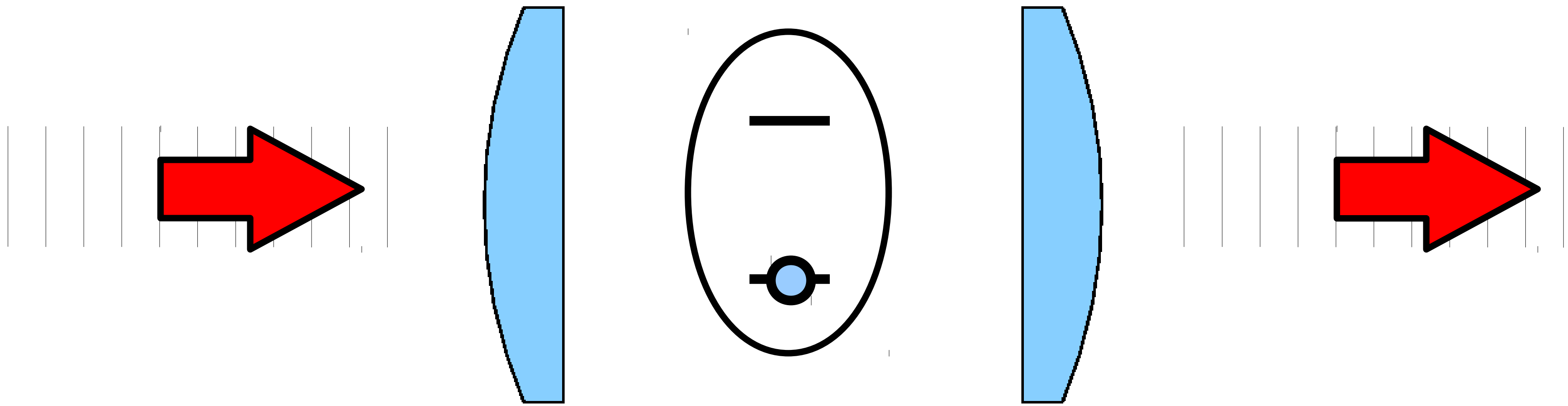
External view of cavity+atom



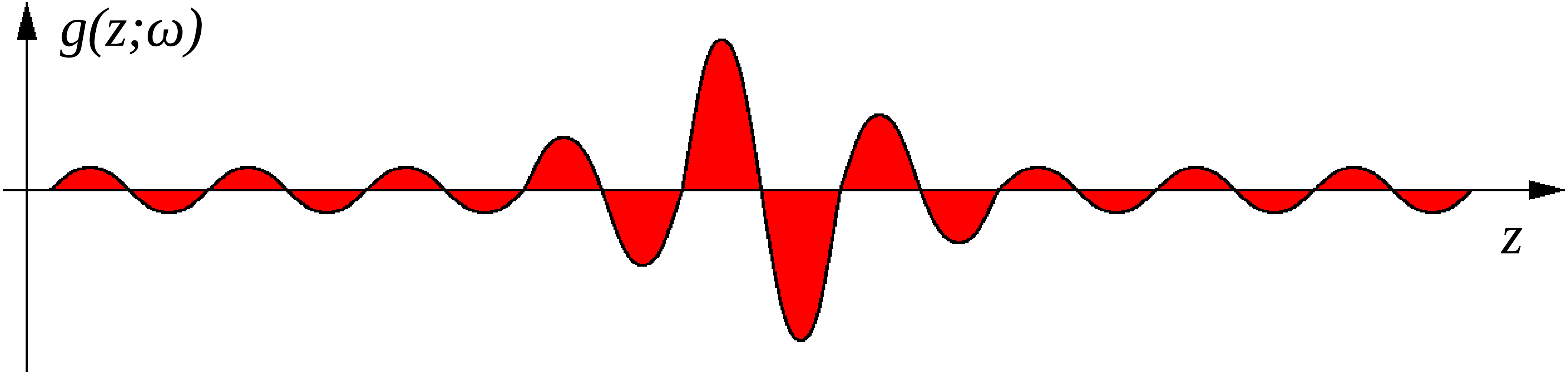
- continuous mode spectrum with enhanced/reduced field mode function:



An alternative approach



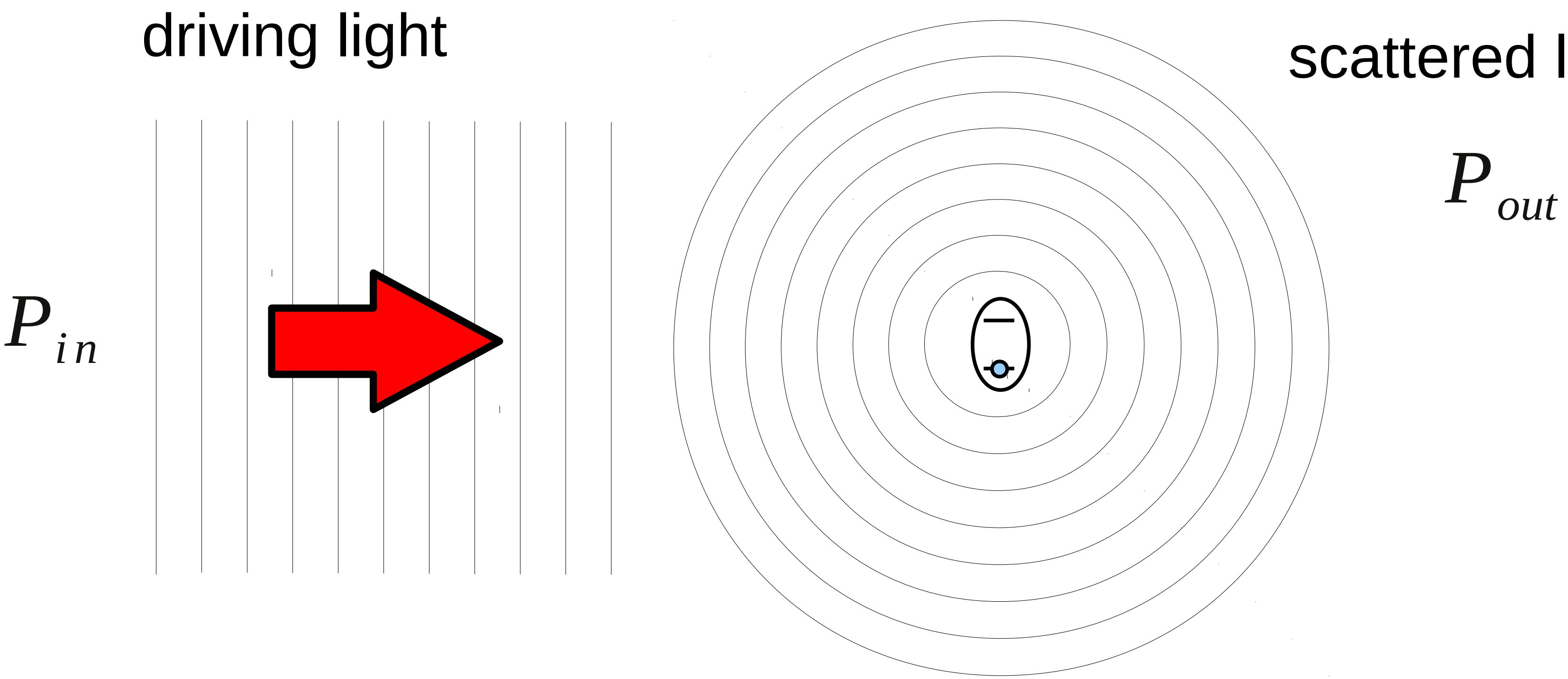
- use a **focusing lens pair** to enhance center mode function:



Resonant scattering

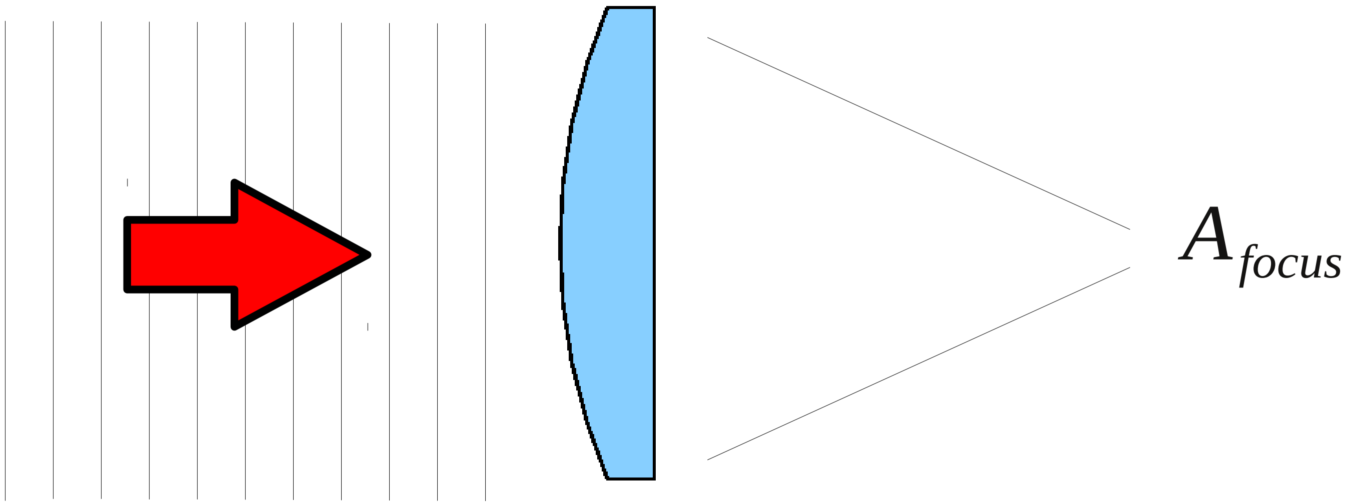
- Resonant scattering cross section of an atom:

$$\sigma_{max} = 3\lambda^2 / 2\pi$$



- Asymptotic limit: $P_{out} = P_{in} \cdot R$ with $R = \frac{\sigma}{A}$

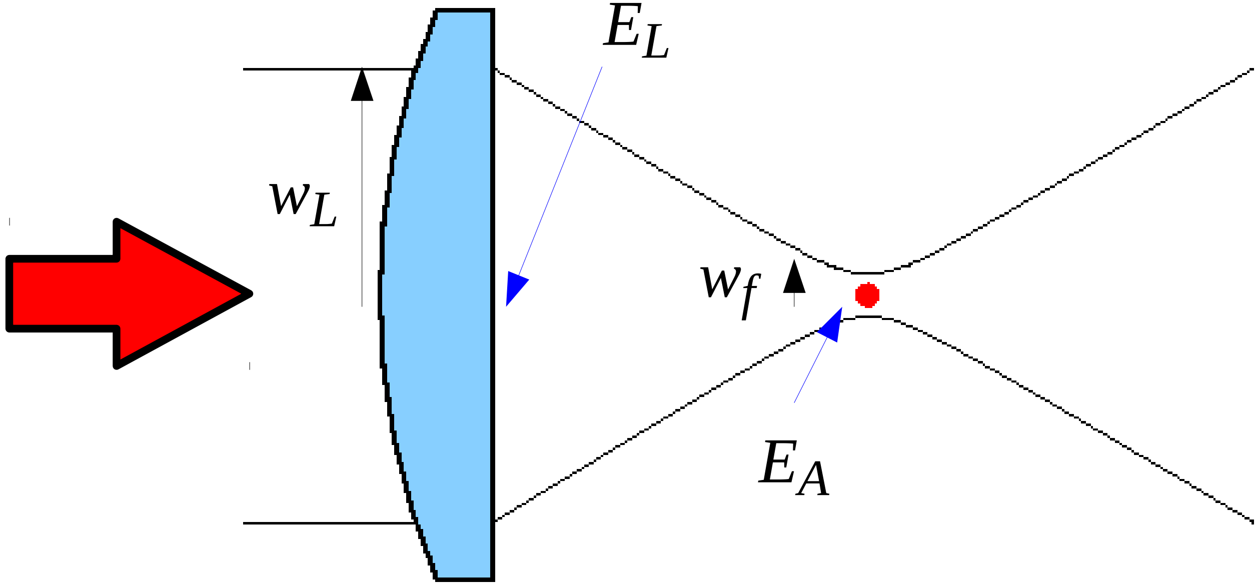
Strong focusing



- Diffraction limit: $A_{focus} \approx \lambda^2 / (NA^2) \cdot something$
- For large numerical aperture: $A_{focus} \approx \sigma_{max}$ or $R \approx 1$

Strong coupling?

Gaussian beams



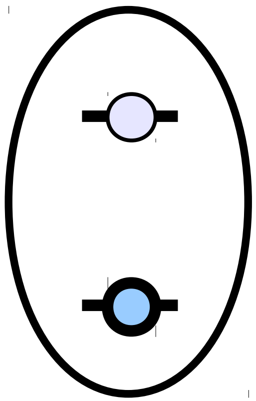
- Collimated beam: $E(\rho) = E_L \cdot e^{-\rho^2/w_L^2}$ $\qquad P_{in} = \frac{1}{4} \epsilon_0 \pi c E_L^2 w_L^2$

- In focus (paraxial approximation): $w_f = \frac{f \lambda}{\pi w_L}$

$$\left(\frac{E_A}{E_L} \right)^2 = \left(\frac{w_L}{w_f} \right)^2$$

Step 1: Scattering from an atom

two - level atom in external driving field (quick & dirty)



- stationary excited state population:

$$\rho_{ee} = \frac{\Omega^2/4}{\delta^2 + \Omega^2/2 + \Gamma^2/4}$$

$$\Omega = E_A |d_{12}|/\hbar \quad \text{Rabi frequency}$$

$$\Gamma = \frac{\omega_{12}^3 d_{12}^2}{3\pi\epsilon_0 \hbar c^3} \quad \text{excited state decay rate}$$

- photon scattering rate $\rho_{ee}\Gamma$ leads to

  scattered power $P_{sc} = 3\epsilon_0 c \lambda^2 E_A^2 / 4\pi$

Field at focus (simple)

paraxial approximation

$$R_{sc} = \frac{P_{sc}}{P_{in}} = \frac{3\lambda^2}{\pi w_L^2}\left(\frac{E_A}{E_L}\right)^2 \approx \frac{3\lambda^2}{\pi w_f^2} = 3u^2 \approx \sigma_{max}/A$$

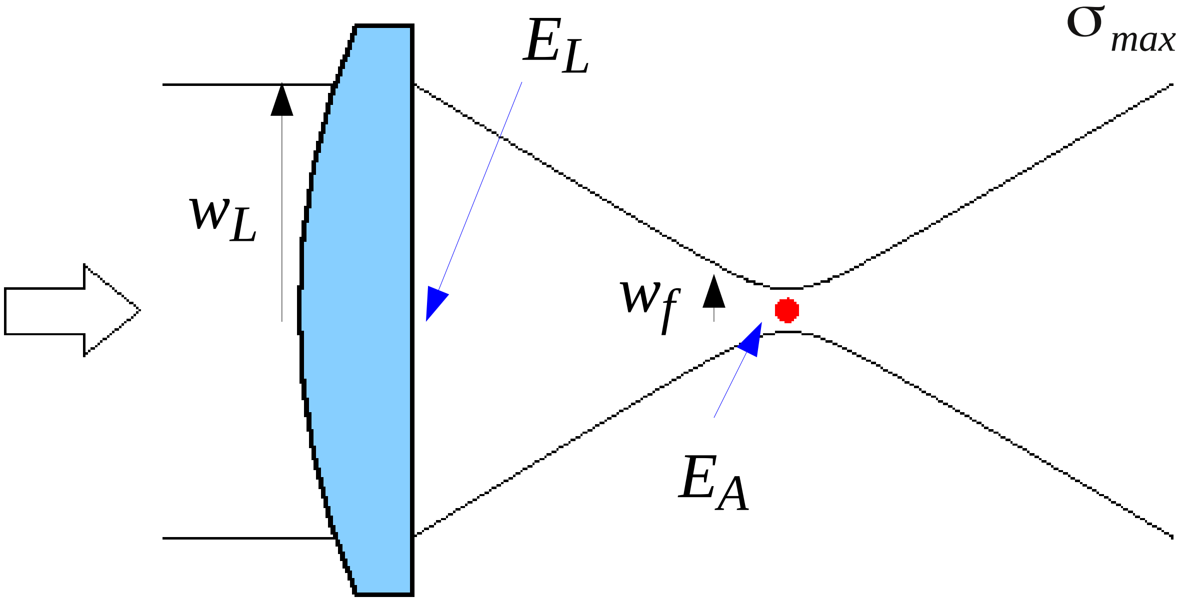
focal area

$$A \approx \pi w_f^2/2$$

focusing strength $u := w_L/f$

atomic scattering cross section

$$\sigma_{max} = 3\lambda^2/2\pi$$

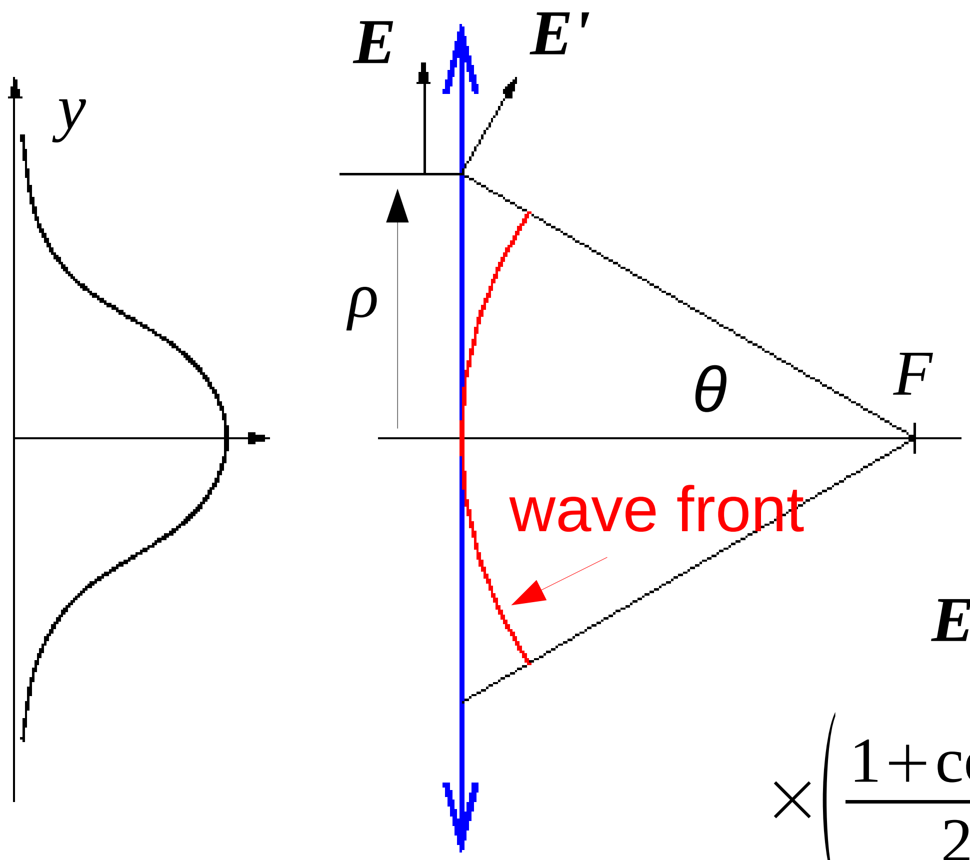


E_A diverges, use full expression for field

Get exact field in focus

Circularly polarized Gaussian beam.....

$$\boldsymbol{E} = E_L \hat{\epsilon}_+ e^{-\rho^2/w_l^2}$$



....transformed by an ideal lens:

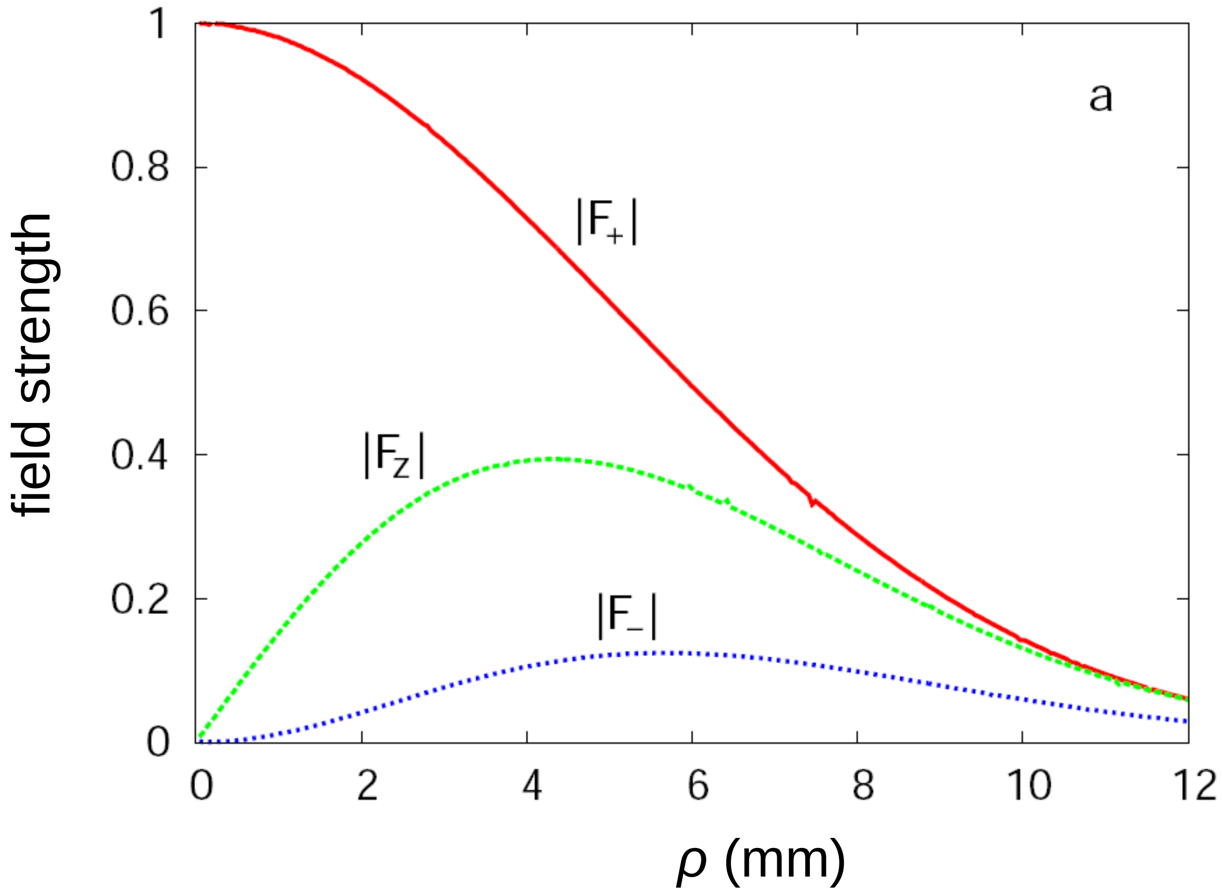
- spherical wave front
- locally transverse
- conserve power through each small area

(Richardson/Wolf criteria, ~1950)

$$\boldsymbol{E}' = E_L e^{-\frac{\rho^2}{w_l^2}} \frac{1}{\sqrt{\cos\theta}} \times e^{-ik\sqrt{\rho^2+f^2}} \times \left(\frac{1+\cos\theta}{2}\hat{\epsilon}_+ + \frac{\sin\theta\, e^{i\phi}}{\sqrt{2}}\hat{z} + \frac{\cos\theta-1}{2} e^{2i\phi}\hat{\epsilon}_- \right)$$

Electrical field after ideal lens

$$\boldsymbol{E}' = E_L e^{-\frac{\rho^2}{w_l^2}} \frac{1}{\sqrt{\cos\theta}} \times e^{-ik\sqrt{\rho^2+f^2}} \times \left(\frac{1+\cos\theta}{2} \hat{\epsilon}_+ + \frac{\sin\theta \, e^{i\phi}}{\sqrt{2}} \hat{z} + \frac{\cos\theta - 1}{2} e^{2i\phi} \hat{\epsilon}_- \right)$$



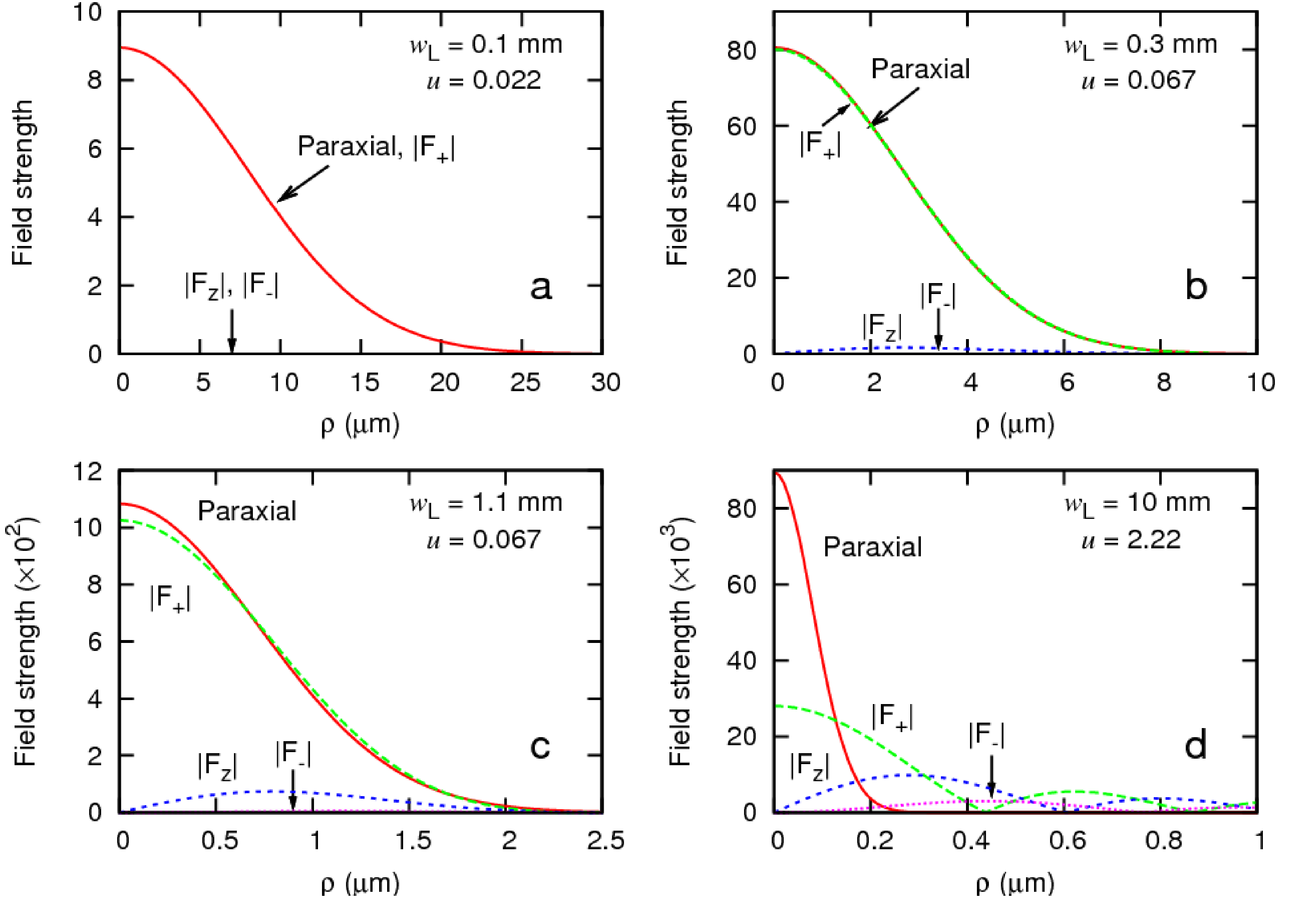
- other polarization components appear

beam parameter:
w_l = 7 mm

focal length:
f = 4.5 mm

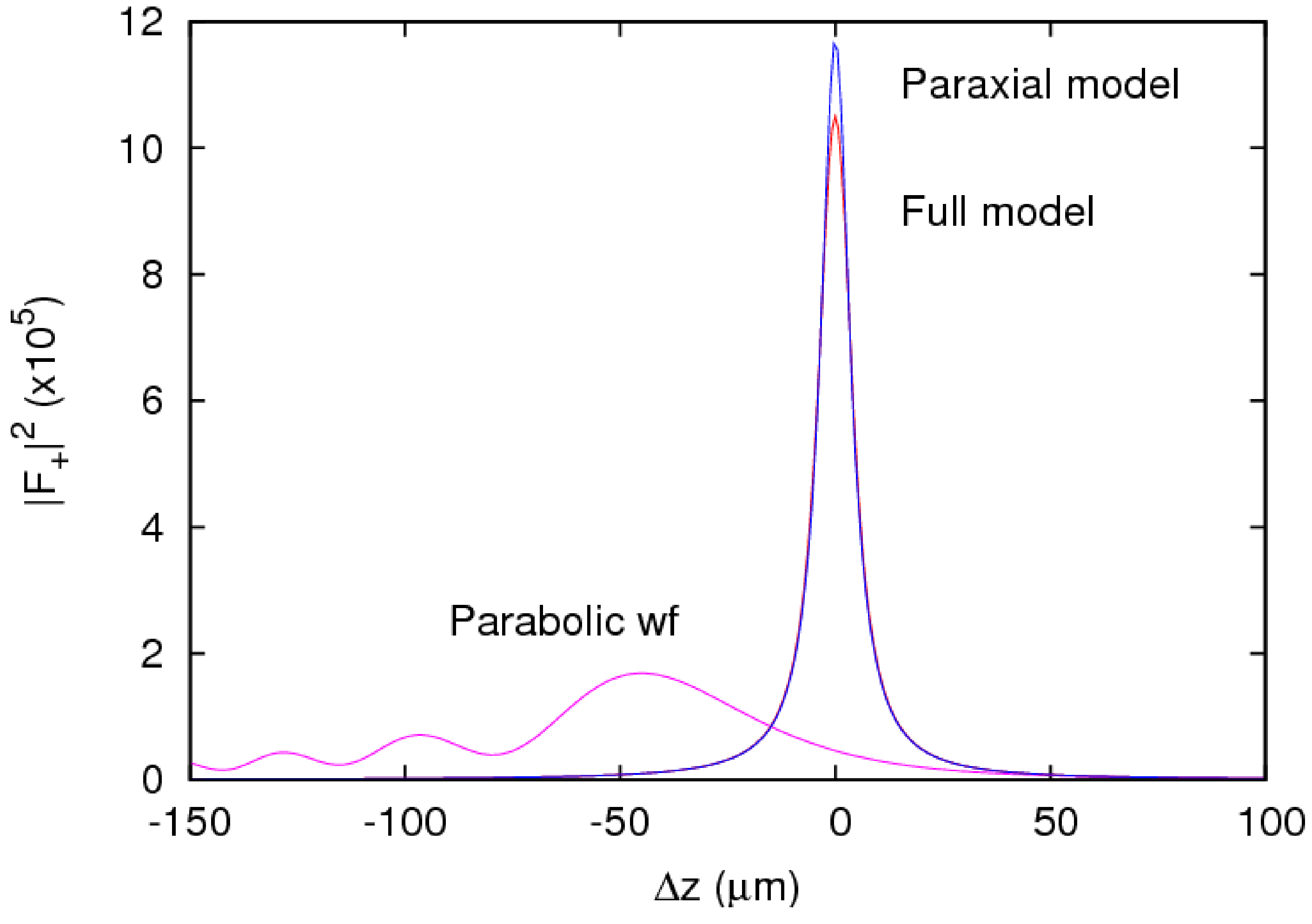
Propagate field to focus

Basic idea: Huygens principle. Numerical results for f = 4.5mm



In focal direction, on axis

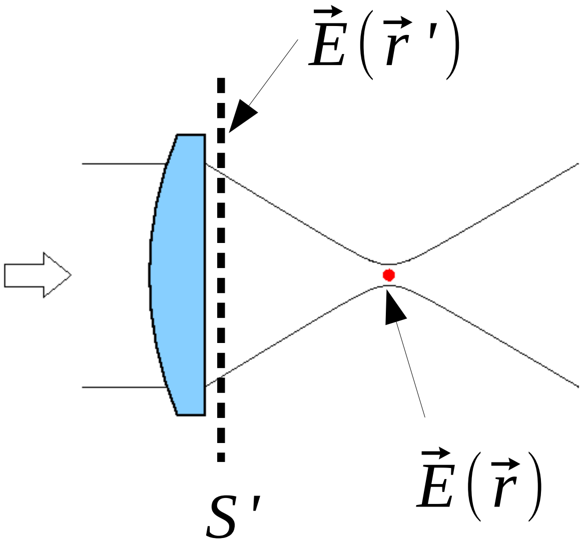
Numerical result for f = 4.5 mm, w_L = 1.1 mm (u = 0.244)



Analytical Result

Propagate Green's function to focal spot, vector version

$$\vec{E}(\vec{r}) = \int_{s'} dA' \Big\{ ikc\left[\vec{n}'\times\vec{B}(\vec{r}\,')\right]G(\vec{r},\vec{r}\,') + \left[\vec{n}'\times\vec{E}(\vec{r}\,')\right]\times\nabla' G(\vec{r},\vec{r}\,') + \left[\vec{n}'\cdot\vec{E}(\vec{r}\,')\right]\nabla' G(\vec{r},\vec{r}\,') \Big\}$$



With scalar Green's function

$$G(\vec{r},\vec{r}\,') = \frac{e^{ik|\vec{r}-\vec{r}\,'|}}{4\pi|\vec{r}-\vec{r}\,'|}$$

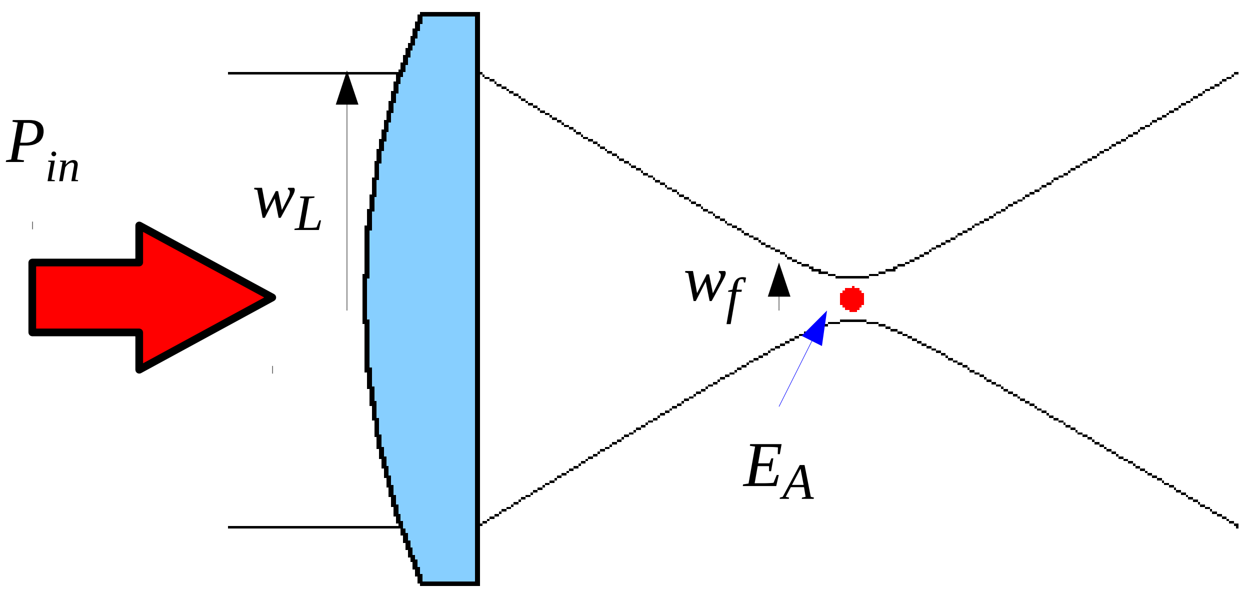
Field to focus (exact)

Exact propagation to focus:

$$\boldsymbol{E}_A(z=f,\rho=0)=\sqrt{\frac{\pi P_{in}}{\epsilon_0 c\lambda^2}}\cdot\frac{1}{u}e^{1/u^2}\left[\sqrt{\frac{1}{u}}\,\Gamma\left(-\frac{1}{4},\frac{1}{u^2}\right)+\sqrt{u}\,\Gamma\left(\frac{1}{4},\frac{1}{u^2}\right)\right]\hat{\epsilon}_+,$$

$$u:=w_L/f$$

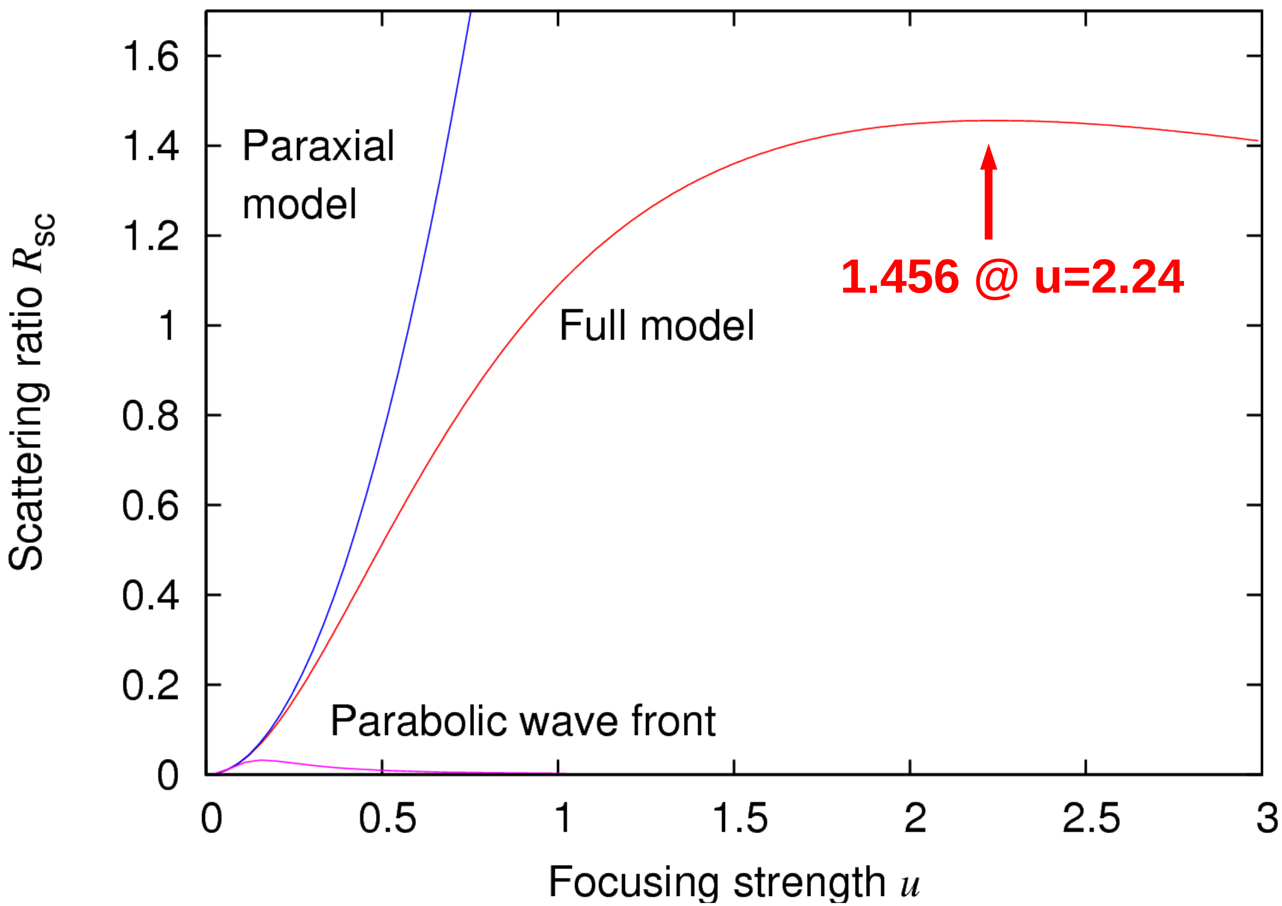
Incomplete Gamma function



Field to focus (exact)

- scattering "ratio" like in plane wave excitation mode:

$$R_{sc} := \frac{P_{sc}}{P_{in}} = \frac{3}{4u^3} e^{-2/u^2} \left[\Gamma\left(-\frac{1}{4}, \frac{1}{u^2}\right) + u\,\Gamma\left(\frac{1}{4}, \frac{1}{u^2}\right) \right]^2$$



???

Richardson/Wolf limit:

$$R_{sc} \leq 2$$

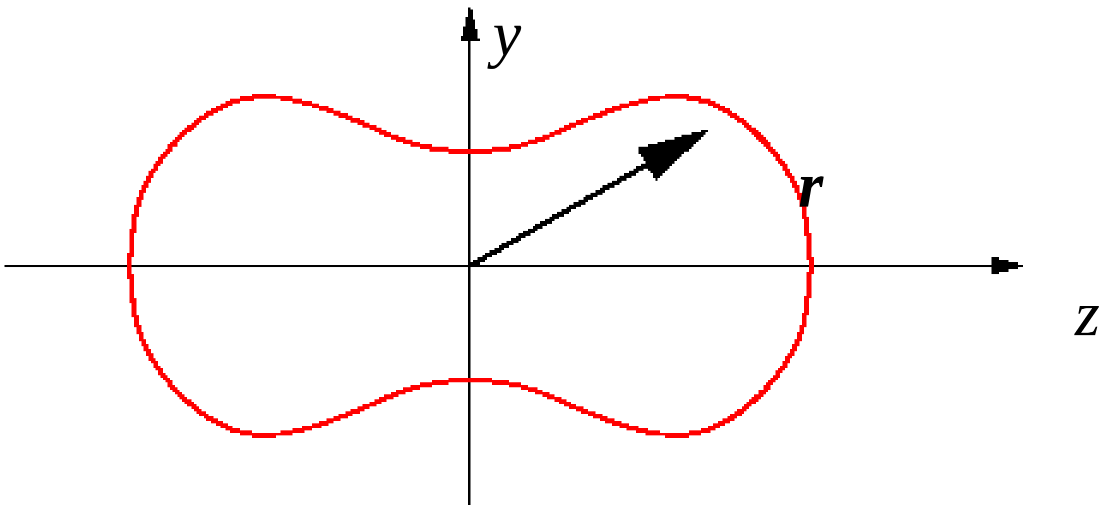
Atomic response

- scattered field has electric dipole characteristic corresponding to σ+ transition:

$$\boldsymbol{E}_{sc}(\boldsymbol{r}) = E_A \frac{3}{2} \frac{e^{ikr+\pi/2}}{kr} \left[\hat{\epsilon}_+ - (\hat{\epsilon}_+ \cdot \hat{r}) \hat{r} \right]$$

$$\hat{r} = \frac{1}{|\boldsymbol{r}|} \boldsymbol{r}$$ radial unit vector

$$\hat{\epsilon}_+ = \frac{\hat{x} + i\,\hat{y}}{\sqrt{2}}$$ circular unit vector

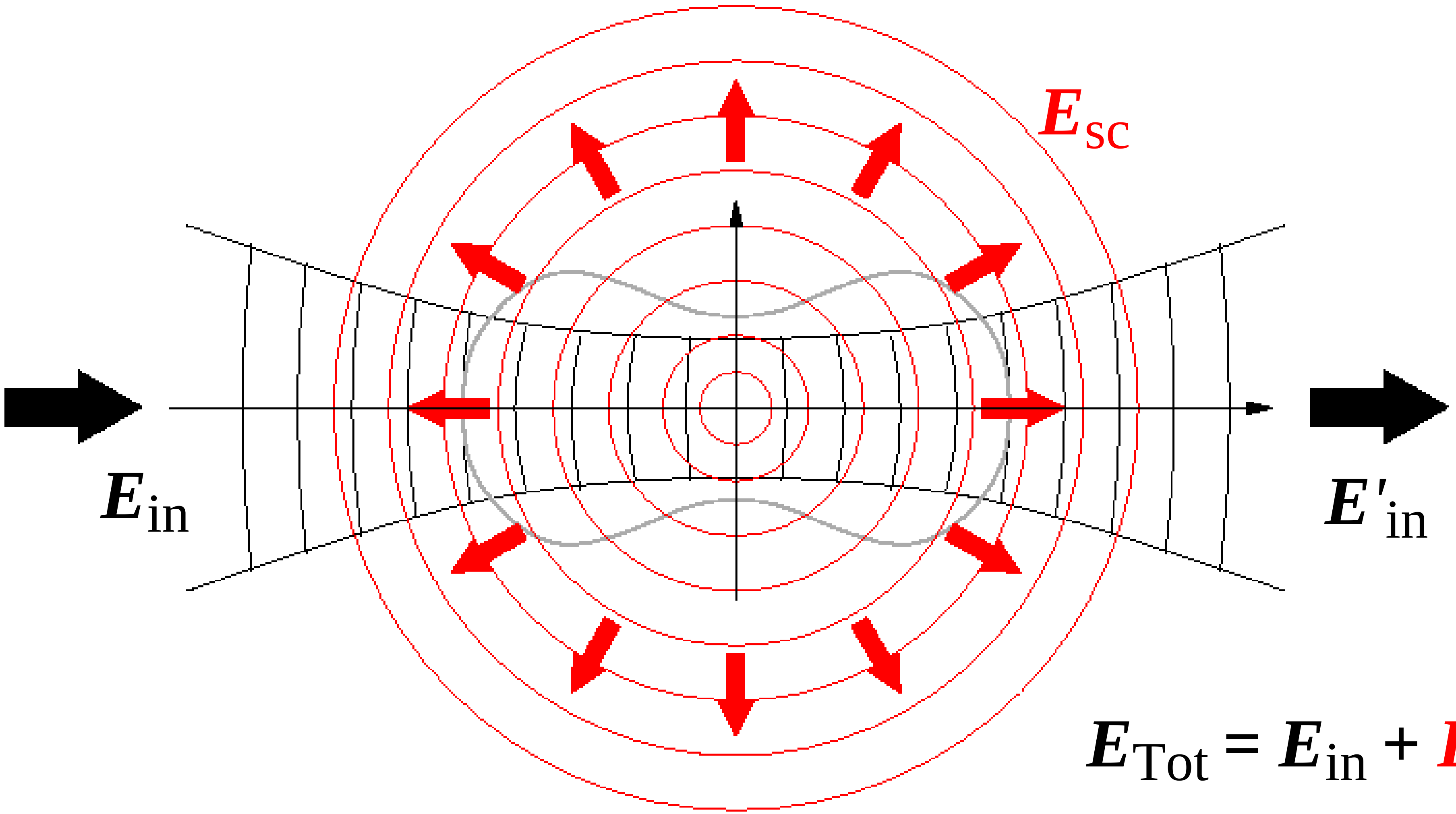


with detuning from resonance:

$$\boldsymbol{E}_{sc} = \boldsymbol{E}_{sc}^0 \cdot \frac{i\,\Gamma}{2\Delta + i\,\Gamma}$$

Combine with probe

scattered field for σ+ transition: $$\boldsymbol{E}_{sc}(\boldsymbol{r}) = E_A \frac{3}{2}\frac{e^{ikr+\pi/2}}{kr}\left[\hat{\epsilon}_+ - (\hat{\epsilon}_+ \cdot \hat{r})\hat{r}\right]$$



$\boldsymbol{E}_{Tot} = \boldsymbol{E}_{in} + \boldsymbol{E}_{sc}$

Collection into Gaussian mode

- Project total field onto Gaussian mode of collection fiber

$$P_{out} = \left| \langle \vec{g}, \vec{E}_{Tot} \rangle \right|^2 \qquad \langle \vec{g}, \vec{E} \rangle := \int_{\vec{x} \in S} \vec{E}_{Tot}(\vec{x}) \cdot \vec{g}(\vec{x}) (\vec{k}_g \cdot \vec{n}) \, dA$$

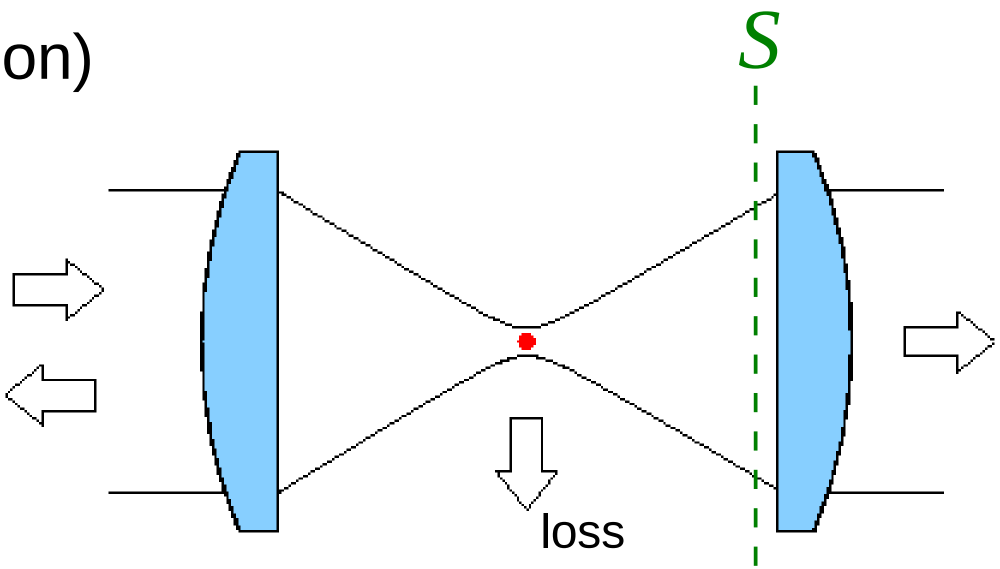
cross section fiber mode

- Forward transmission:

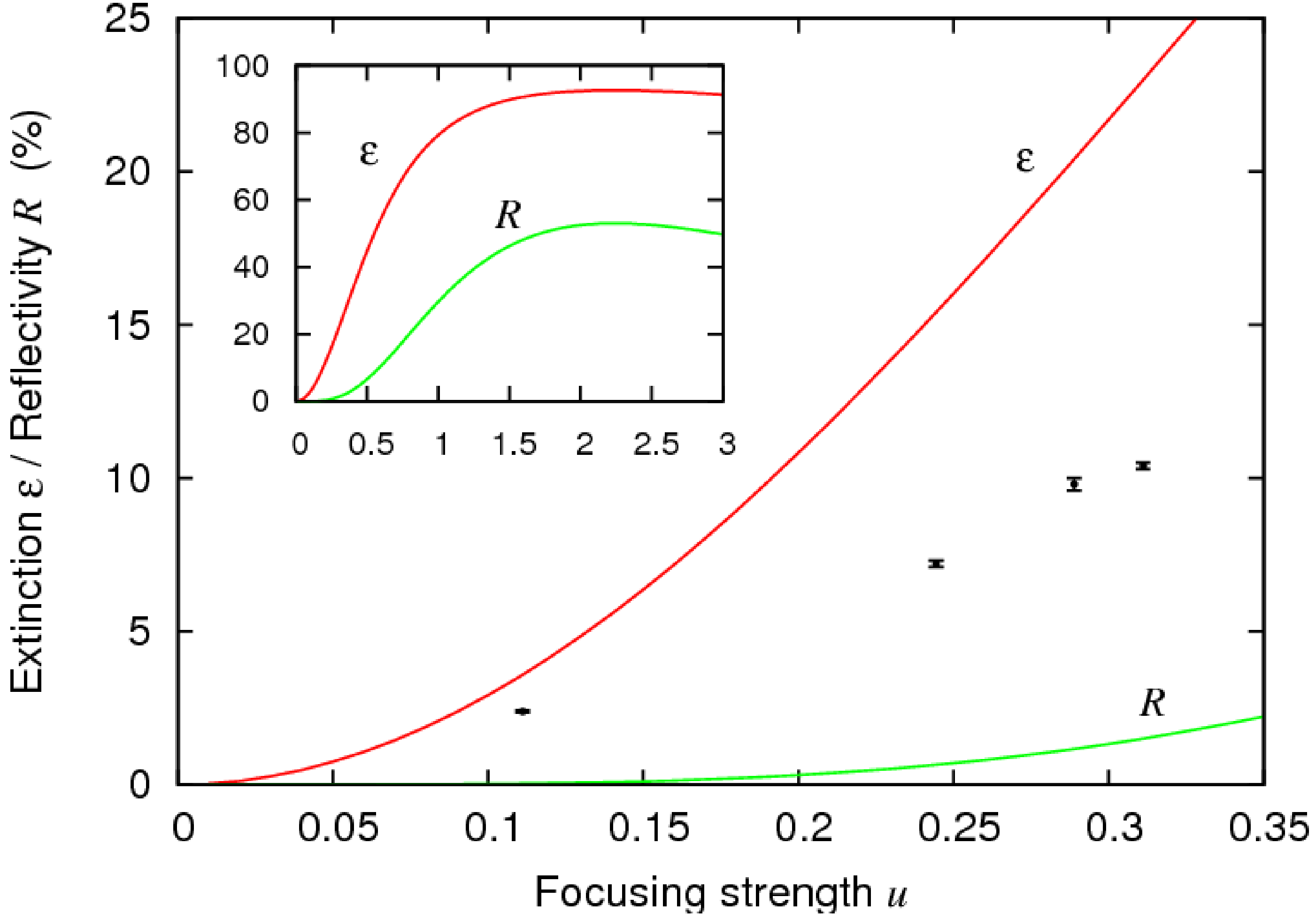
$$1 - \epsilon = \frac{P_{out}}{P_{in}} = \left| 1 - \frac{P_{sc}/P_{in}}{2} \right|^2$$

- Reflectivity (backward direction)

$$R = \frac{(P_{sc}/P_{in})^2}{4}$$



Collection into Gaussian mode

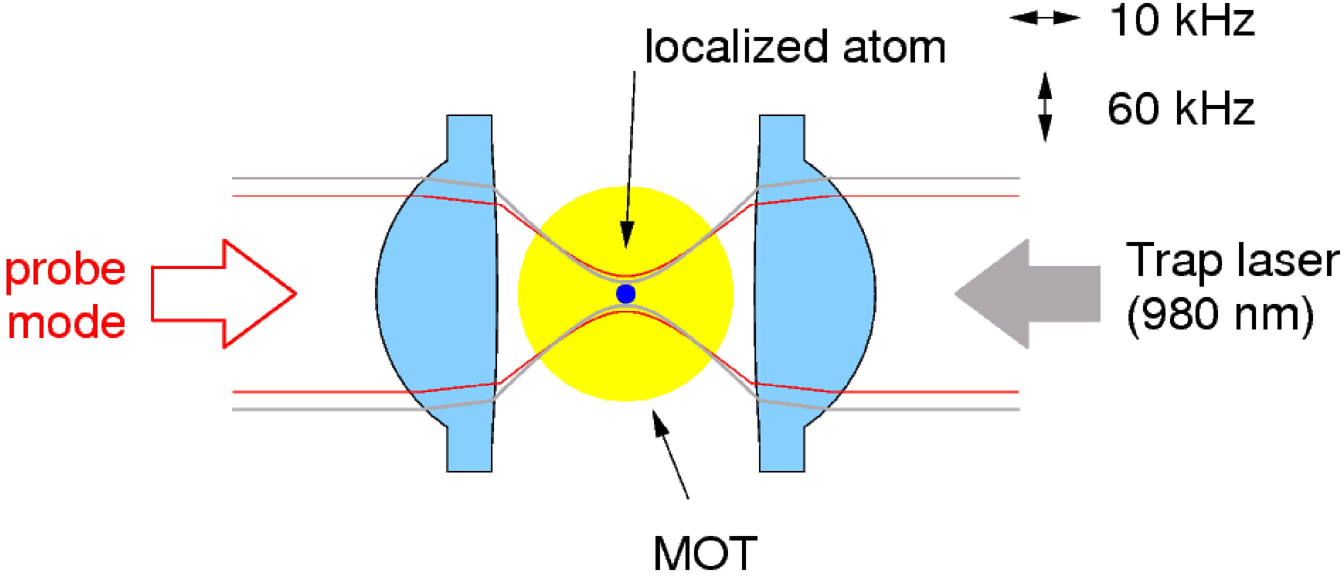


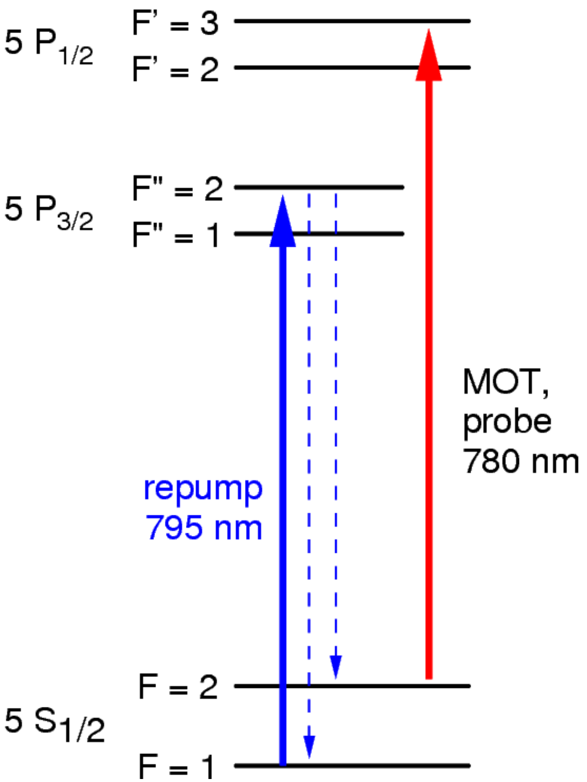
Scattering probabilities now < 1 !

Want a break?

Experiment

One atom in an optical dipole trap, loaded from a MOT

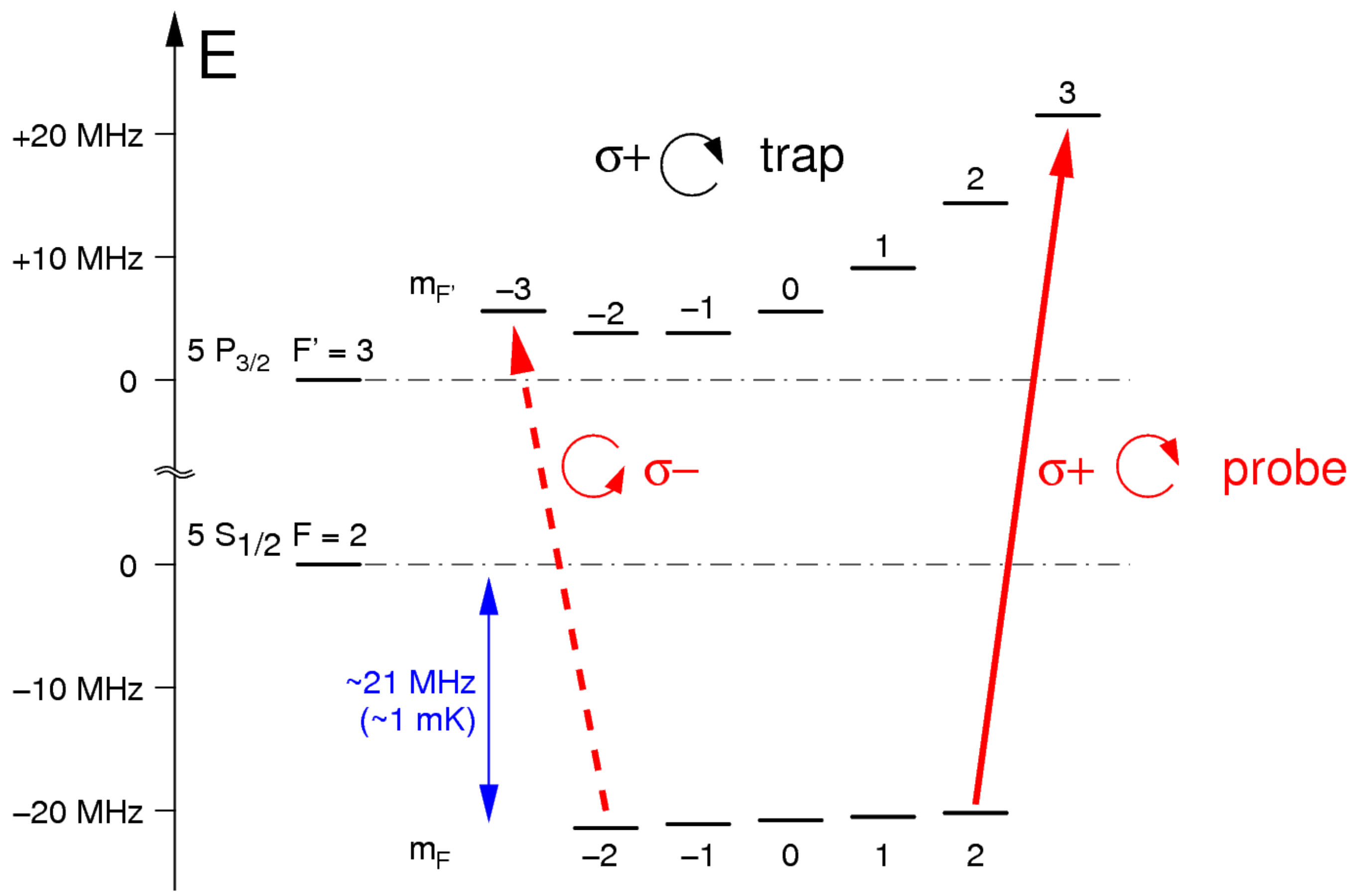




- use Rubidium-87 atom because it is convenient

M. K. Tey, Z. Chen, S.A. Aljunid, B. Chng, F. Huber, G. Maslennikov,C. K.
*nature physics **4**, 924 (2008)*

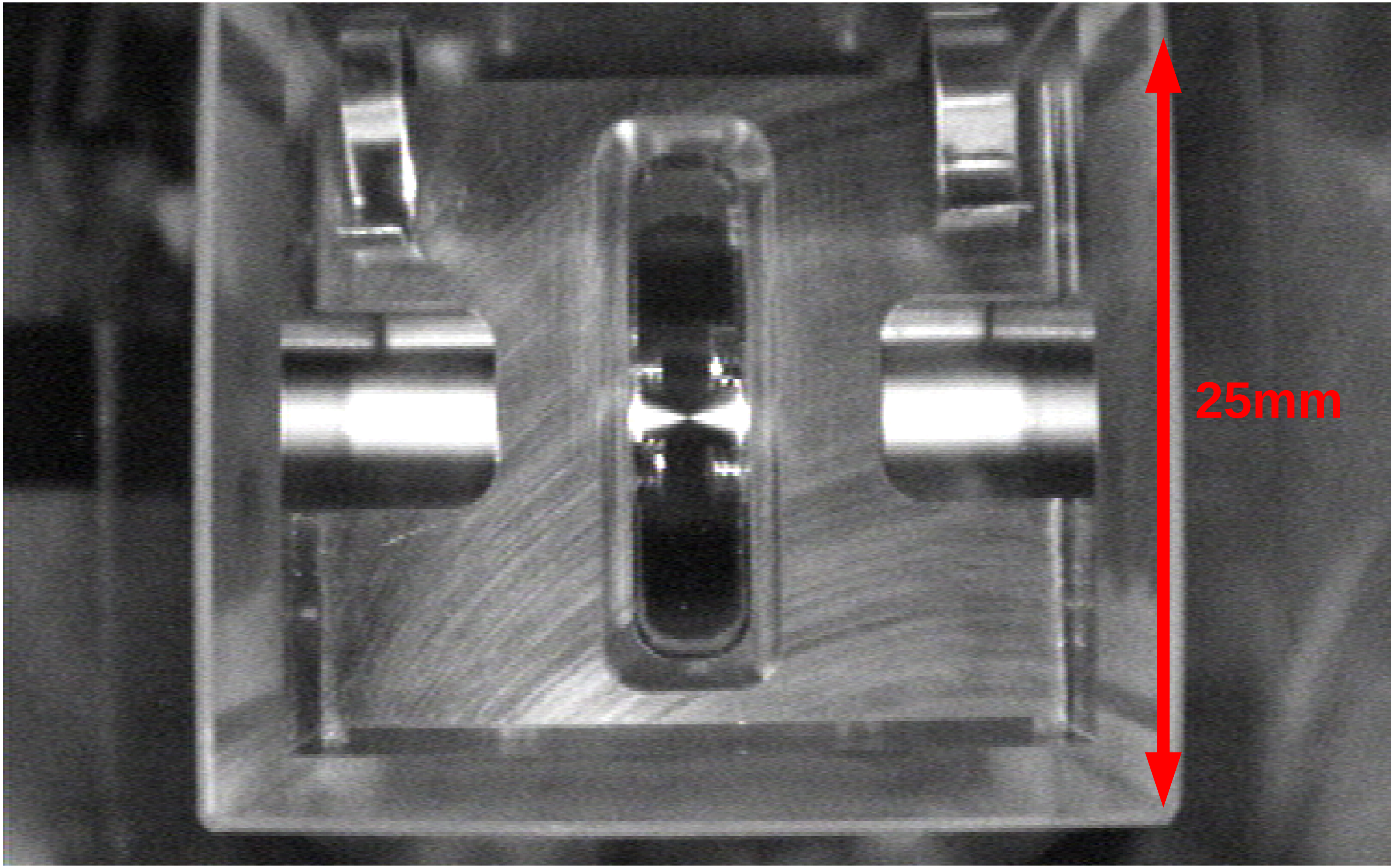
Atomic levels in a dipole trap



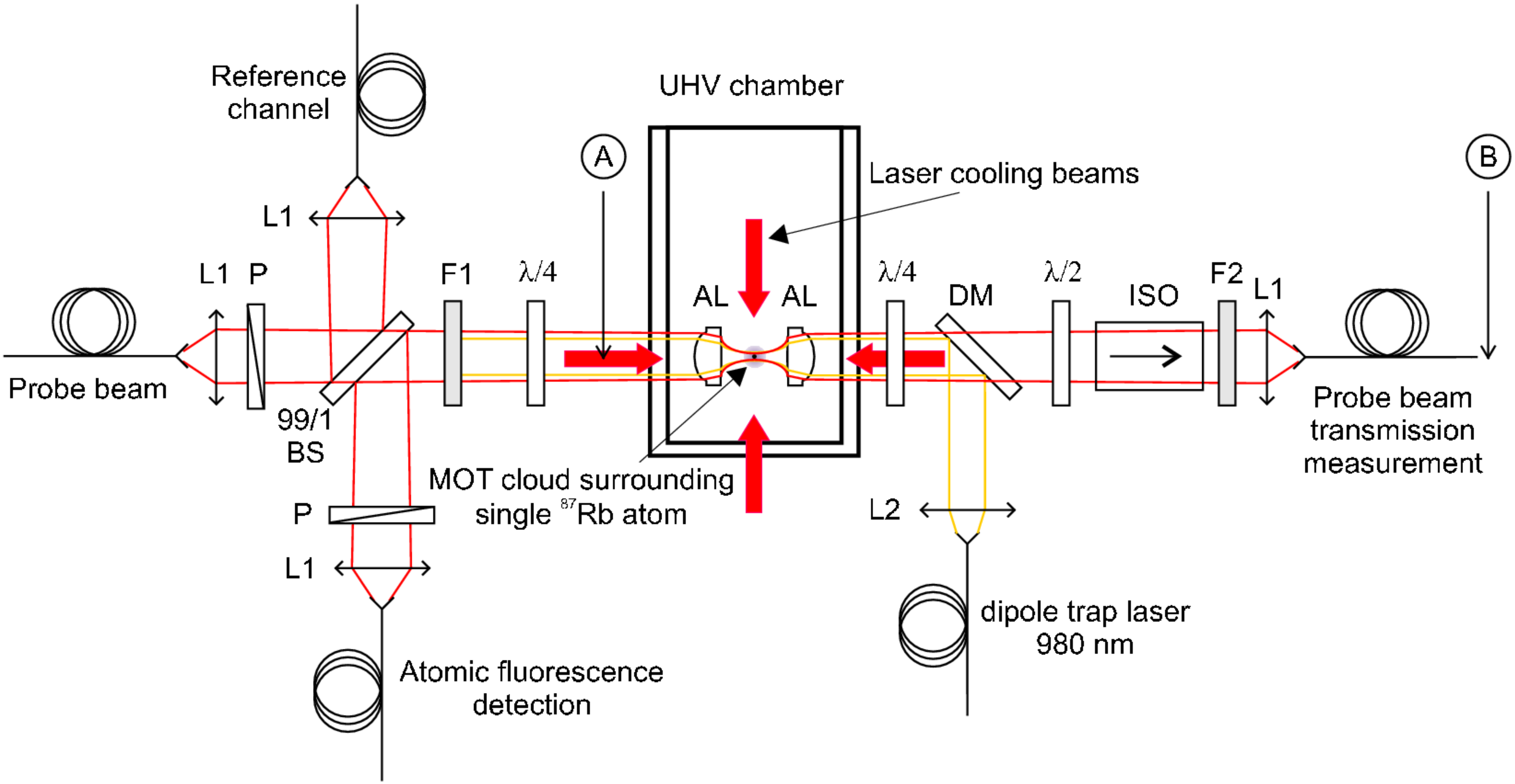
- optically pump with the probe beam into 2-level system

Focusing geometry...

...as seen by a CCTV camera at high Rb pressure

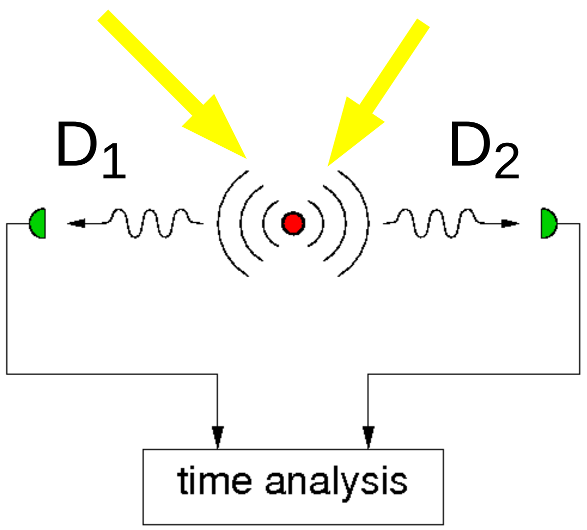


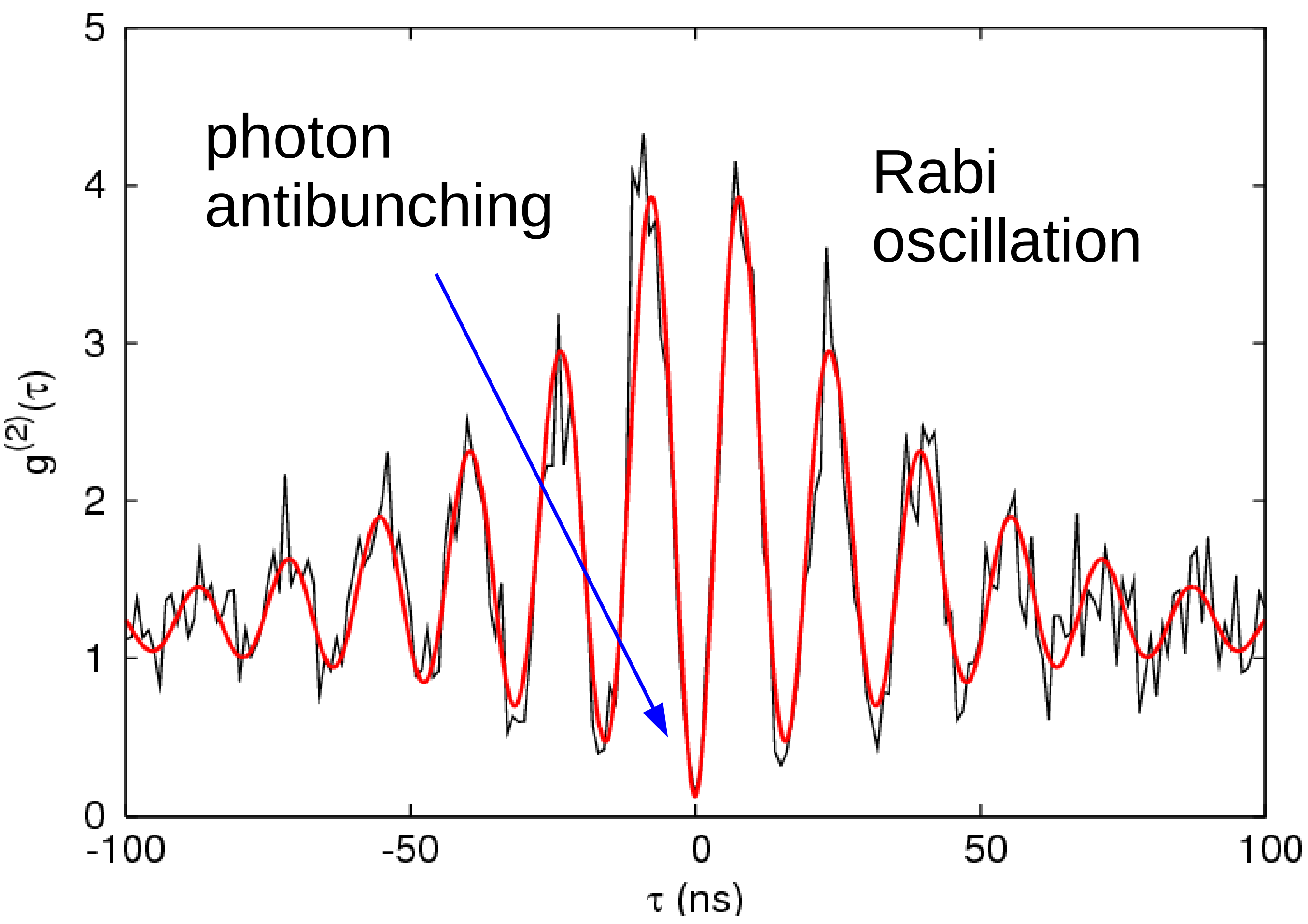
Almost the real experiment...



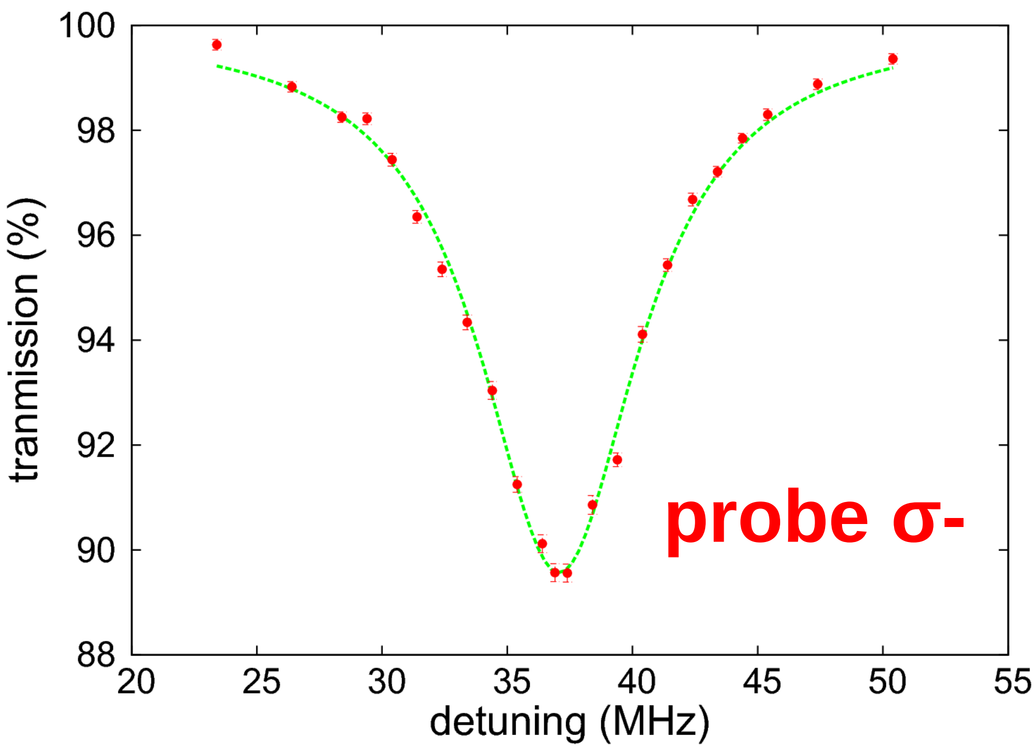
Single atom evidence

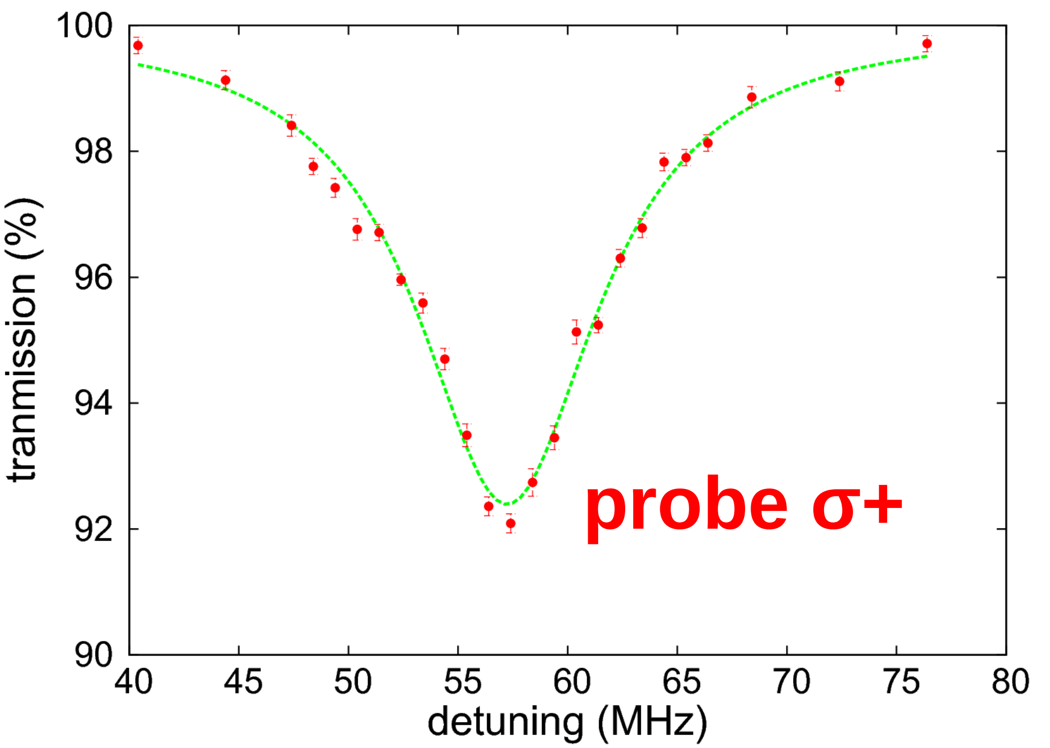
(almost) Hanbury-Brown—Twiss experiment on atomic fluorescence during cooling





Transmission measurement



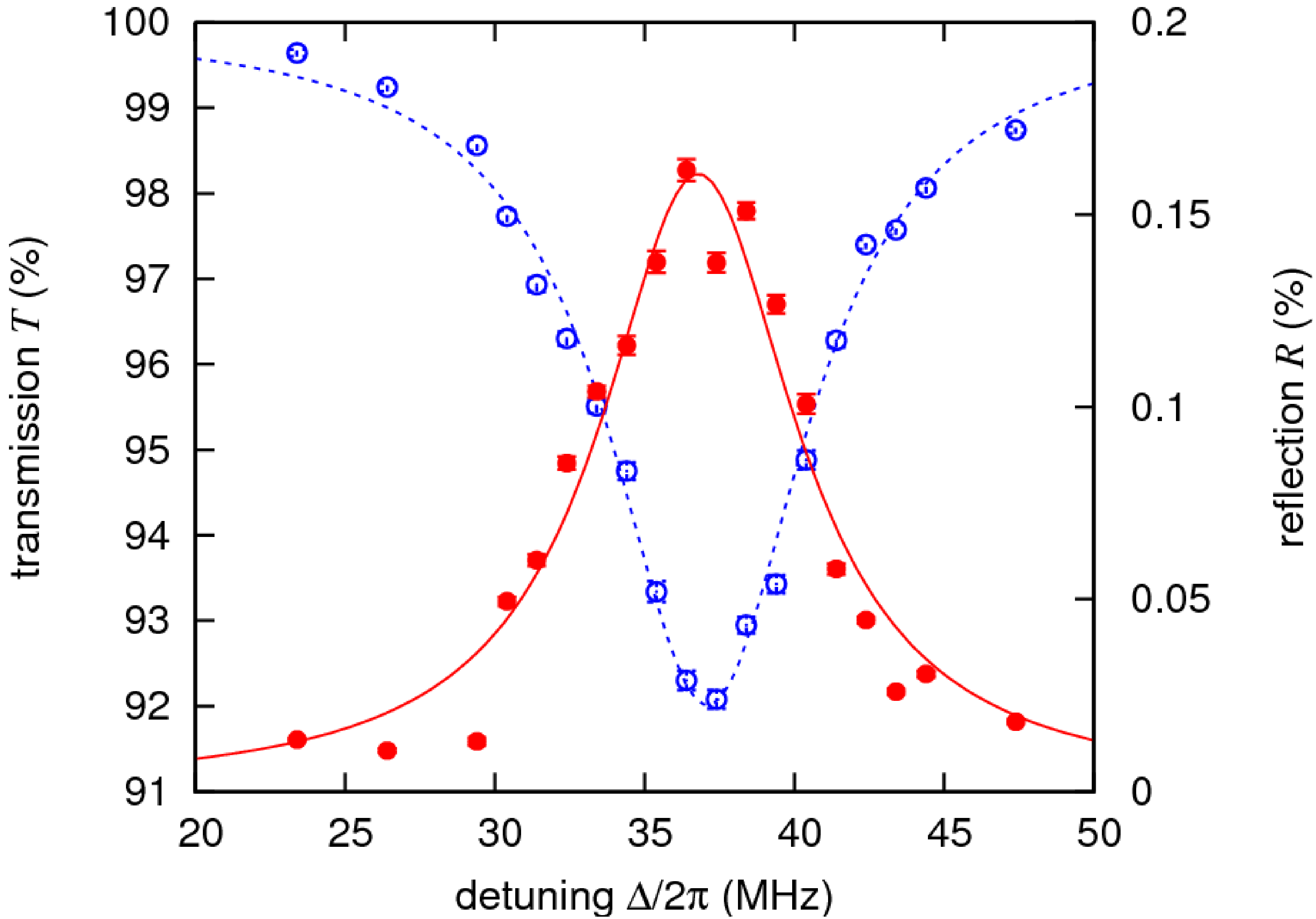


- almost natural line width of atomic transition
- different resonances for different probe polarizations

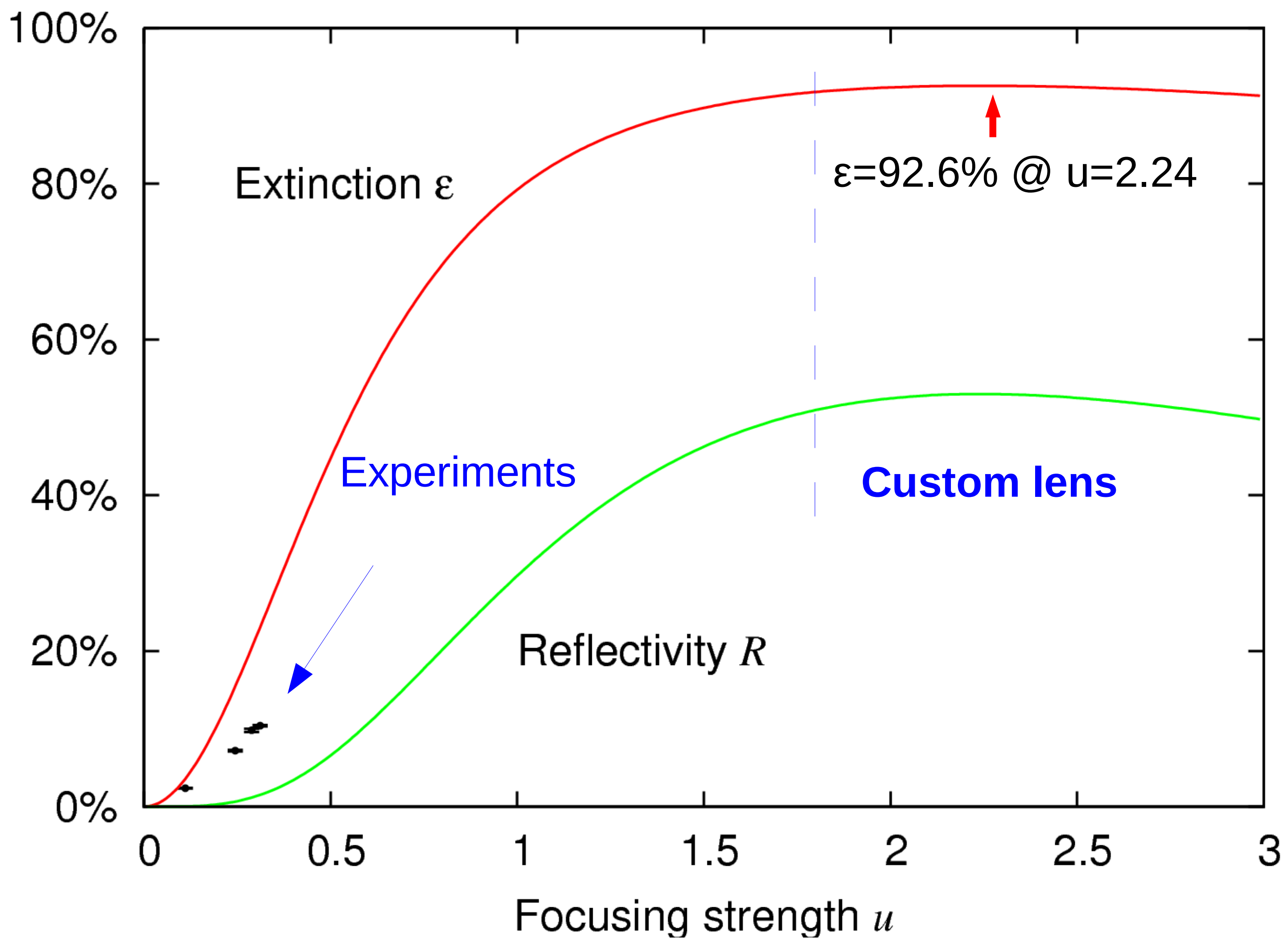
M. K. Tey, Z. Chen, S.A. Aljunid, B. Chng, F. Huber, G. Maslennikov, C. K.
nature physics ***4****, 924 (2008)*

Reflection & Transmission

(σ- probe)



How far does this go?



M.K. Tey, G. Maslennikov, T.C.H. Liew, et al., New J. Phys. ***11****, 040311 (2009)*

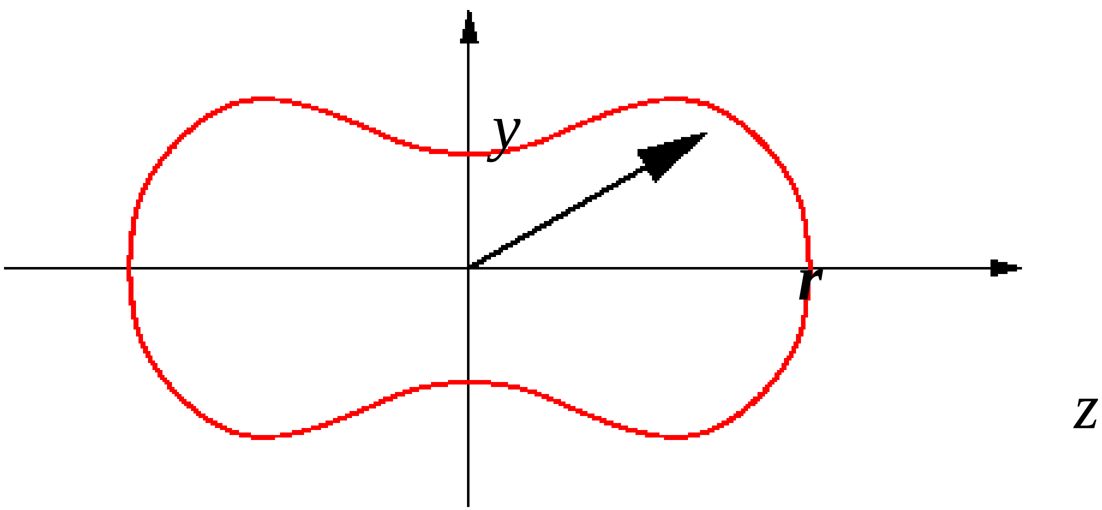
Off-resonant scattering

exciting field

natural line width

$$\boldsymbol{E}_{sc}(\boldsymbol{r},\Gamma) = E_A \frac{3}{2}\frac{e^{ikr+\pi/2}}{kr}\left[\hat{\epsilon}_+ - (\hat{\epsilon}_+\cdot\hat{r})\hat{r}\right] \cdot \frac{i\Gamma}{2\Delta+i\Gamma}$$

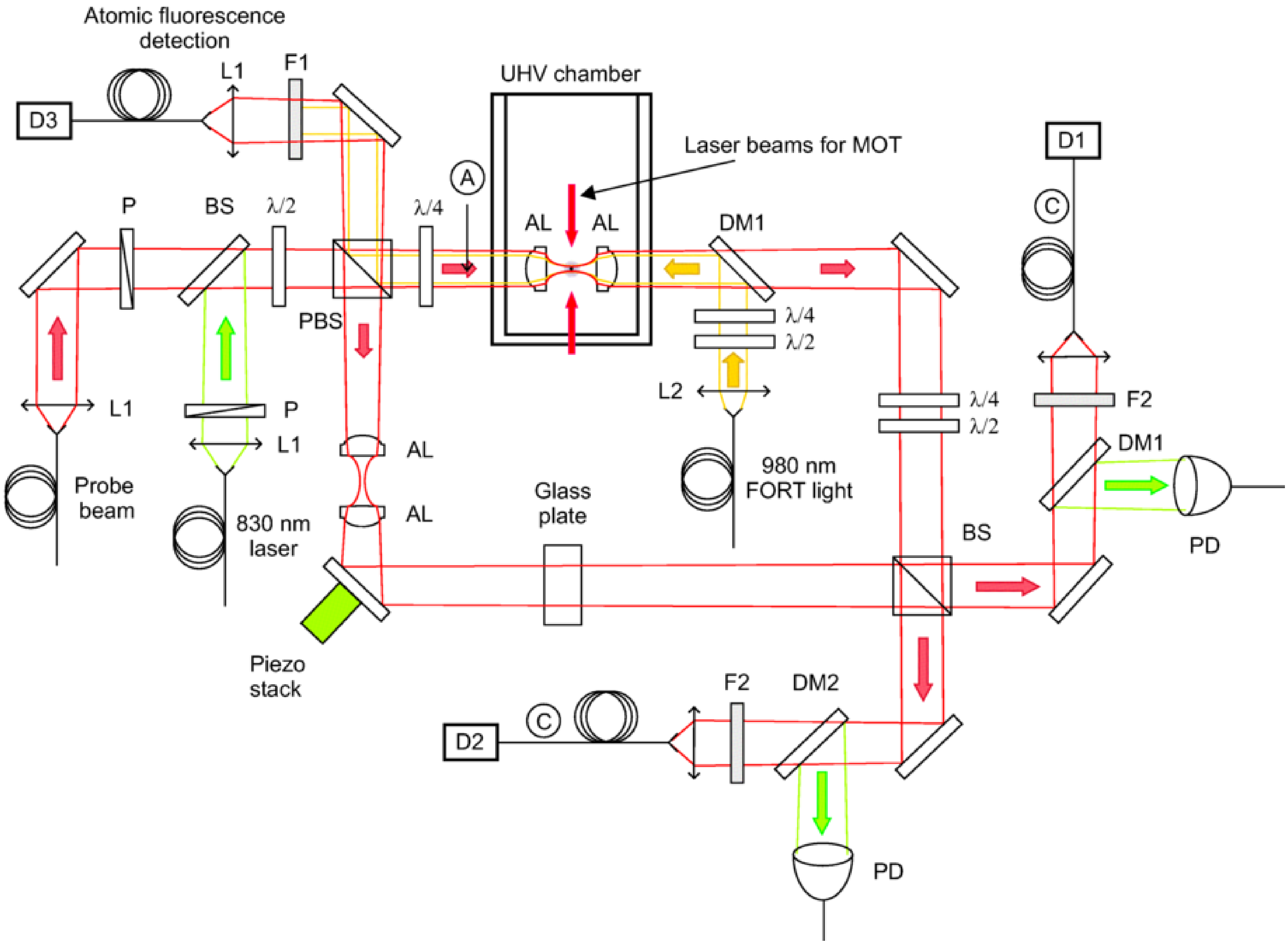
detuning



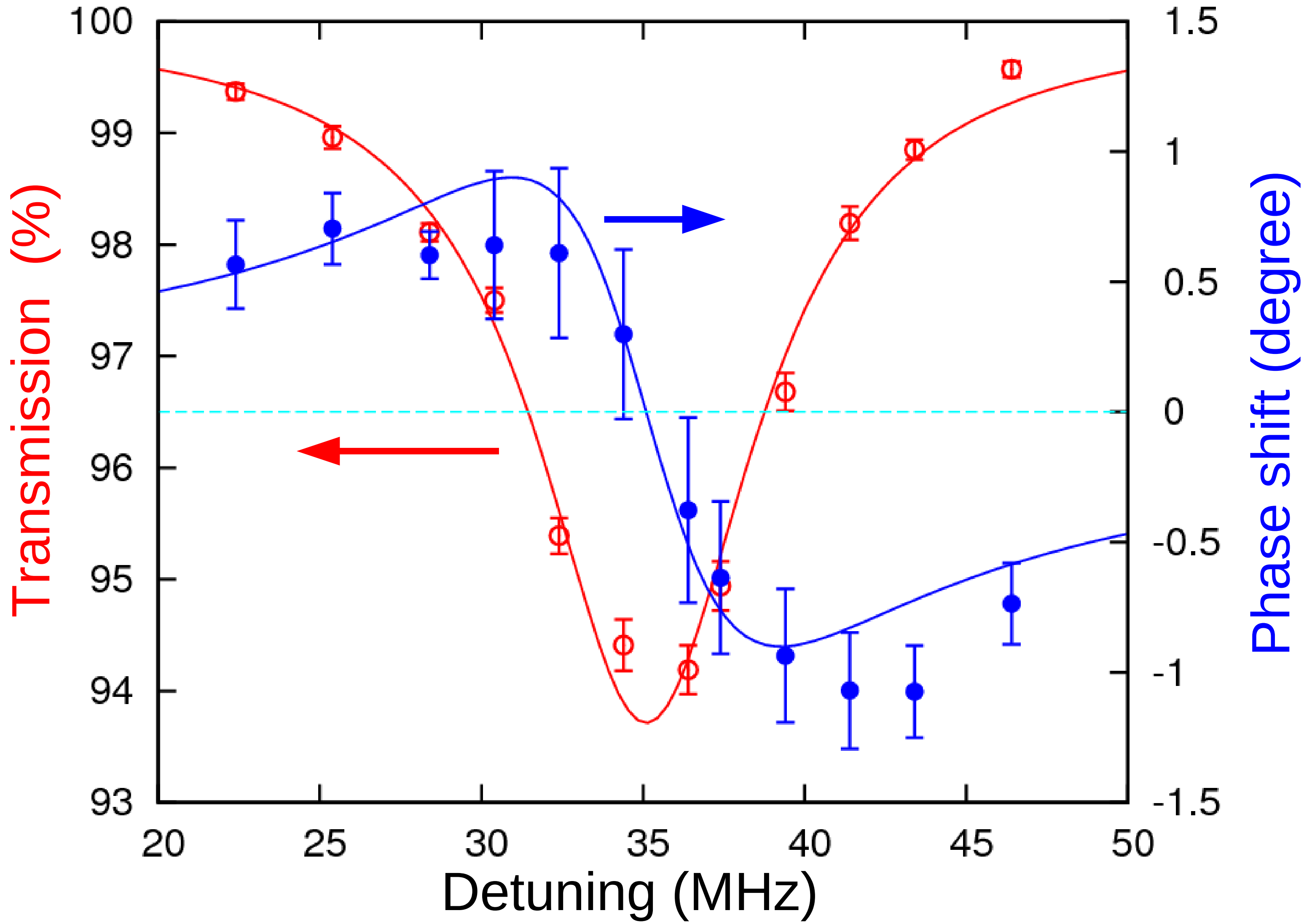
- Complex scattering amplitude leaves **phase shift** in combined excitation + scattered field

Phase shift measurement

Mach-Zehnder interferometer with one atom



Phase shift / Transmission

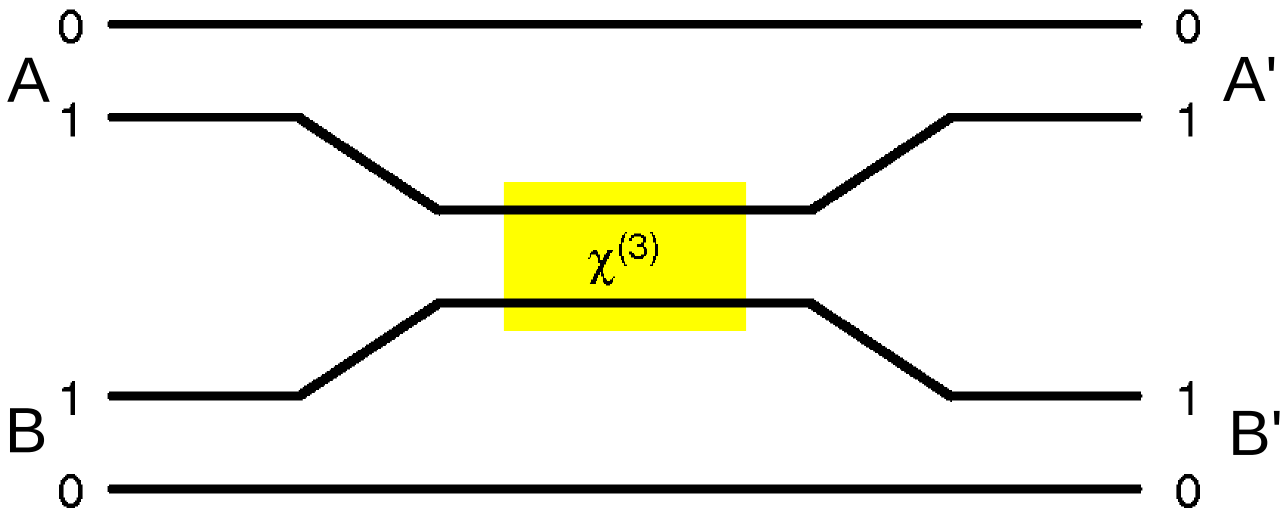


phase shift within factor 2..3 of prediction by stationary atom model!

*S.A. Aljunid et al. PRL **103**, 153601 (2009)*

Photonic Phase Gate Concept

- universal 2-qubit operations, require large optical nonlinearity



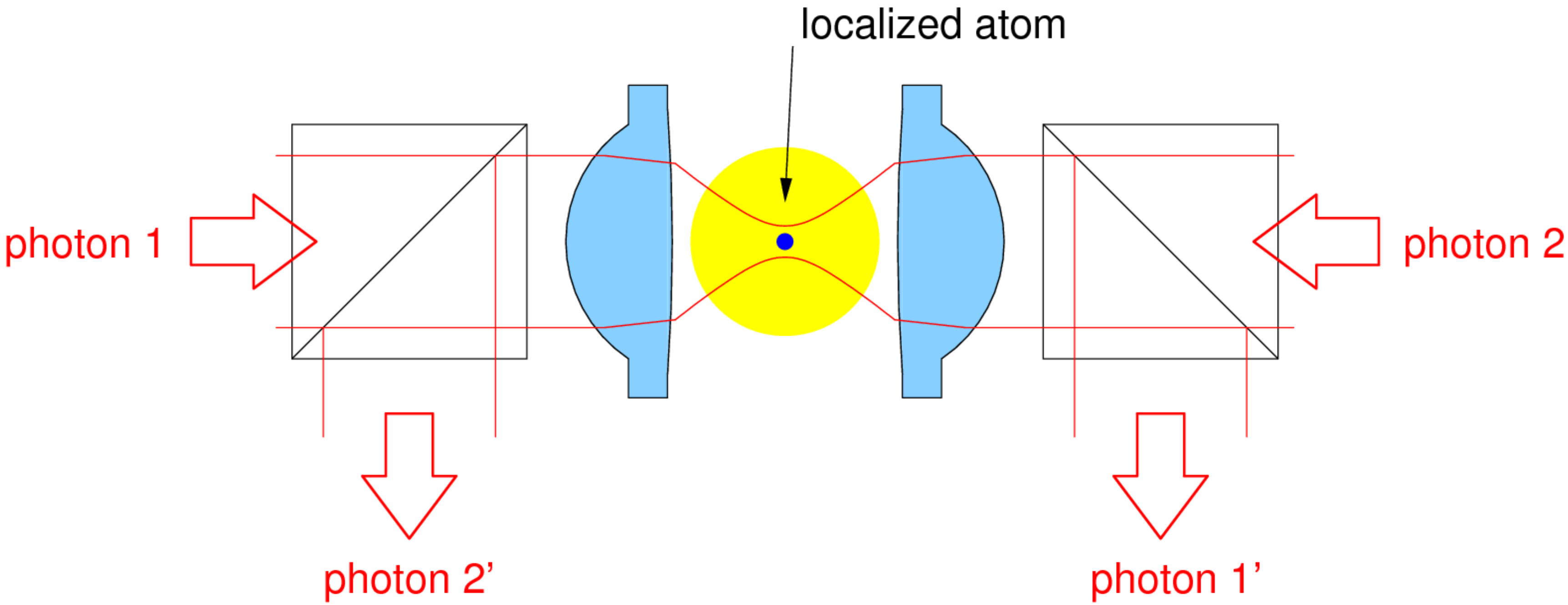
A, B	A', B'
0,0	0,0
0,1	0,1
1,0	1,0
1,1	(1,1) $e^{i\varphi}$

- hopeless with typical bulk nonlinearities
- possible with atoms close to resonance:

S. Harris & team, Stanford: atomic clouds
M. Lukin & team, Harvard: atoms in fibers

Dreams...

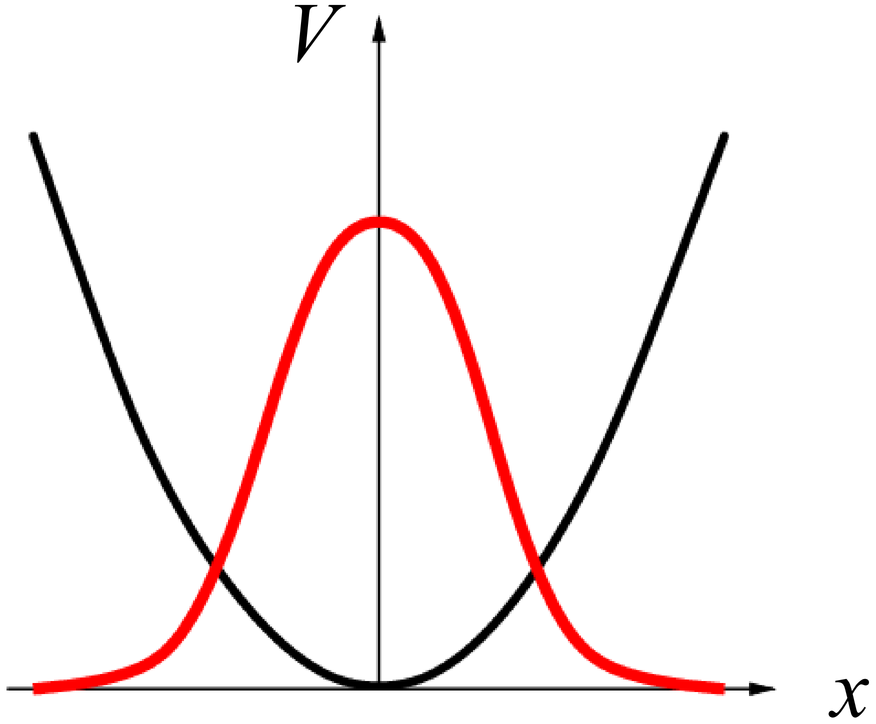
- Try to see conditional phase gate....



need photons with compatible bandwidth

Next steps in the real world

- Atom does not sit nicely in our trap:

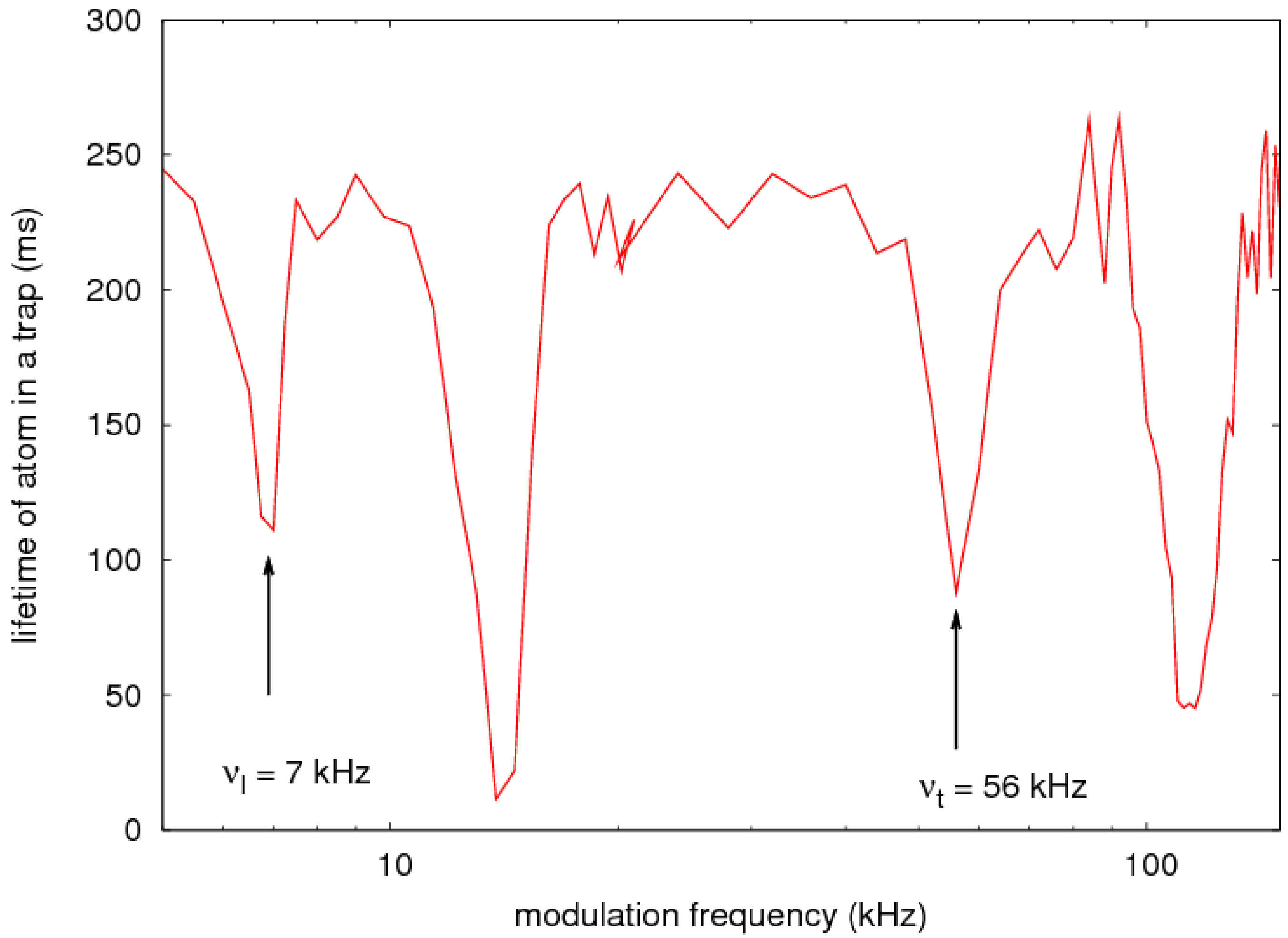


For T= 35 μK:

Δ x = 160 nm
Δ z = 1.3 μm

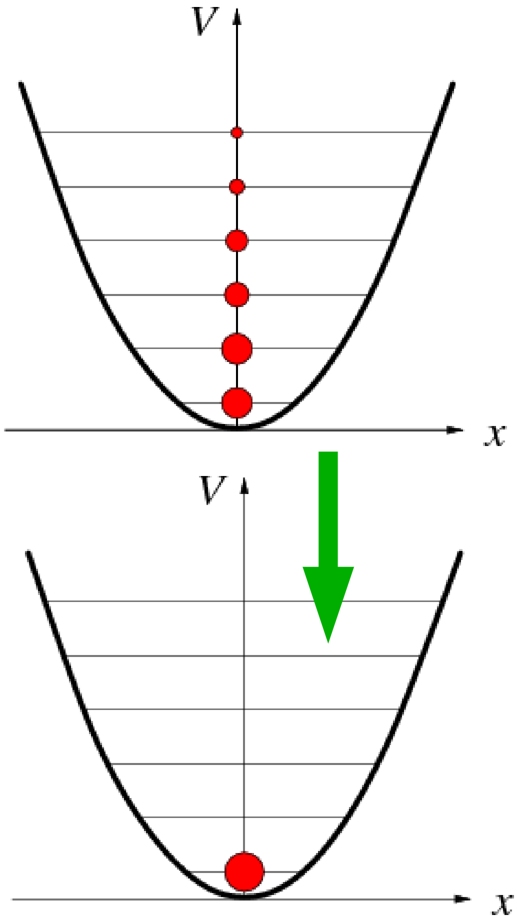
Motional Sidebands

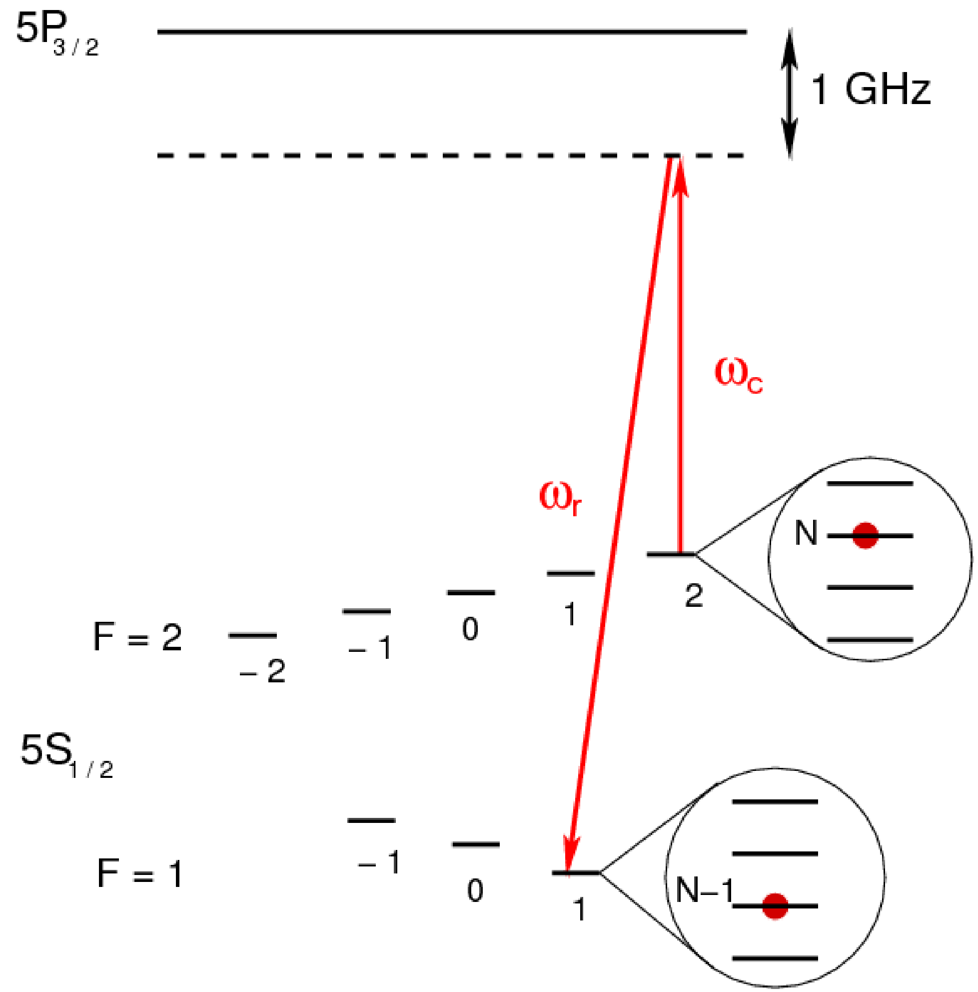
Find exact trap frequencies via parametric heating



Raman Sideband Cooling

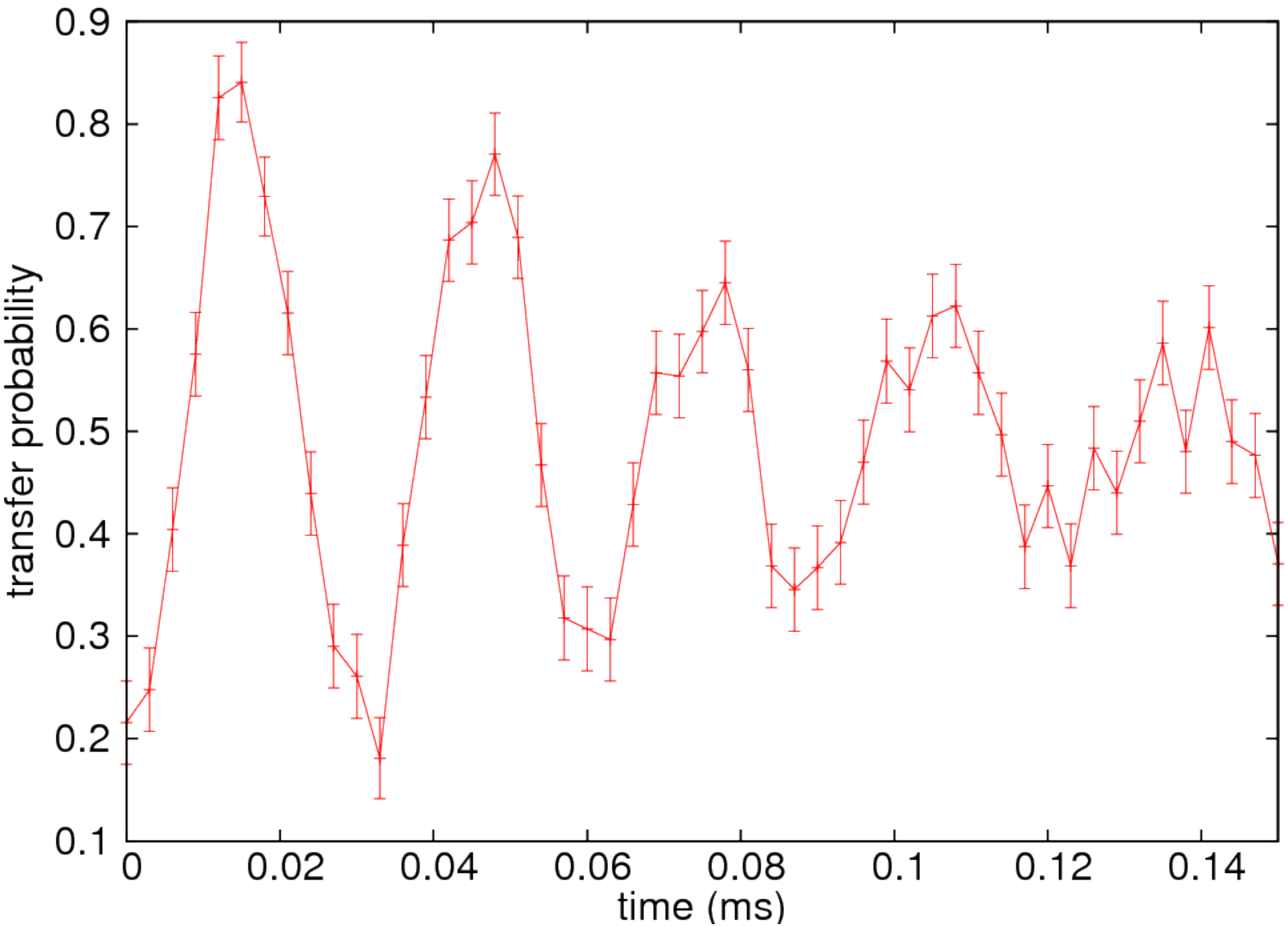
- Reduce vibrational quanta directly:





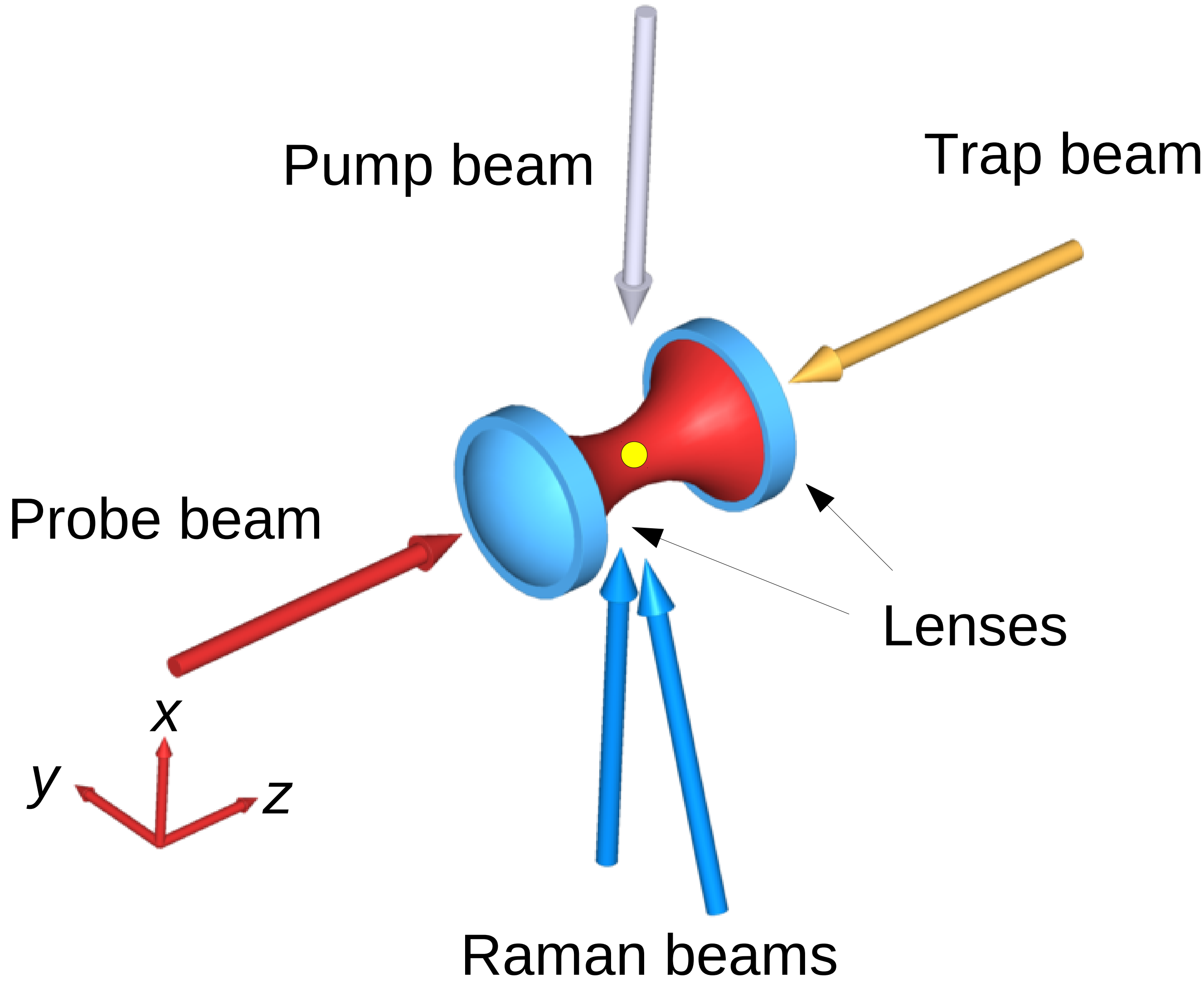
First steps: Raman transitions

Atom state manipulation : Raman Rabi oscillations

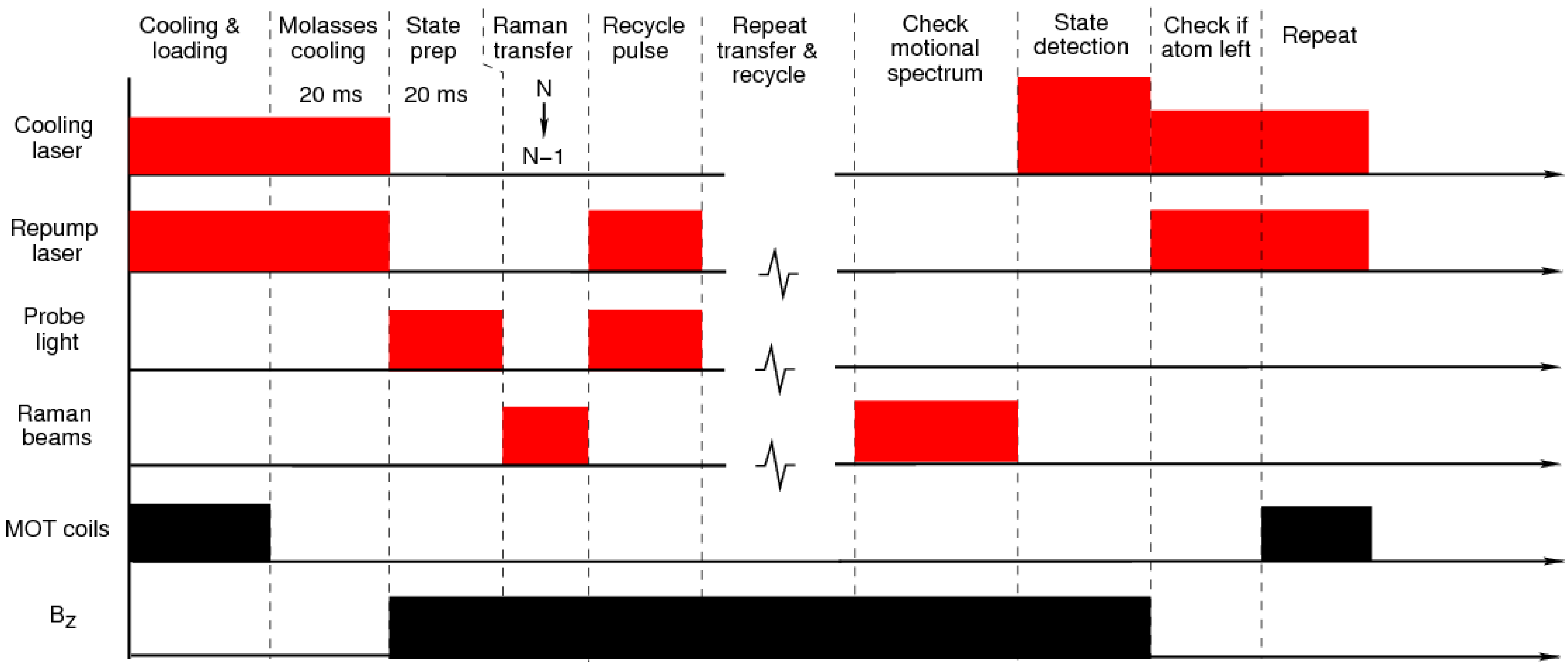


Decoherence time ~ 100 µs

Raman cooling geometry

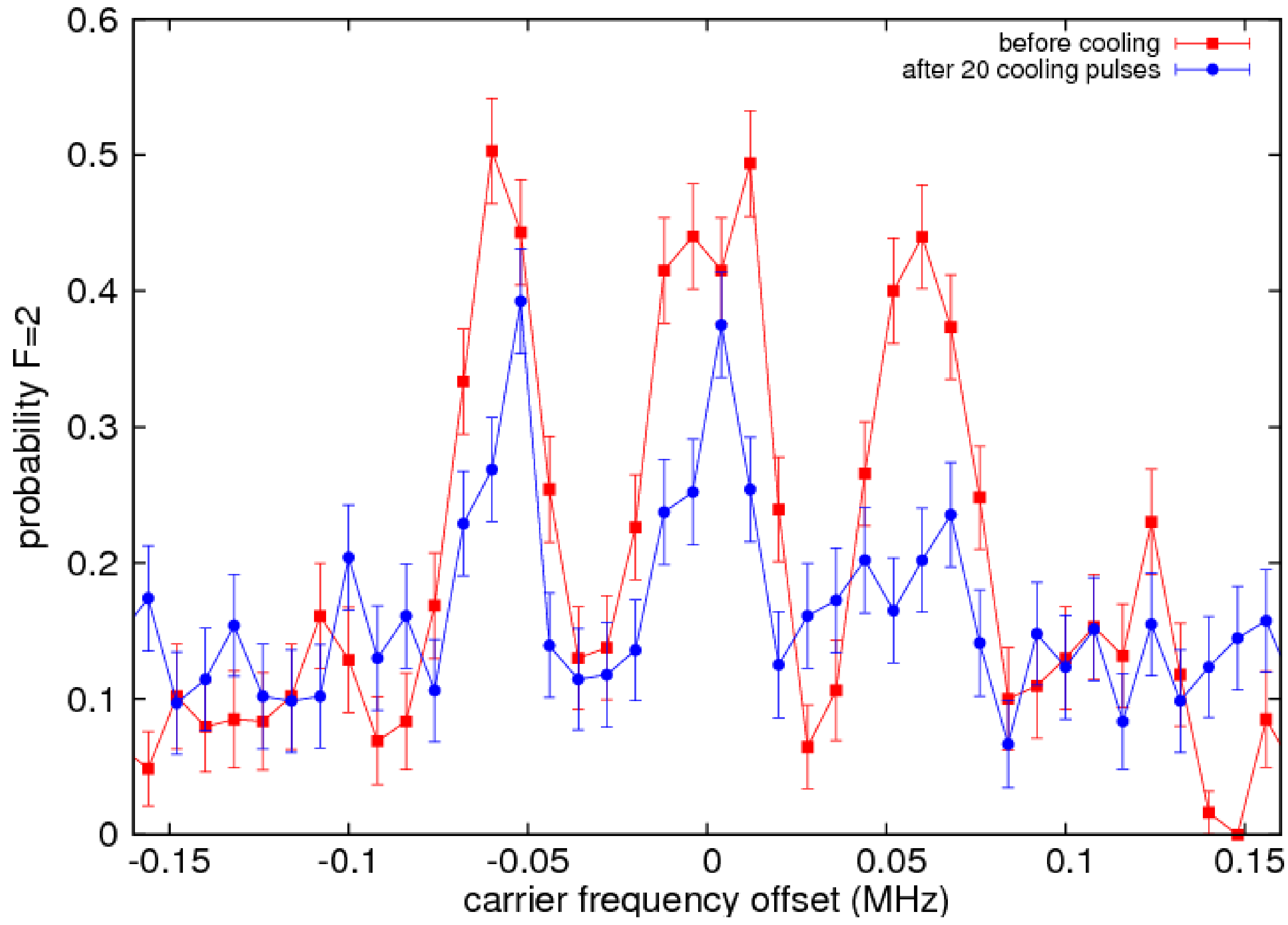


Cooling Sequence



Raman sideband transitions

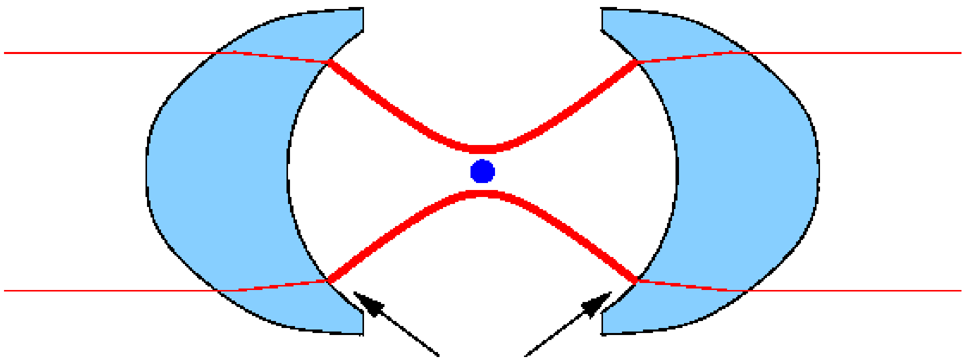
- Motional state manipulation of transverse motion



- From this data: $\langle \hat{n} \rangle \approx 0.5$

Combine focusing & cavity

- Get easier into “strong coupling” regime



S.E. Morrin, C.C. Yu,
T.W. Mossberg,
*PRL **73**, 1489 (1994)*

A. Haase, B. Hessmo,
J. Schmiedmayer,
*Opt. Lett. **31**, 268 (2006)*

Recently: many nice papers from Jakob Reichel group

Electrical field operator (single freq):

$$\hat{\boldsymbol{E}}(x,y,z)=i\sqrt{\frac{\hbar\omega\pi}{\epsilon_0 L 3\lambda^2}R_{sc}}\left(\boldsymbol{g}(x,y,z)\hat{a}^+-\boldsymbol{g}^*(x,y,z)\hat{a}\right)$$

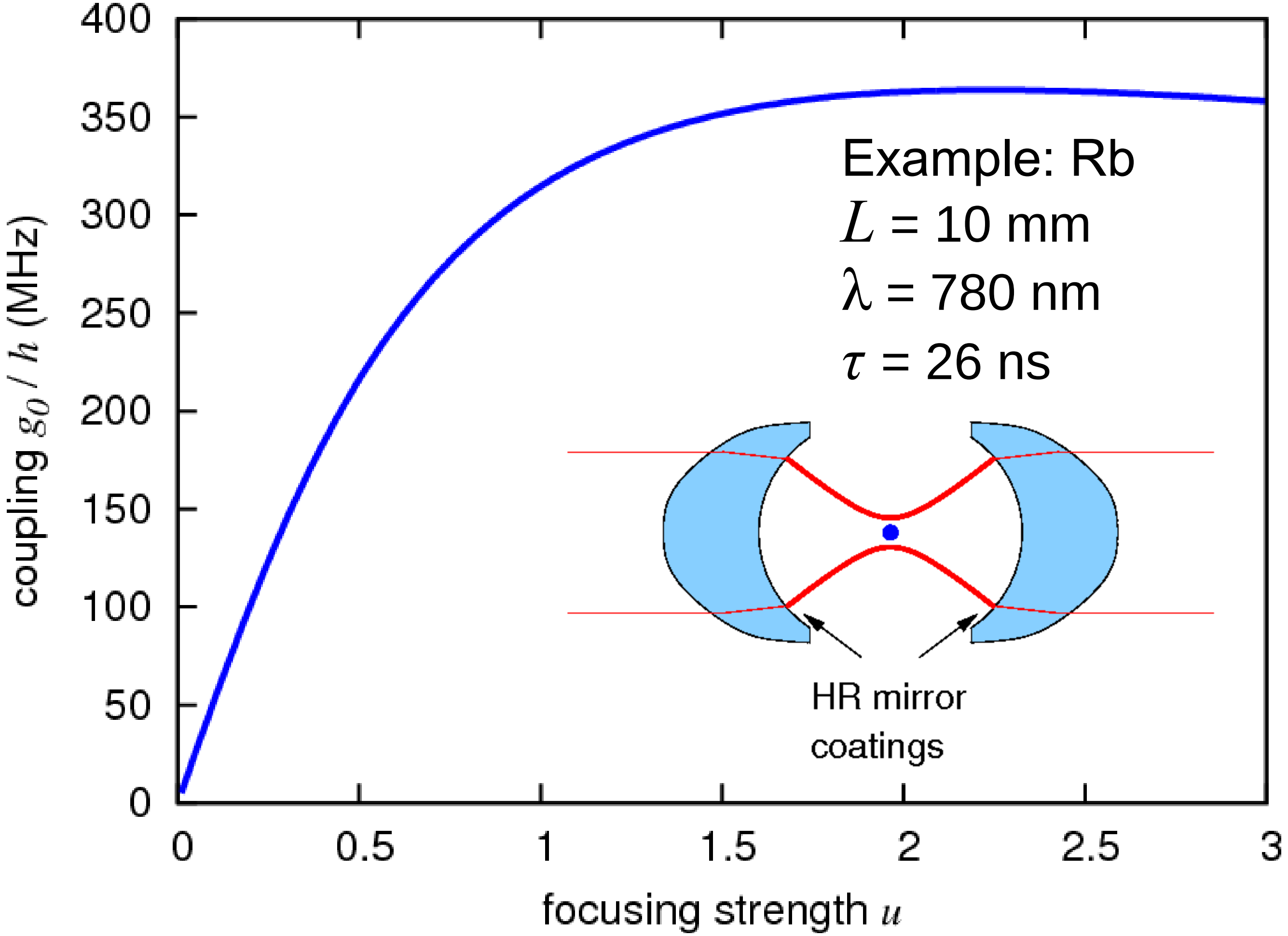
Scattering ratio, 0...2

mode function, g=1 at focus

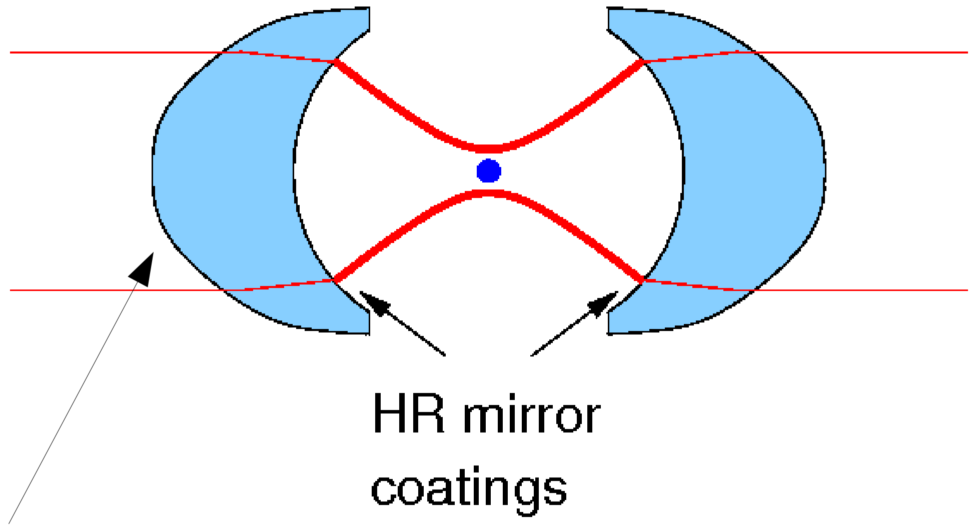
Effective mode volume: $V=L\lambda^2/R_{sc}$

Weak cavity – strong coupling?

Coupling strength: $g_0 = \hbar\sqrt{\frac{\pi c R_{sc}}{\tau L}}$



Coupling to outside modes



Ideal “anaclastic” lens with ellipsoidal surface:

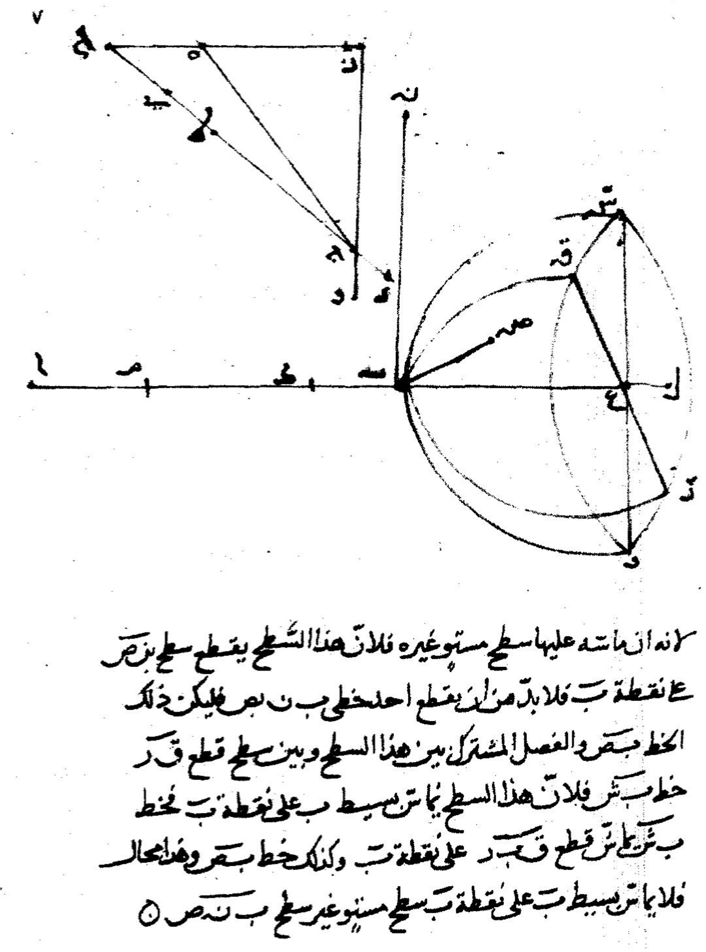
Half axis in longitudinal direction: $fn/(n+1)$

Half axis in radial direction: $f\sqrt{(n-1)/(n+1)}$

S.A. Aljunid, B. Chng, J. Lee, K. Durak et al., J. Mod. Opt. ***58****, 299 (2011)*

Not exactly a new idea...

- Ibn Sahl, ~ 984: optimal focusing



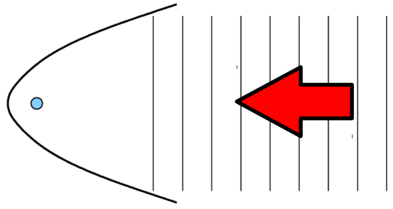
- Today's version of an anaclastic lens

Related work

- Interaction with molecules
  Vahid Sandoghdar group - ETHZ, now MPL Erlangen

- Interaction with quantum dots
  Atac Imamoglu group - ETHZ

- Larger solid angle: ion trap in parabolic mirror
  Gerd Leuchs Group - MPL Erlangen



Large mode overlap
with π transition

- Fiber cavities for small transverse optical modes
  Jakob Reichel group - LKB

Comparison to cavity QED

- Could strong focusing replace cavities for strong coupling?

  Probably not: imperfect mode match
  Gaussian modes --- atomic dipole modes

- Can strong focusing help in cavity QED experiments?

  Probably yes: field enhancement by focusing
  can lower cavity finesse for a given coupling strength

- What is the balance of technical problems?

  high NA lenses vs. high finesse mirrors (similar effort?)

End of Part 1 - Thank you!

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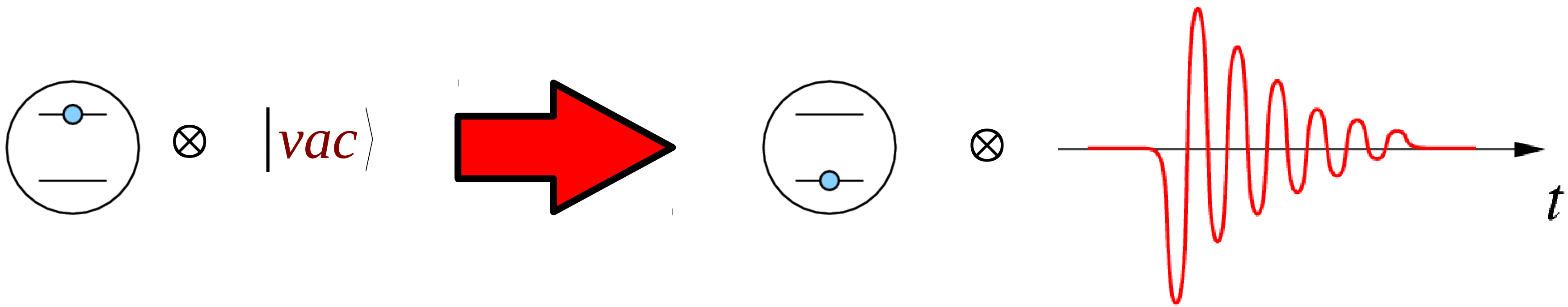
Former members:
Meng Khoon Tey (now UIBK)
Timothy Liew (now Lausanne)

Spontaneous Emission

- Weisskopf-Wigner solution: excited atom at *t=0*



$$|\Psi(t)\rangle = a(t)|e\rangle|vac\rangle + \int d\rho\, b_\rho(t)|g\rangle|n_\rho=1, n_{\rho'\neq\rho}=0\rangle$$

$$a(t) = e^{-\gamma t/2}, \qquad \gamma = 1/\tau$$

$$b_\rho(t) = \frac{w_{eg}^{\rho}}{\hbar} \cdot \frac{e^{-\gamma t/2} - e^{i(\omega_\rho - \omega_{eg})t}}{i\gamma/2 + \omega_{eg} - \omega_\rho}$$

Warning: Not Original symbols

*V. Weisskopf and E. Wigner, Proc. Roy. Soc. London (A), **114**, 243, 710 (1927)*

Asymptotic Weisskopf-Wigner

- For $t \gg \tau$, field and atomic excitation separate, and we have a field state

$$\left|\Psi_F(t)\right\rangle = \left(\int d\rho\, b_\rho(t)\,\hat{a}_\rho^+\right)\left|vac\right\rangle =: \hat{A}^+(t)\left|vac\right\rangle$$

$$b_\rho(t) = \frac{w_{eg}^\rho}{\hbar}\cdot\frac{e^{-\gamma t/2} - e^{i(\omega_\rho - \omega_{eg})t}}{i\gamma/2 + \omega_{eg} - \omega_\rho} \rightarrow \frac{w_{eg}^\rho}{\hbar}\,\frac{-e^{i\Delta t}}{i\gamma/2 - \Delta}$$

detuning

Mode index $\rho = (k, m)$ is over spherical waves with a far field (for $r \gg 2\pi/k$):

$$\vec{g}_\rho(r,\theta,\phi) \propto \Re\left[\frac{e^{-ikr}}{kr}\right]\vec{\epsilon}_r \times \vec{X}_{1,m}(\theta,\phi)$$

Vector spherical harmonics

More spontaneous emission

- The field state $\hat{A}^{+}(t)|vac\rangle$ is what is left after the atom lost its excitation.

  Let's call it the spontaneously emitted photon.

- It has a quadrature electrical field component

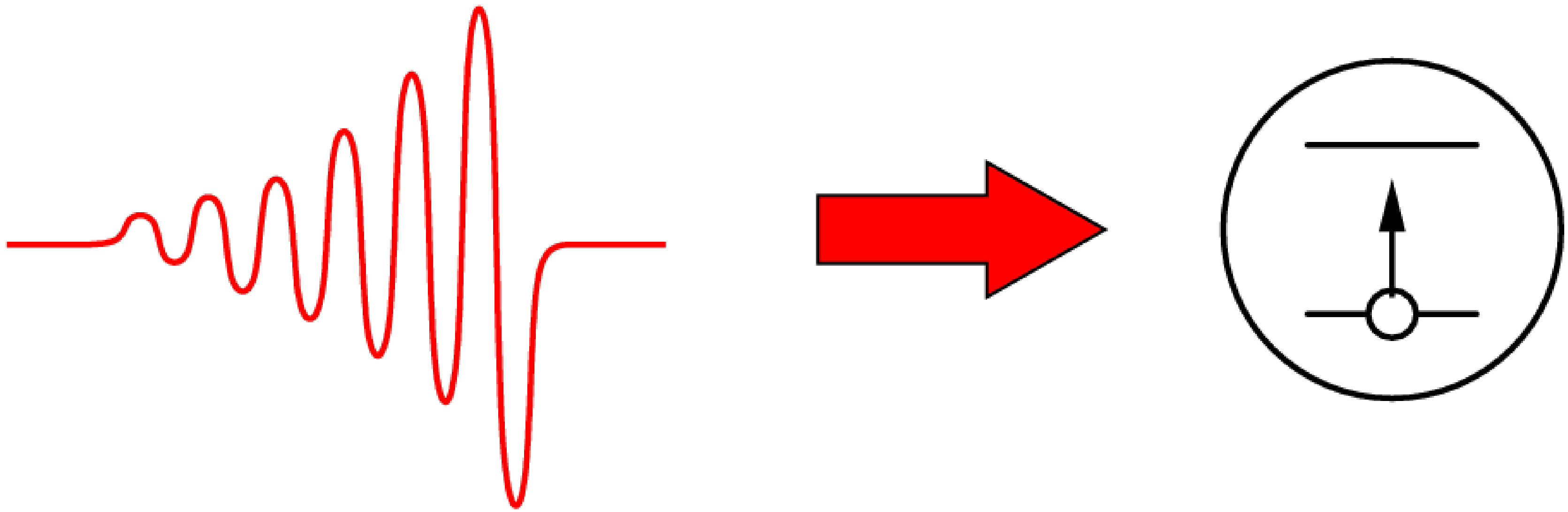
$$\vec{E}(r,\theta,\phi,t) \propto \Re\left[\frac{e^{-ik_0 r}}{k_{0r}}\right] e^{-\frac{\gamma}{2c}(ct-r)} \Theta(ct-r)\vec{\epsilon}_r \times \vec{X}_{1,m}(\theta,\phi)$$

Step function

which looks like the classical field emitted by a damped oscillating electrical dipole.

Reverse Spontaneous Emission

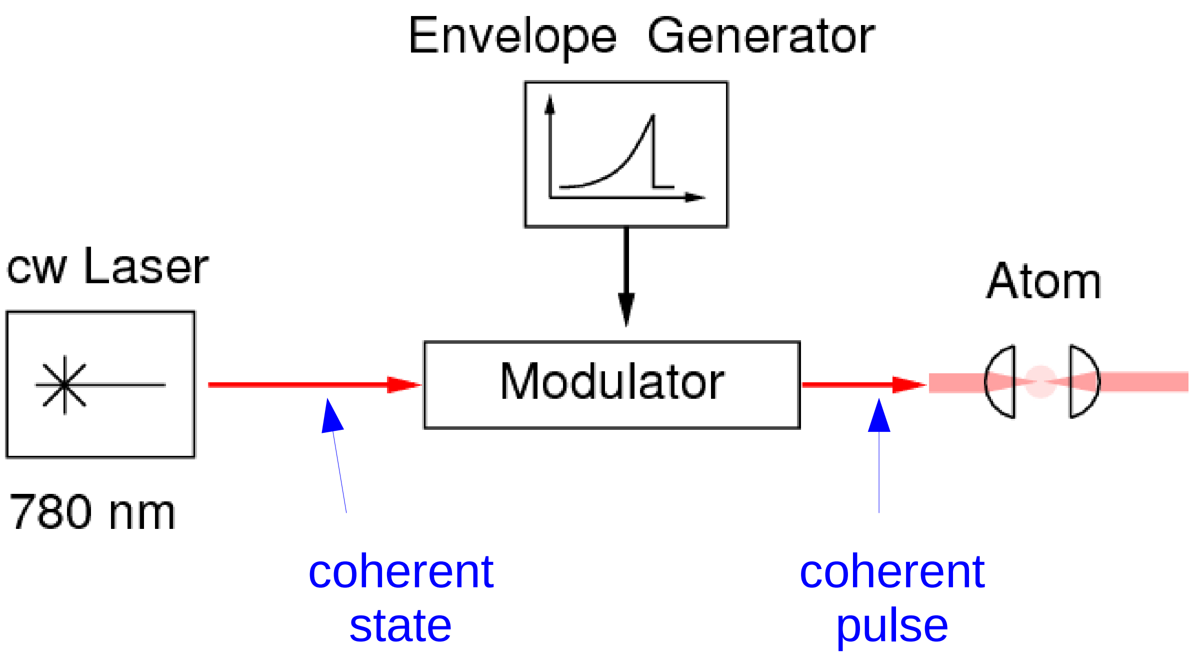
- Optimal absorption process:
  Time-reversed Wigner-Weisskopf solution
- Requires photon with a shaped mode



M.Sondermann, R. Maiwald, H. Konermann et al. Appl. Phys. B ***89****, 489 (2007)*

Creating a “reverse” photon

- A single field excitation is difficult to make
- Let's start with shaping a coherent state

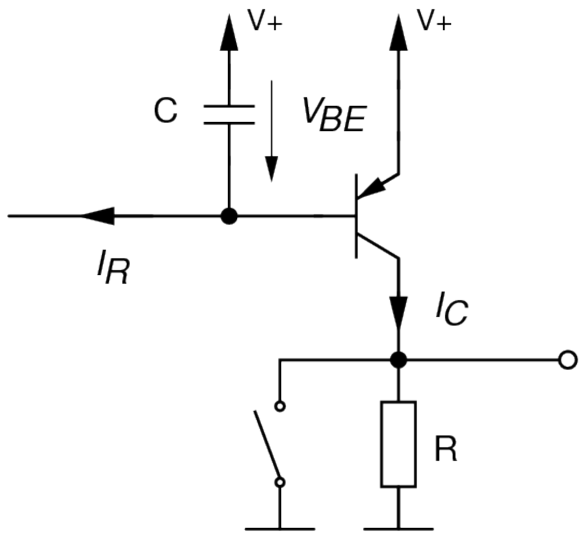


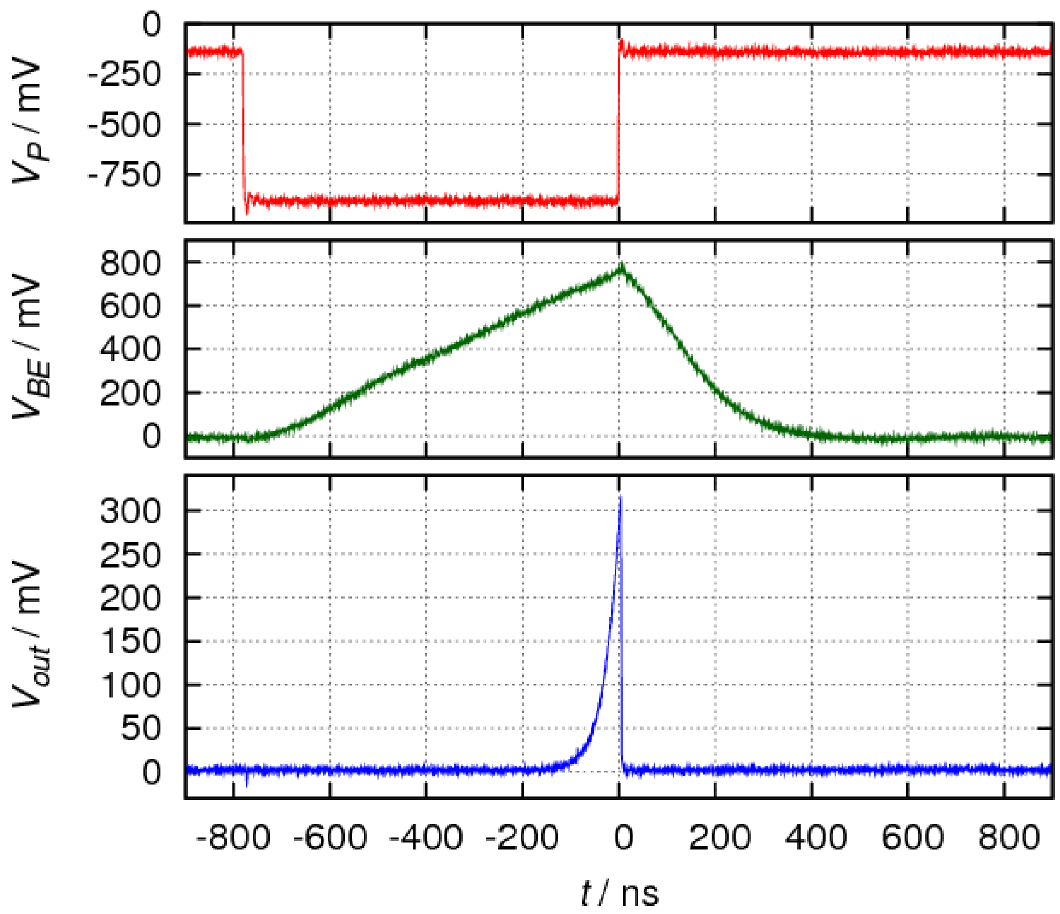
Y. Wang, L. Sheridan, V. Scarani, Phys. Rev. A ***83****, 063842 (2011)*

Generation of Envelope

- Linear slope, use transistor transfer function

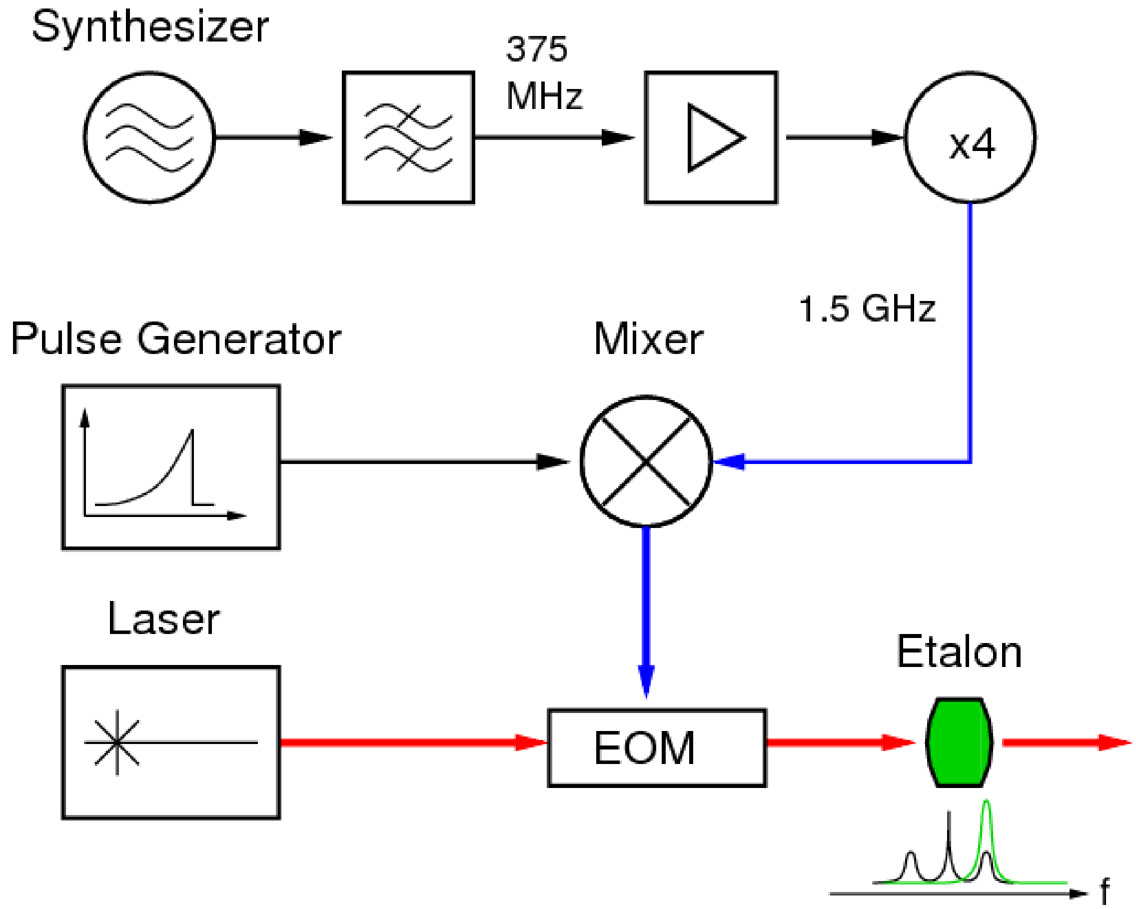
$$I_C = I_0\left(e^{eV_{BE}/kT} - 1\right) \approx I_0 e^{V_{BE}/V_T}$$



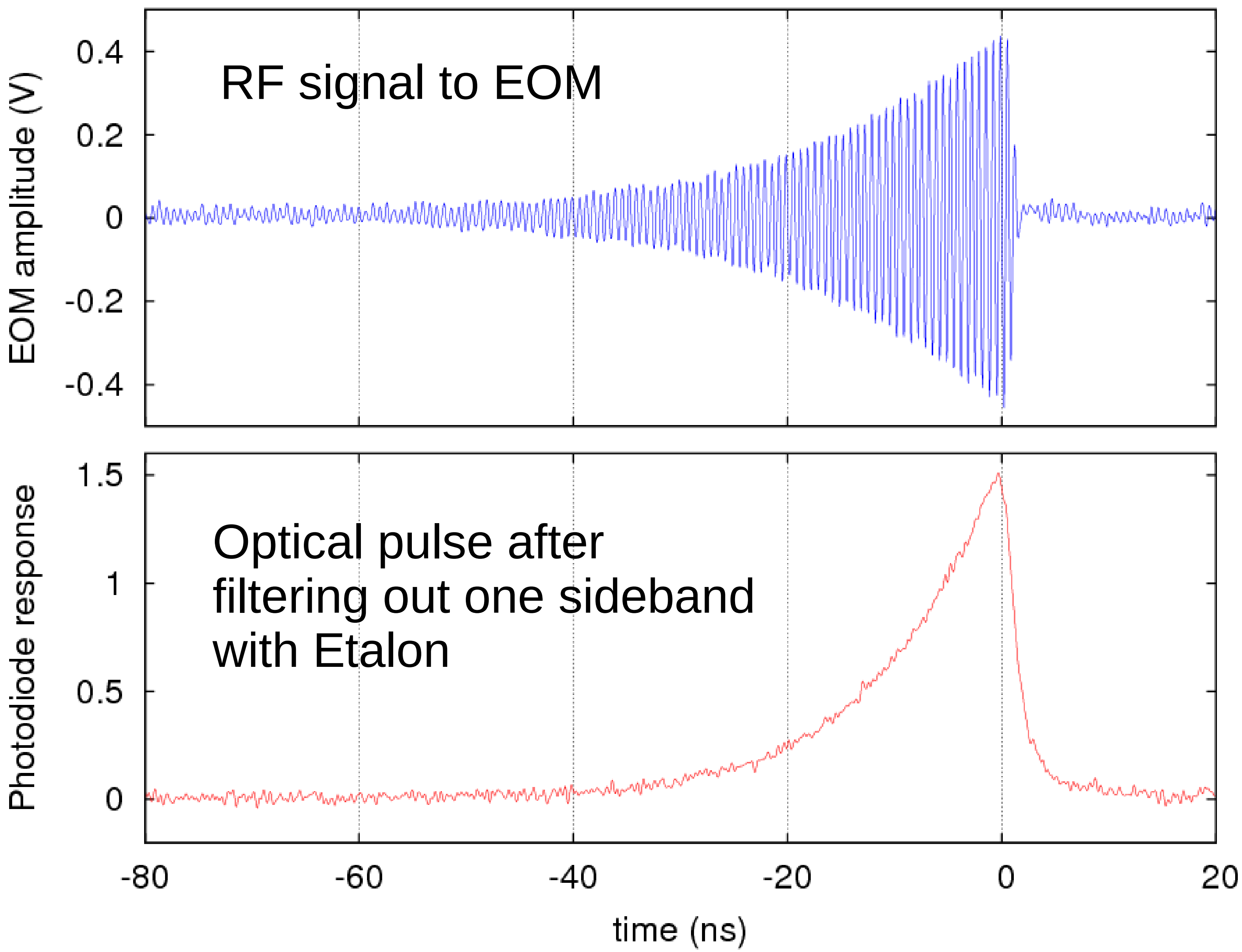


Generation of optical pulse I

- Generate electrical pulse
- Modulate RF carrier
- Generate optical sideband with EOM and filter with Etalon

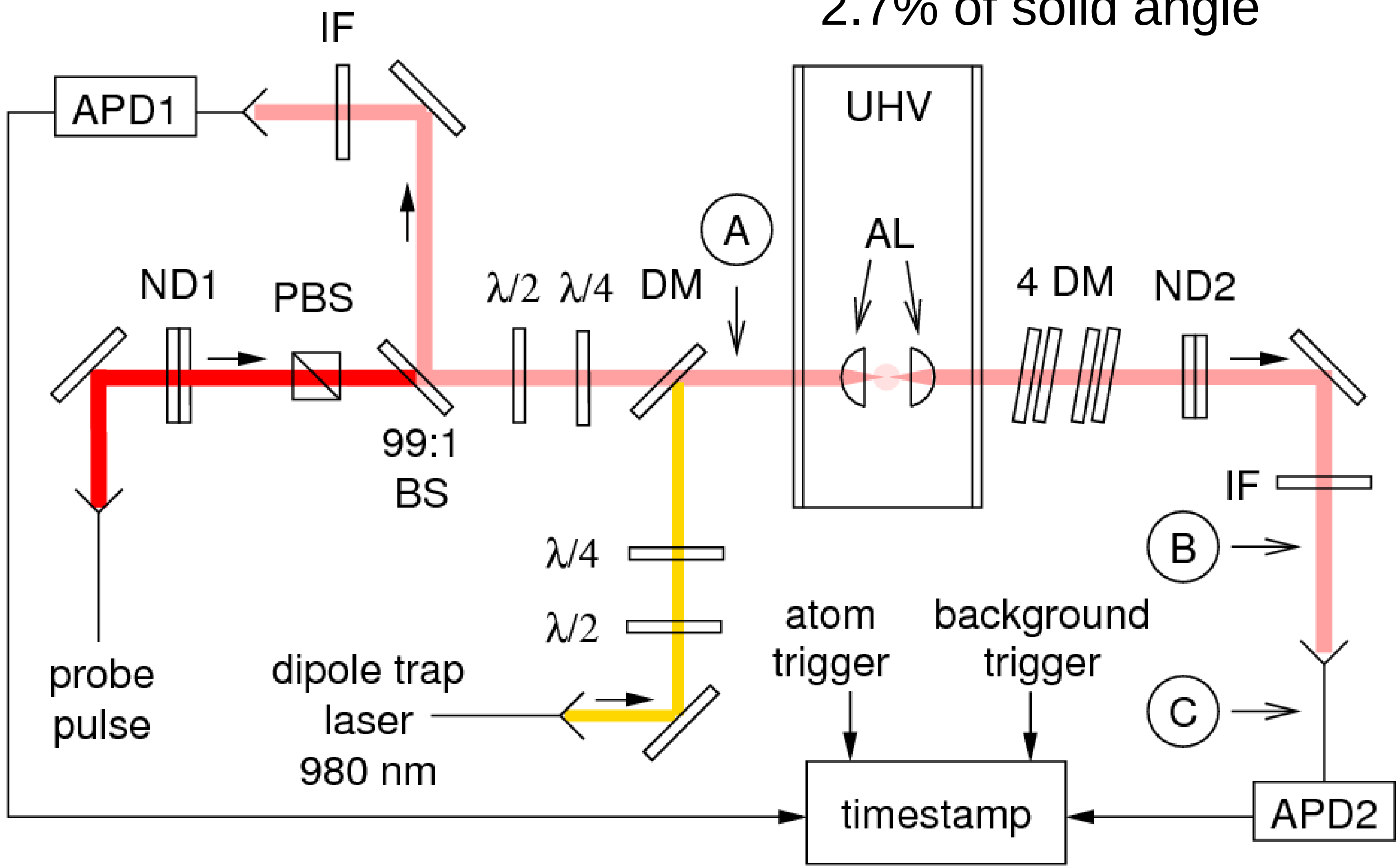


Generation of optical pulse II

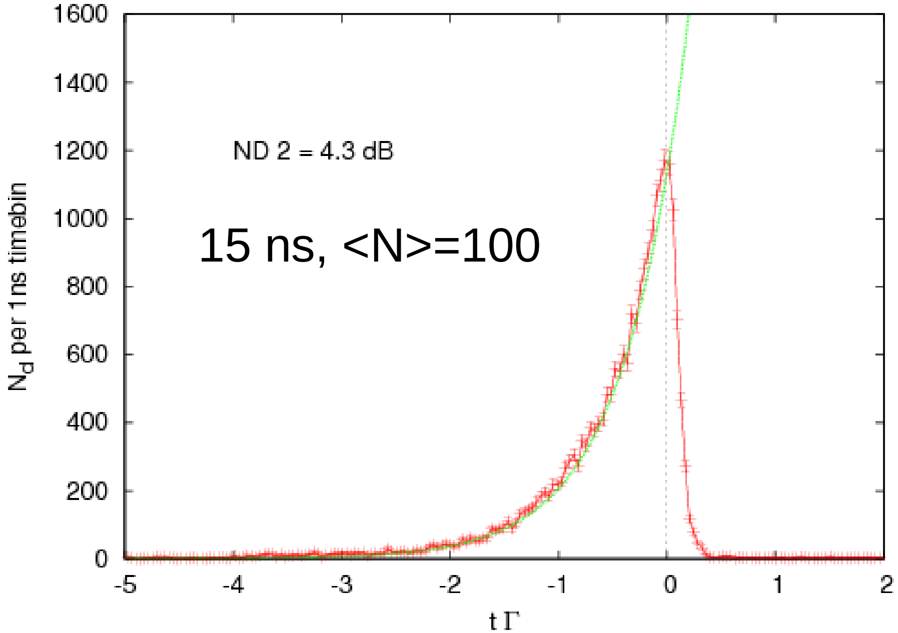


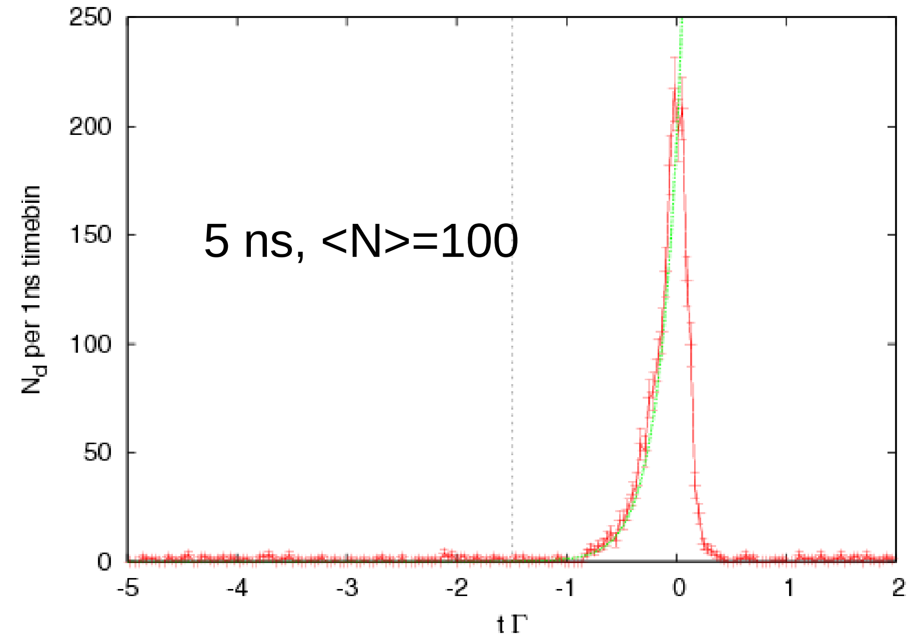
Experimental setup

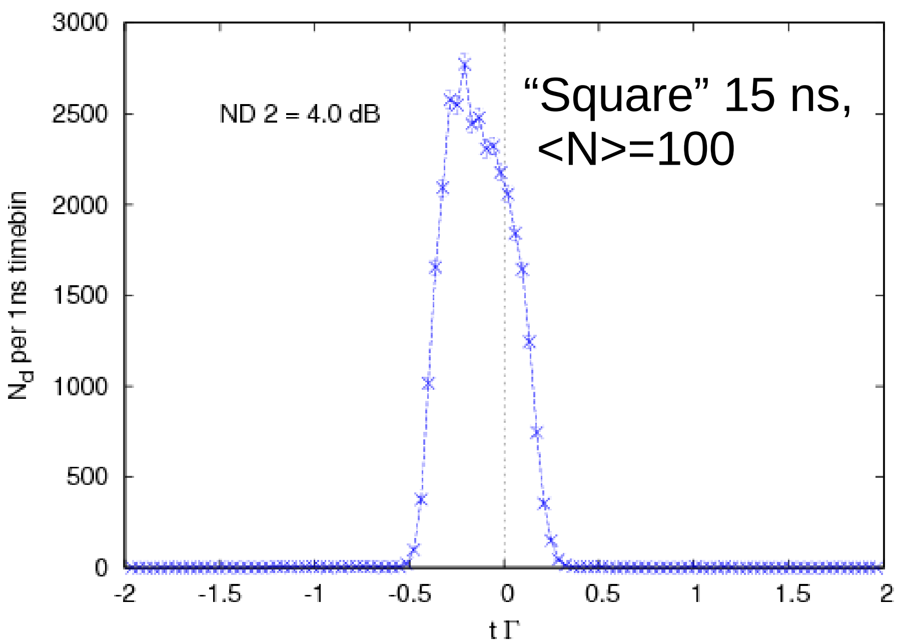
- Capture of fluorescence: 2.7% of solid angle

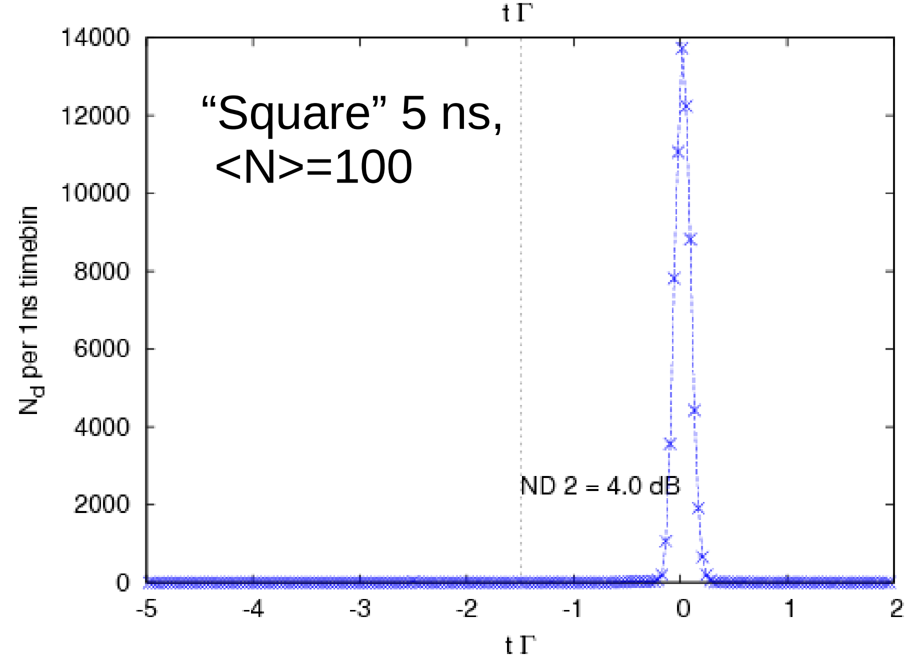


Different Pulse Shapes

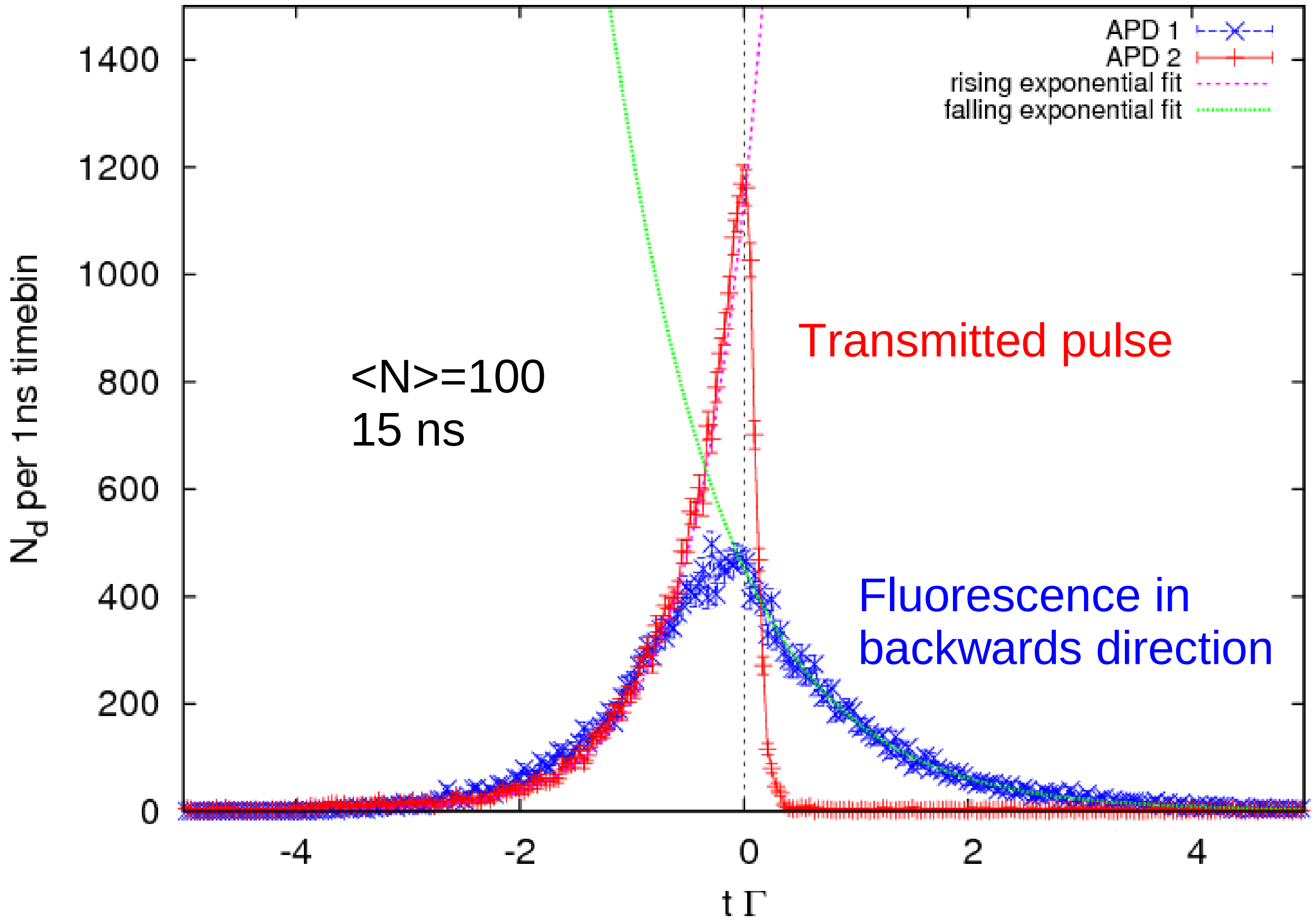






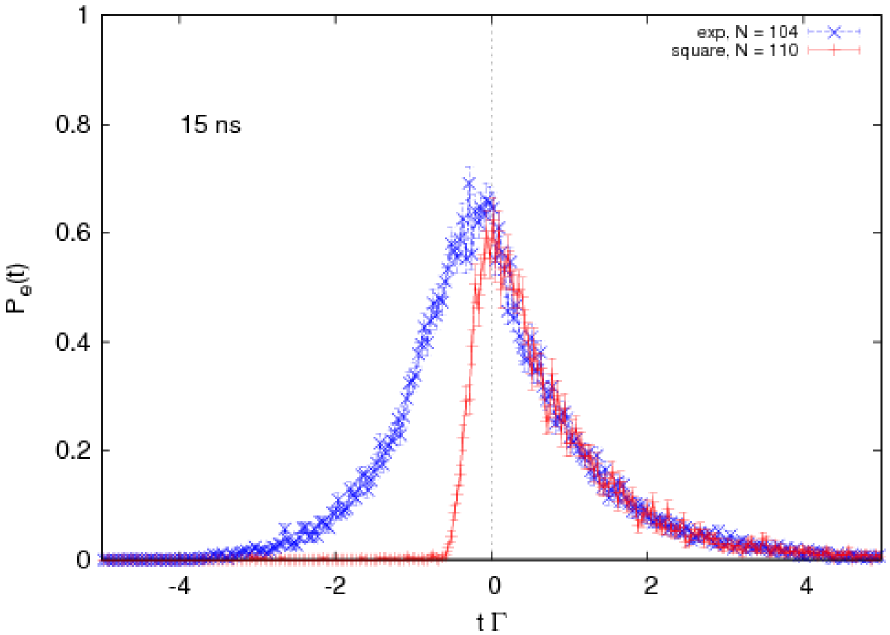


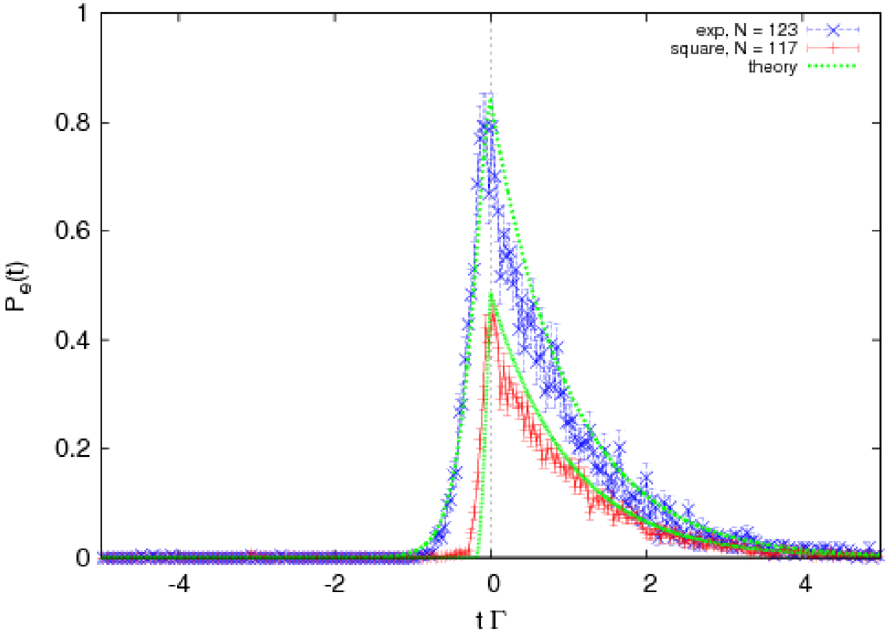
Atomic fluorescence

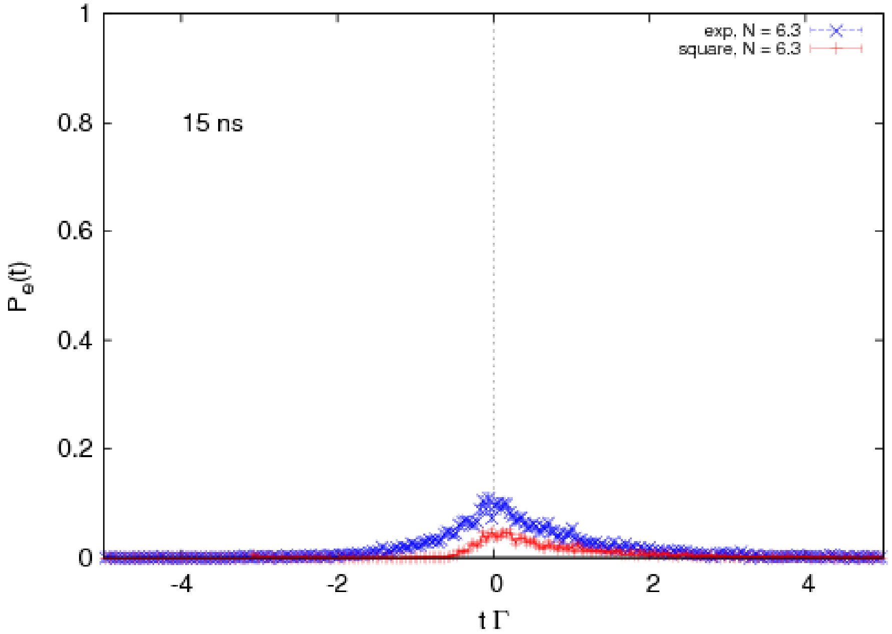


- Excitation probability rises and falls exponentially

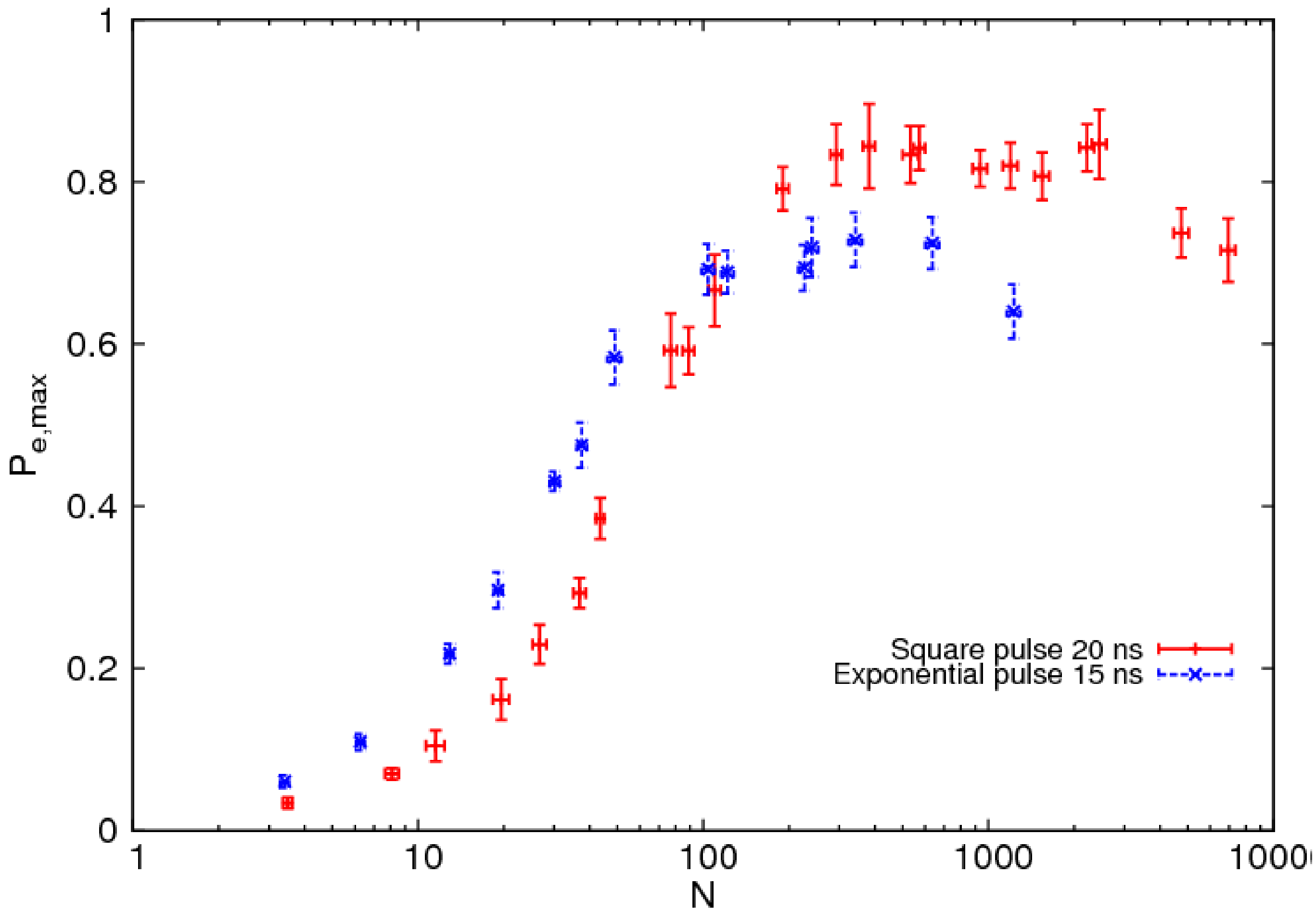
Calibrate excitation probability





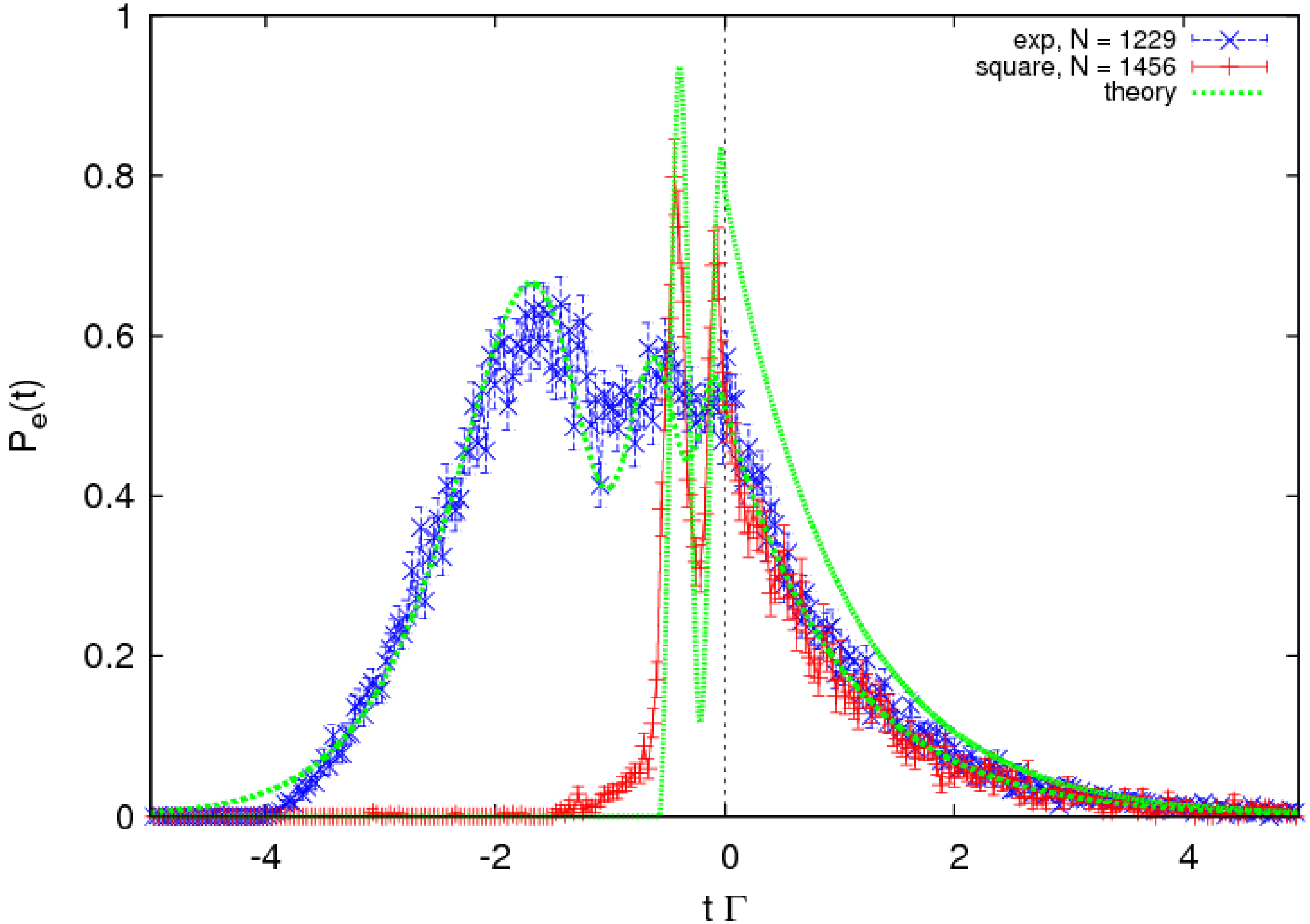


Systematics



- Saturation of excitation with ~ 100 photons
- Rising exponential pulse shale does (a bit) better than square

Stronger fields



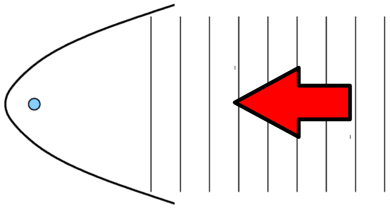
- Onset of Rabi oscillations

Conclusion

- We are still far from exciting with a single photon
- Continuous excitation and subsequent decay is expected
- Limited spatial mode overlap is main obstacle for seeing strong excitation

Outlook

- Large spatial overlap should help a lot!



Large mode overlap
with π transition

- Shaped single photon pulses still an open problem

  Solved with 3-level cavity-QED systems and
  Raman transitions?

End of Part 2 - Thank you!

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