

# *Substantial scattering of photons by a Single Atom*

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Centre for  
Quantum  
Technologies



## **Motivation:**

- Atoms and photons are good for different quantum information tasks – allow an exchange of quantum information between them
- Understand elementary interaction between flying qubits and single atoms
- Explore possibilities of controlled phase gates & friends for photonic qubits

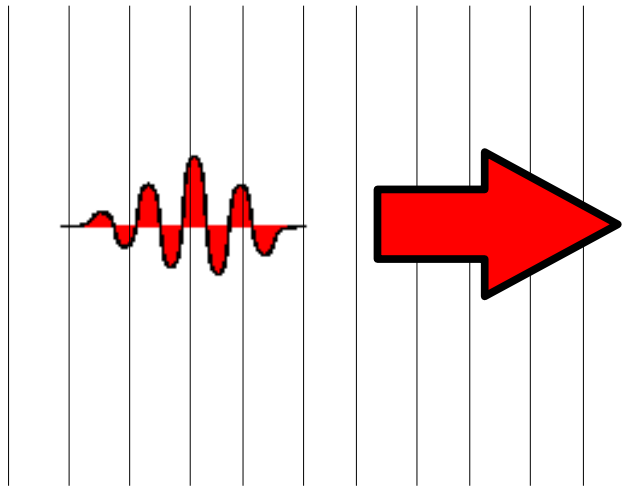
## **Key idea:**

- Try to **mode-match** traveling qubit modes to field modes of spontaneous emission of a single atom

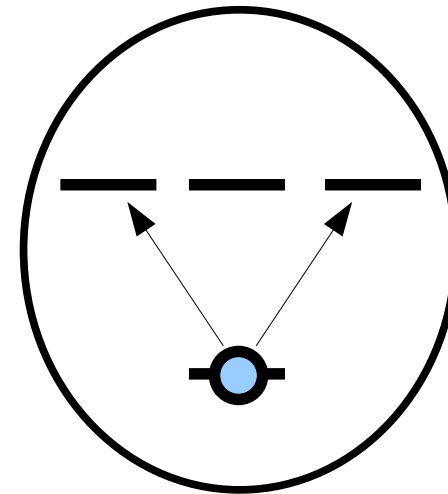
# Why is this interesting?



- e.g. transfer of information from flying qubits into a quantum memory



$$|\Psi_L\rangle = \alpha|L\rangle + \beta|R\rangle$$



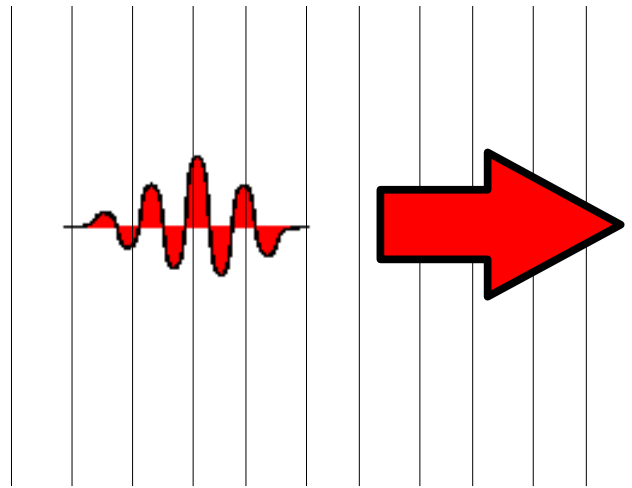
$$|\Psi_A\rangle = \alpha|m=-1\rangle + \beta|m=+1\rangle$$

- requires internal states of atom and an **absorption process**

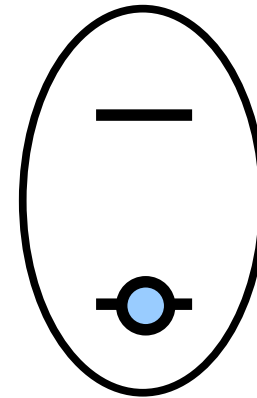
# *The basic problem*



- Get strong coupling between an atom and a light field on the single photon level

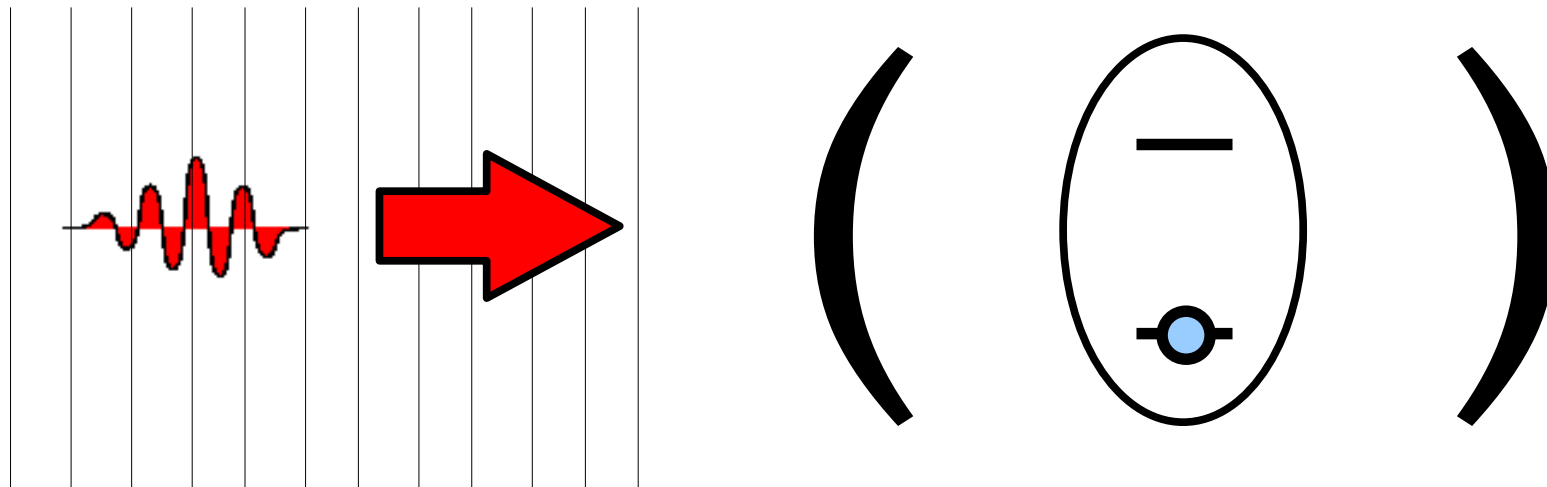


**electromagnetic field / photon**



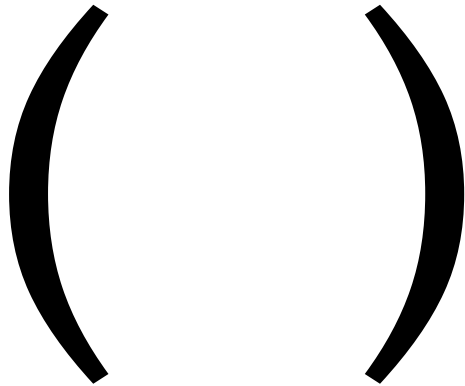
**2-level atom**

# *One solution: Use a cavity*



- High electrical field strength even for a single photon
- Preferred spontaneous emission into the cavity mode
- A cavity can enhance the interaction between a propagating external mode and an atom

# Why cavities are nice



- It's clear what photons in a cavity are  
discrete mode spectrum, 'textbook' energy eigenstates for the electromagnetic field

$$\hat{H}_{field} = \frac{\epsilon_0}{2} \int (\hat{\mathbf{E}}^2 + c^2 \hat{\mathbf{B}}^2) dV = \hbar \omega (\hat{n} + \frac{1}{2})$$

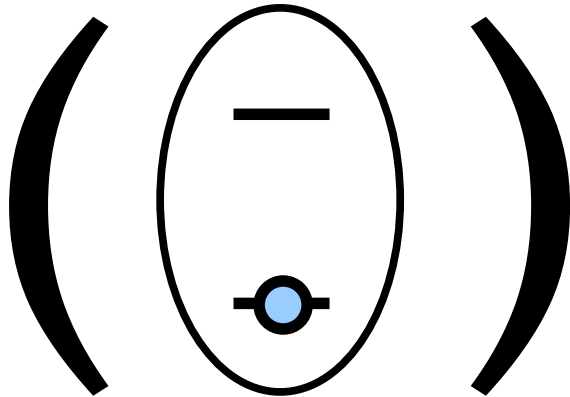
Electrical field operator (single freq):

$$\hat{\mathbf{E}}(x, y, z) = i \sqrt{\frac{\hbar \omega}{2\pi \epsilon_0 V}} (\mathbf{g}(x, y, z) \hat{a}^+ - \mathbf{g}^*(x, y, z) \hat{a})$$

mode function, e.g.

$$\mathbf{g}(x, y, z) = \mathbf{e} \sin kz e^{-\frac{x^2+y^2}{w^2}}$$

# Atom in a cavity



- atom Hamiltonian

$$\hat{H}_{atom} = E_g |g\rangle\langle g| + E_e |e\rangle\langle e|$$

- electric dipole interaction

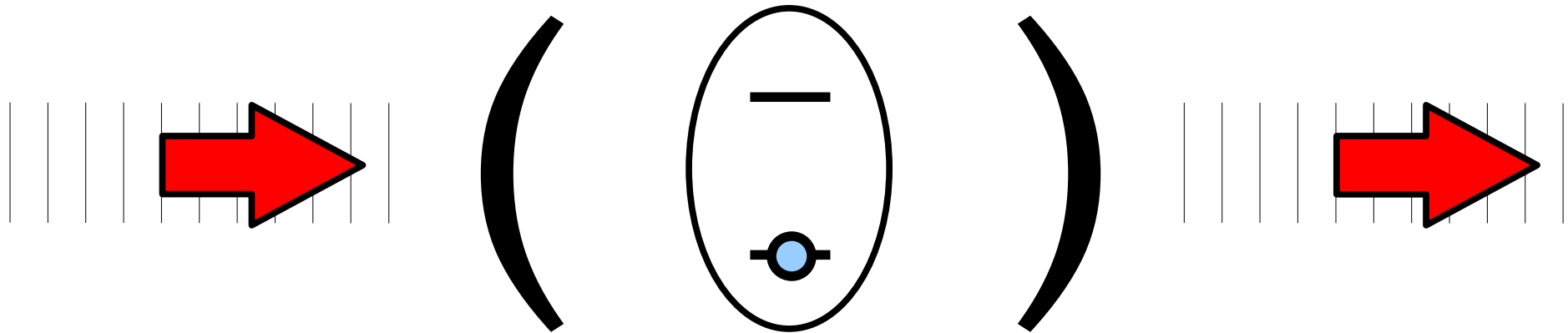
$$\hat{H}_I = \hat{\mathbf{E}} \cdot \hat{\mathbf{d}} \quad \text{with} \quad \hat{\mathbf{d}} = \mathbf{e} d_{eff} (|e\rangle\langle g| + |g\rangle\langle e|)$$

- (treat other field mode as losses)...

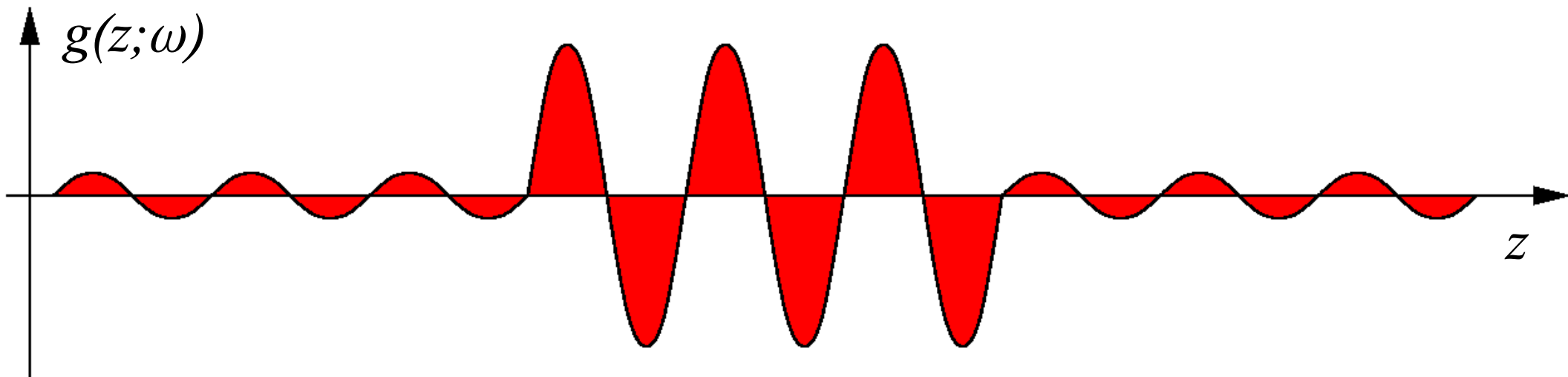
**.....Jaynes-Cummings model with all its aspects**

- treat external fields as perturbation/spectator of internal field

# *External view of cavity+atom*

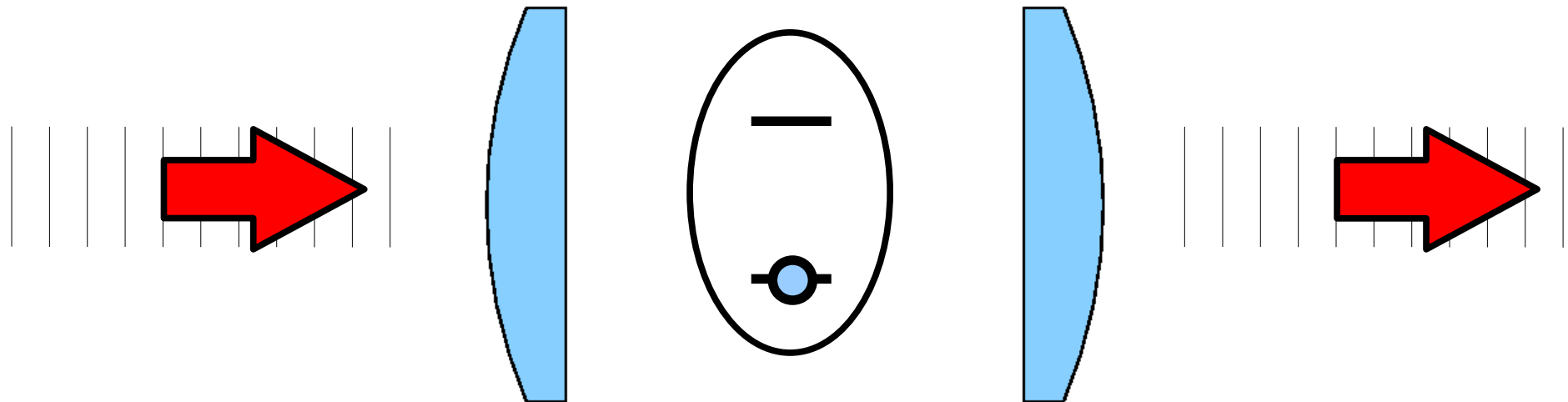


- continuous mode spectrum with enhanced/reduced field mode function:

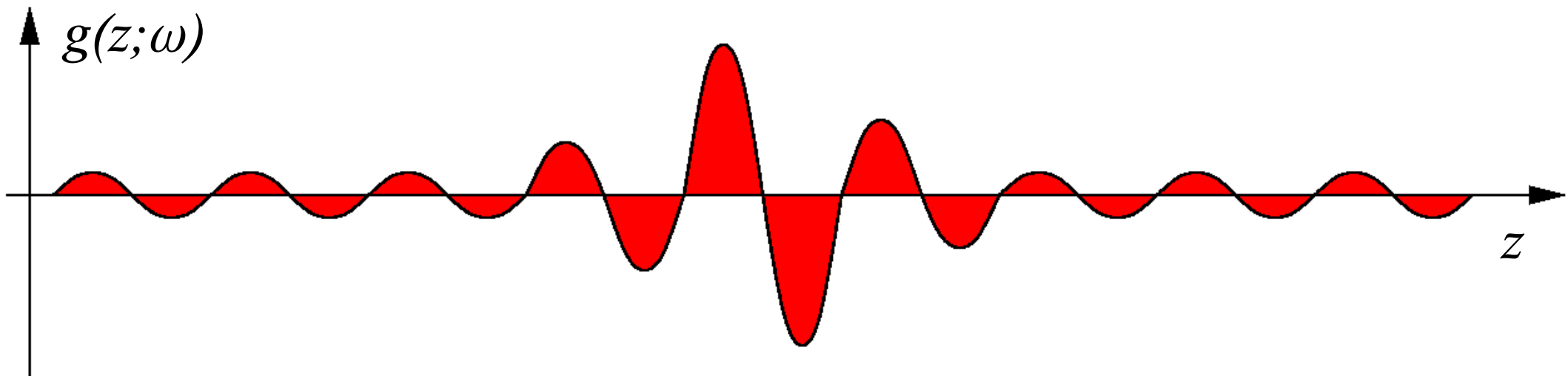




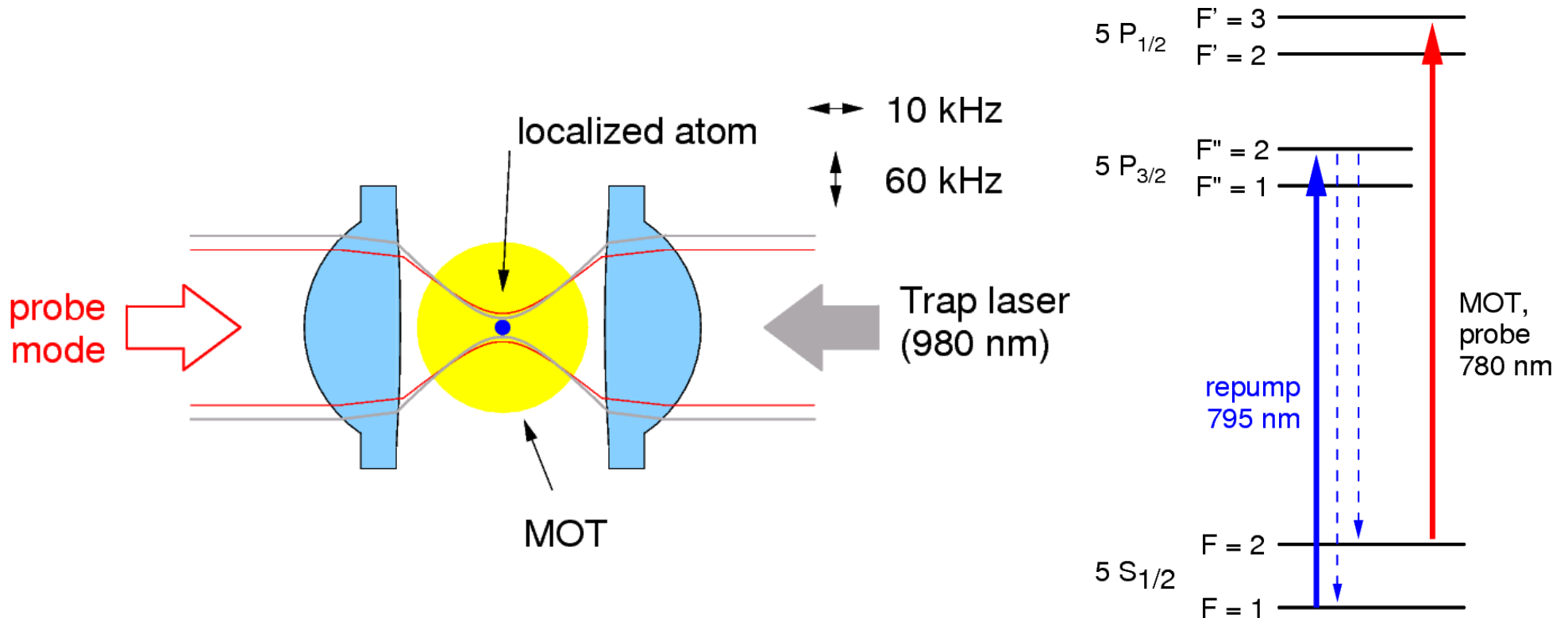
# *An alternative approach*



- use a **focusing lens pair** to enhance center mode function:



## One atom in an optical dipole trap, loaded from a MOT

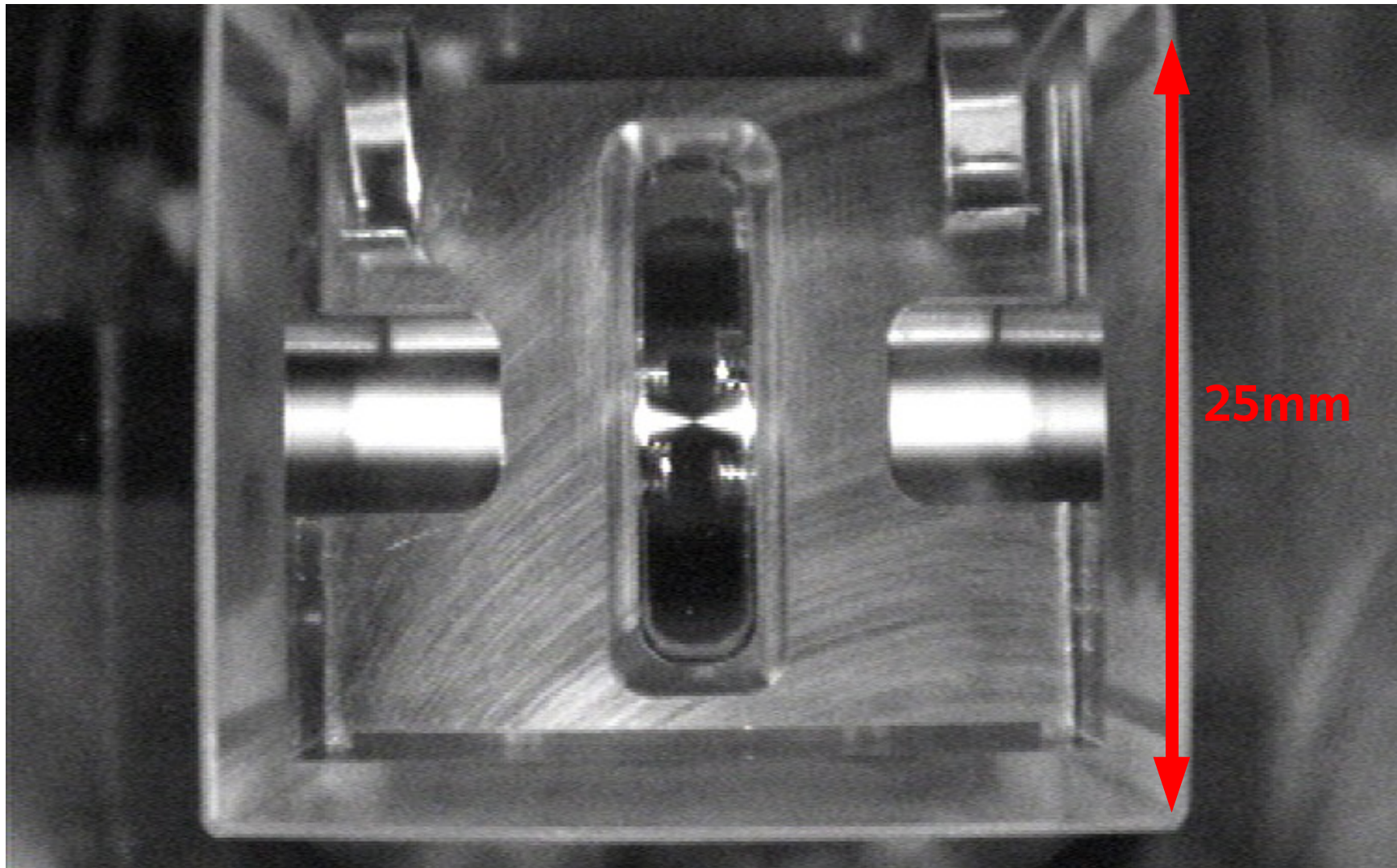


- use Rubidium-87 atom because it is convenient

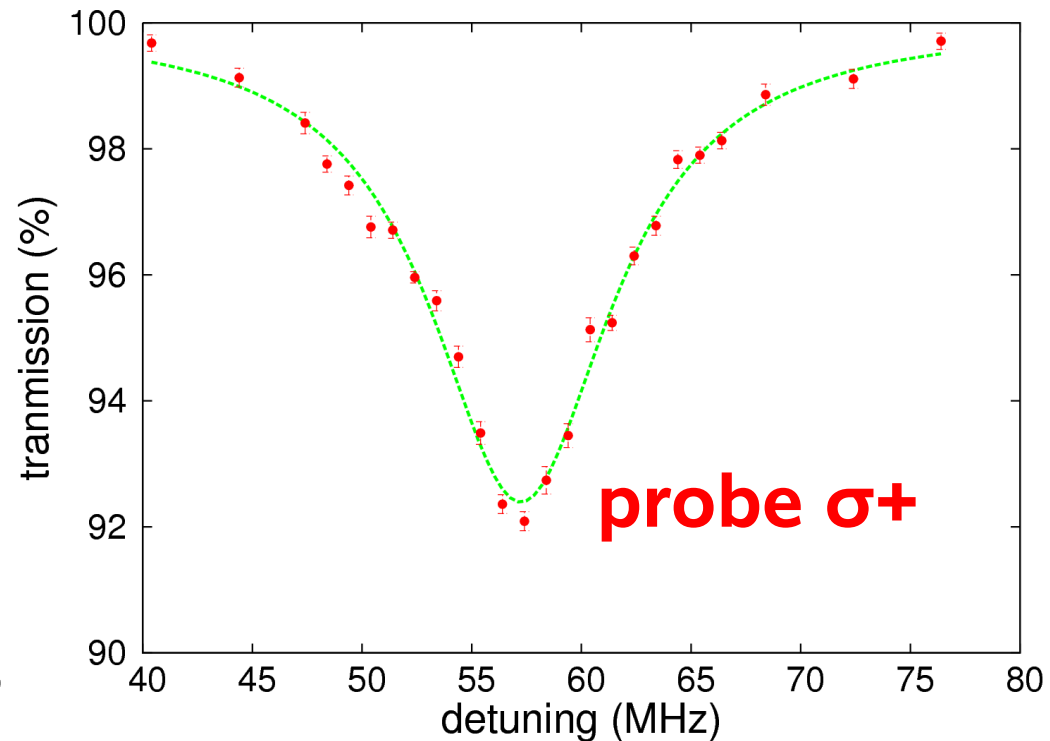
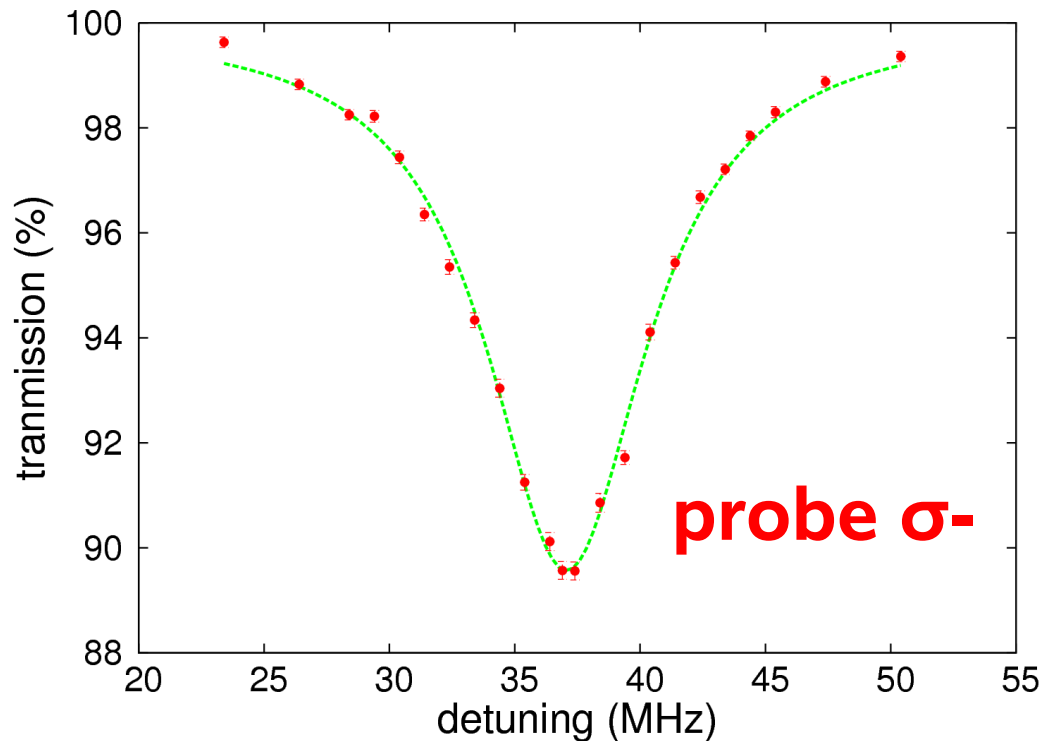
# *Focusing geometry...*



**...as seen by a CCTV camera at high Rb pressure**

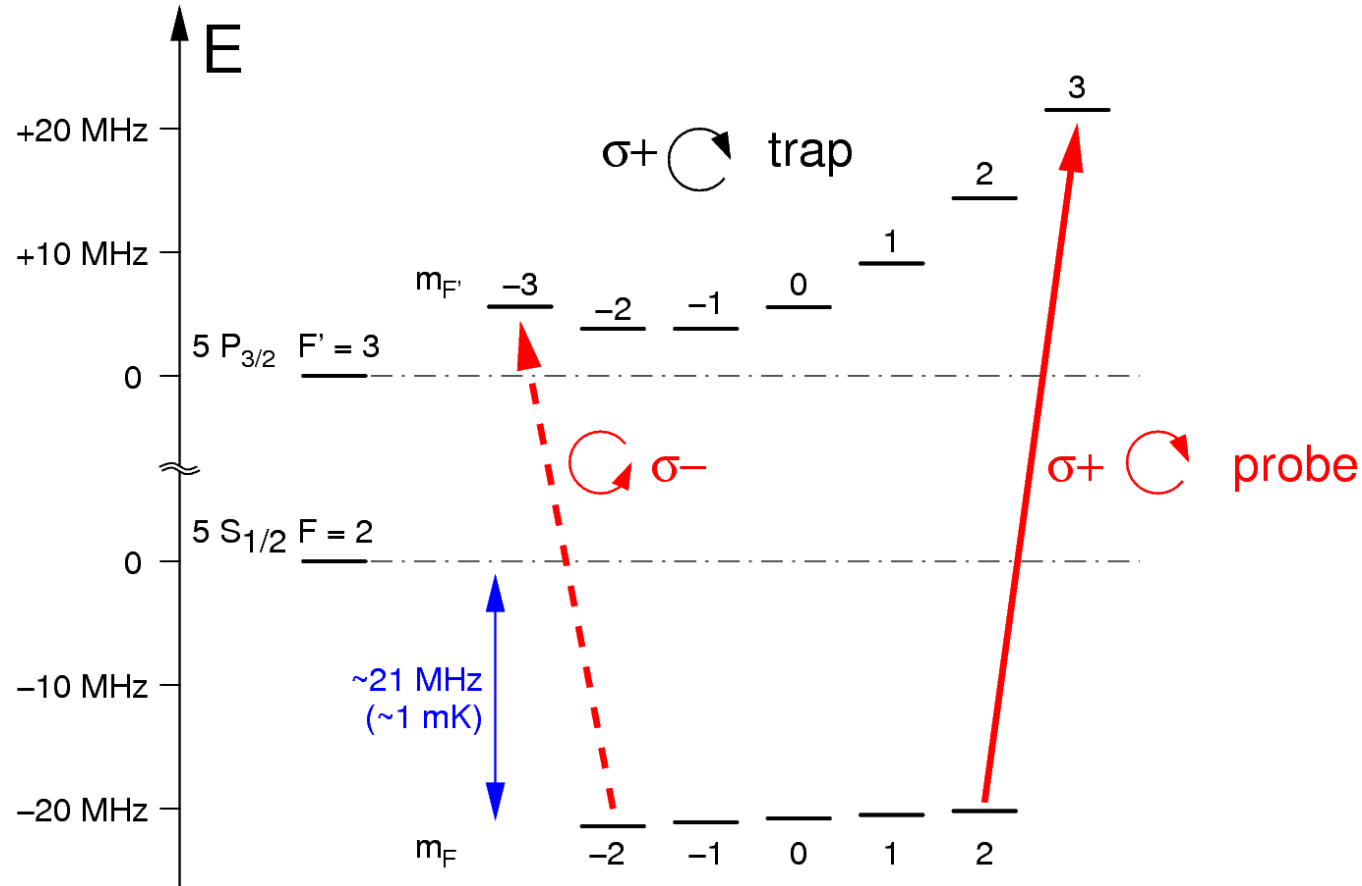


# Transmission results



- almost natural line width of atomic transition
- different resonances for different probe polarizations

# Atomic levels in a dipole trap

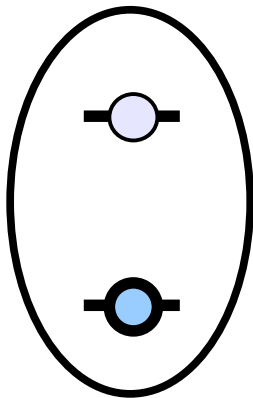


- optically pump with the probe beam into 2-level system

# Step 1: Scattering from an atom



## two - level atom in external driving field (quick & dirty)



- stationary excited state population:

$$\rho_{ee} = \frac{\Omega^2/4}{\delta^2 + \Omega^2/2 + \Gamma^2/4}$$

$$\Omega = E_A |d_{12}| / \hbar \quad \text{Rabi frequency}$$

$$\Gamma = \frac{\omega_{12}^3 d_{12}^2}{3\pi \epsilon_0 \hbar c^3} \quad \text{excited state decay rate}$$

- photon scattering rate  $\rho_e \Gamma$  leads to

$$\text{scattered power } P_{sc} = 3\epsilon_0 c \lambda^2 E_A^2 / 4\pi$$

# Simple model II



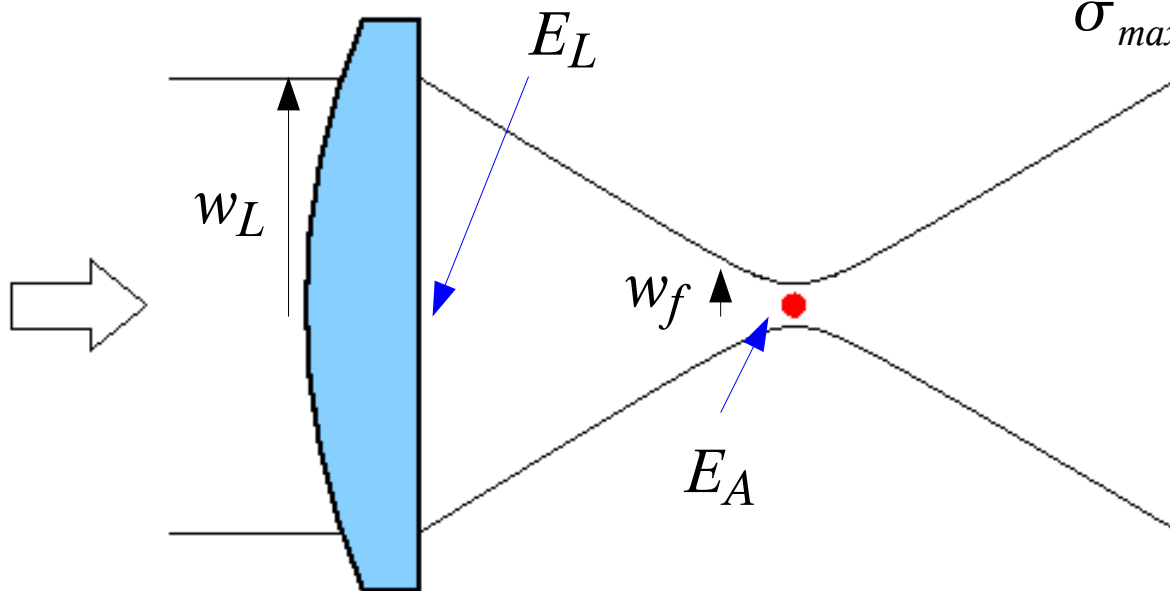
$$R_{sc} = \frac{P_{sc}}{P_{in}} = \frac{3\lambda^2}{\pi w_L^2} \left( \frac{E_A}{E_L} \right)^2 \stackrel{\text{paraxial approximation}}{\approx} \frac{3\lambda^2}{\pi w_f^2} = 3u^2 \approx \sigma_{max} / A$$

focal area  
 $A \approx \pi w_f^2 / 2$

focusing strength  $u := w_L / f$

atomic scattering cross section

$$\sigma_{max} = 3\lambda^2 / 2\pi$$



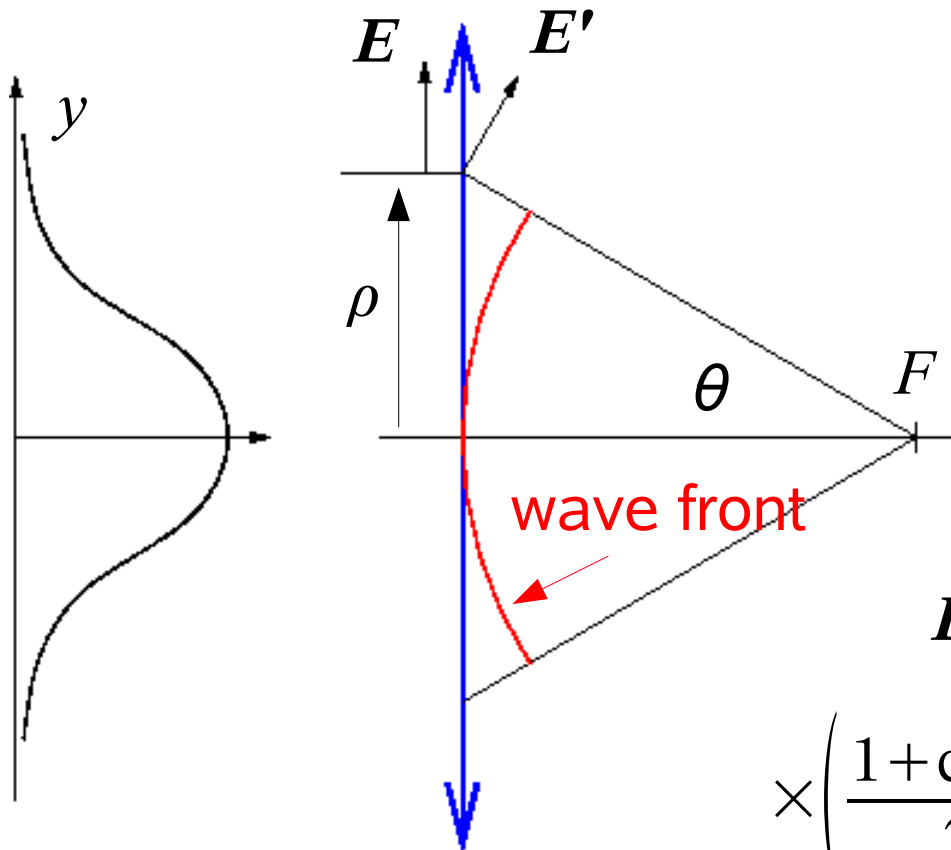
$E_A$  diverges, use full expression for field

# Step 2: Get exact field in focus



Circularly polarized Gaussian beam.....

$$\mathbf{E} = E_L \hat{\mathbf{e}}_+ e^{-\frac{\rho^2}{w_l^2}}$$



....transformed by an ideal lens:

- spherical wave front
- locally transverse
- conserve power through each small area

(Richardson/Wolf criteria, ~1950)

$$\mathbf{E}' = E_L e^{-\frac{\rho^2}{w_l^2}} \frac{1}{\sqrt{\cos \theta}} \times e^{-ik\sqrt{\rho^2 + f^2}} \times \left( \frac{1 + \cos \theta}{2} \hat{\mathbf{e}}_+ + \frac{\sin \theta e^{i\phi}}{\sqrt{2}} \hat{\mathbf{z}} + \frac{\cos \theta - 1}{2} e^{2i\phi} \hat{\mathbf{e}}_- \right)$$



# Propagate field to focus



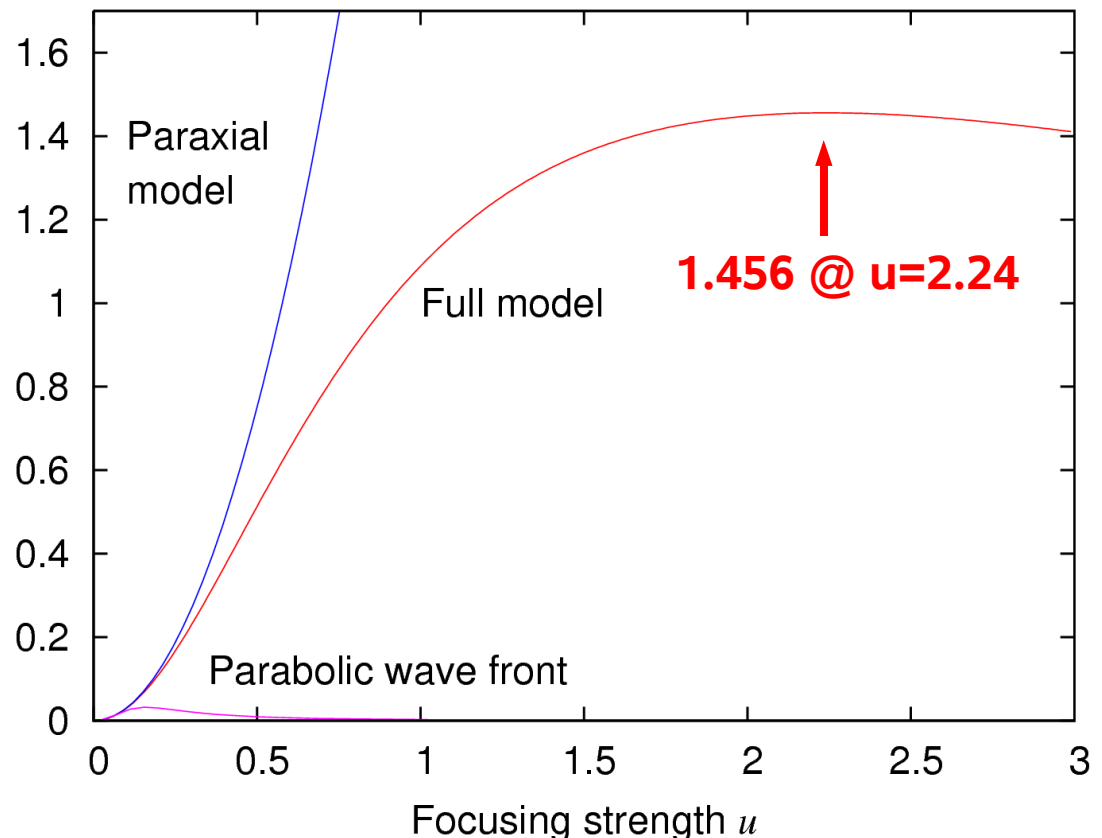
Exact propagation to focus:

$$\mathbf{E}_A(z=f, \rho=0) = \sqrt{\frac{\pi P_{in}}{\epsilon_0 c \lambda^2}} \cdot \frac{1}{u} e^{1/u^2} \left[ \sqrt{\frac{1}{u}} \Gamma\left(-\frac{1}{4}, \frac{1}{u^2}\right) + \sqrt{u} \Gamma\left(\frac{1}{4}, \frac{1}{u^2}\right) \right] \hat{\mathbf{e}}_+,$$

leads to “scattering ratio”

$$R_{sc} := \frac{P_{sc}}{P_{in}} = \frac{3}{4u^3} e^{-2/u^2} \times \left[ \Gamma\left(-\frac{1}{4}, \frac{1}{u^2}\right) + u \Gamma\left(\frac{1}{4}, \frac{1}{u^2}\right) \right]^2$$

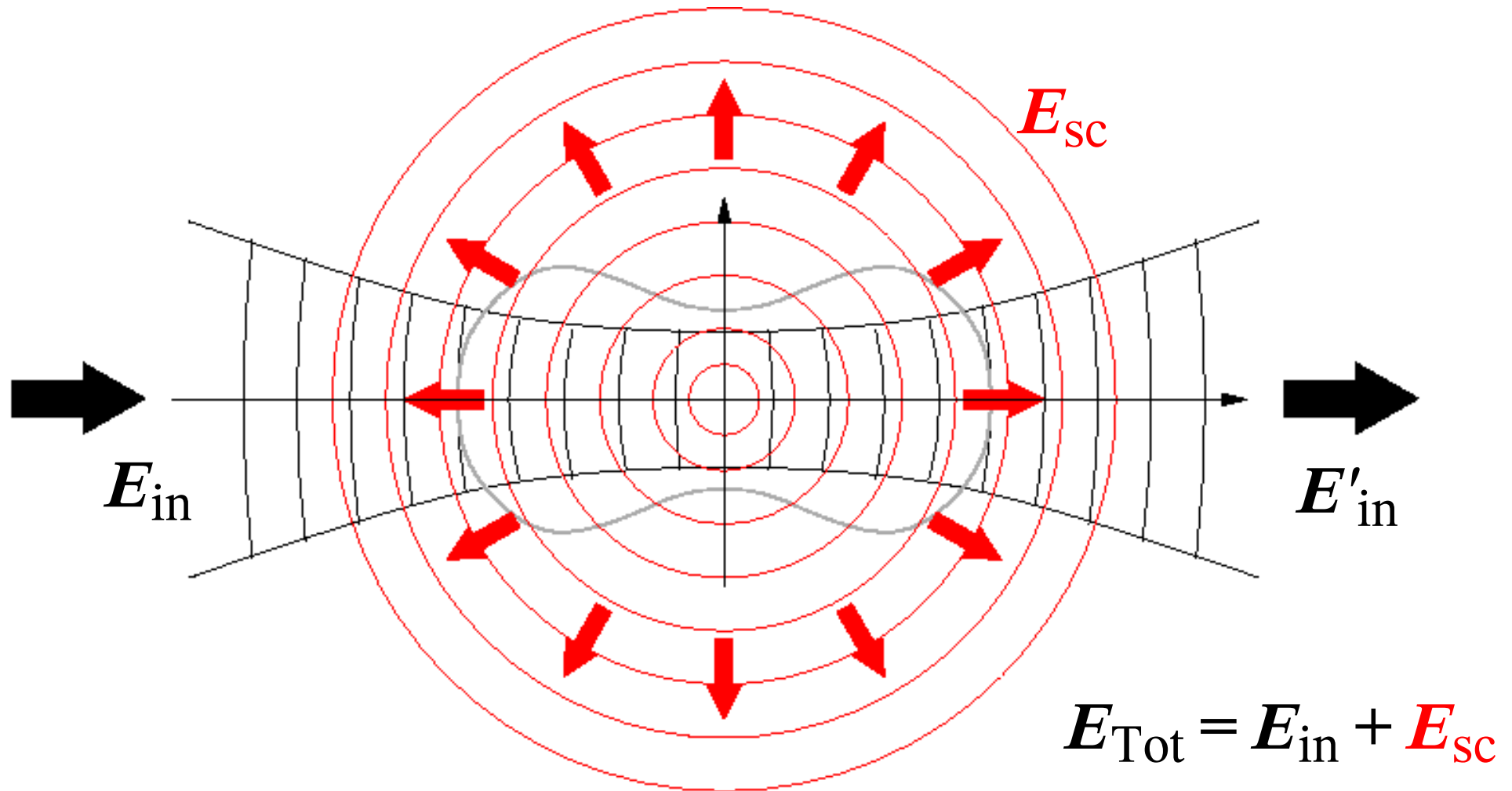
Scattering ratio  $R_{sc}$



# Step 3: Combine with probe



scattered field for  $\sigma+$  transition: 
$$\mathbf{E}_{sc}(\mathbf{r}) = E_A \frac{3}{2} \frac{e^{ikr + \pi/2}}{kr} \left[ \hat{\mathbf{e}}_+ - (\hat{\mathbf{e}}_+ \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} \right]$$



# Collection into Gaussian mode



- Project total field onto Gaussian mode of collection fiber

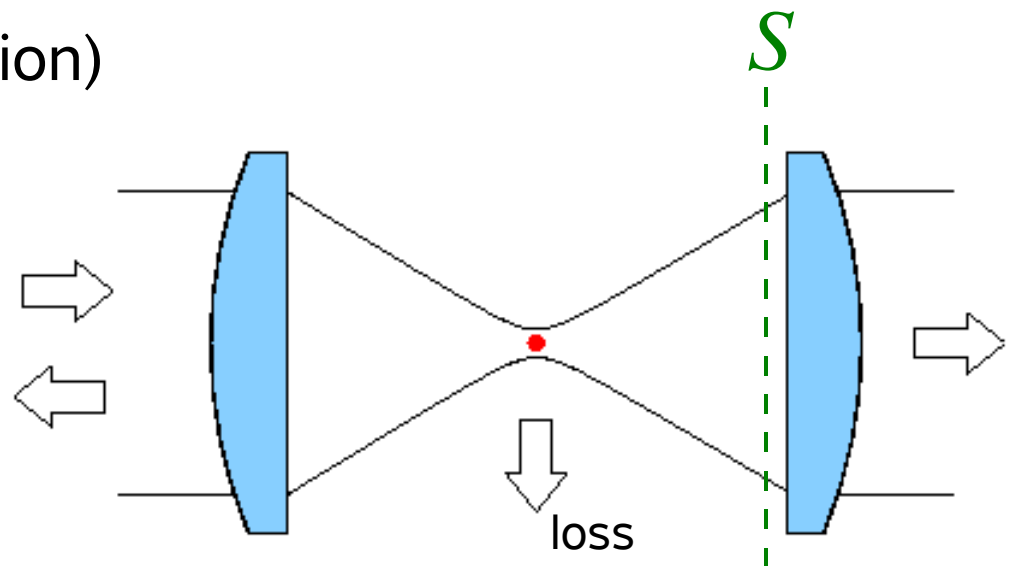
$$P_{out} = \left| \langle \vec{g}, \vec{E}_{Tot} \rangle \right|^2 \quad \langle \vec{g}, \vec{E} \rangle := \int_{\vec{x} \in S} \vec{E}_{Tot}(\vec{x}) \cdot \vec{g}(\vec{x}) (\vec{k}_g \cdot \vec{n}) dA$$

- Forward transmission: cross section fiber mode

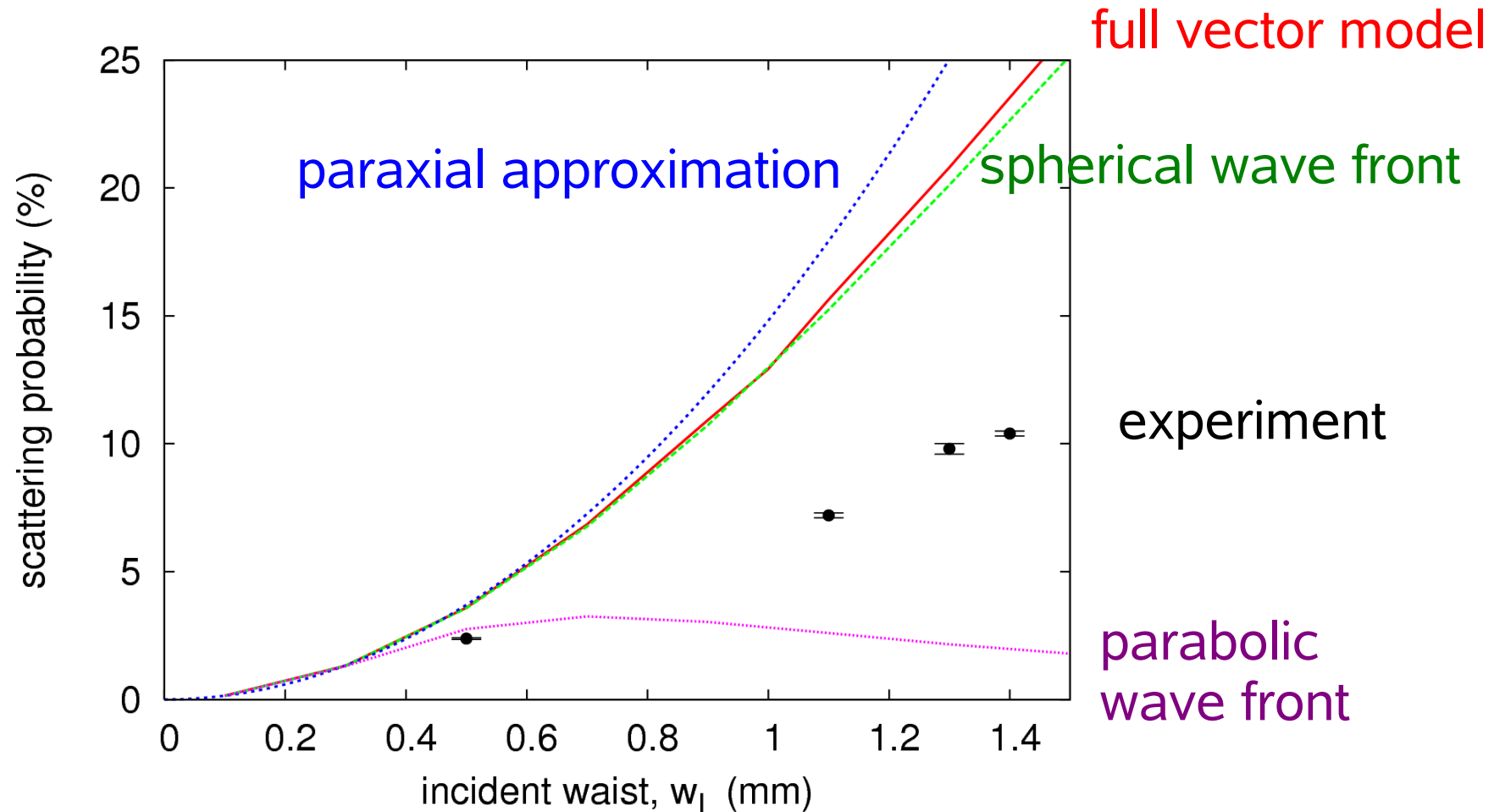
$$1 - \epsilon = \frac{P_{out}}{P_{in}} = \left| 1 - \frac{P_{sc}/P_{in}}{2} \right|^2$$

- Reflectivity (backward direction)

$$R = \frac{(P_{sc}/P_{in})^2}{4}$$



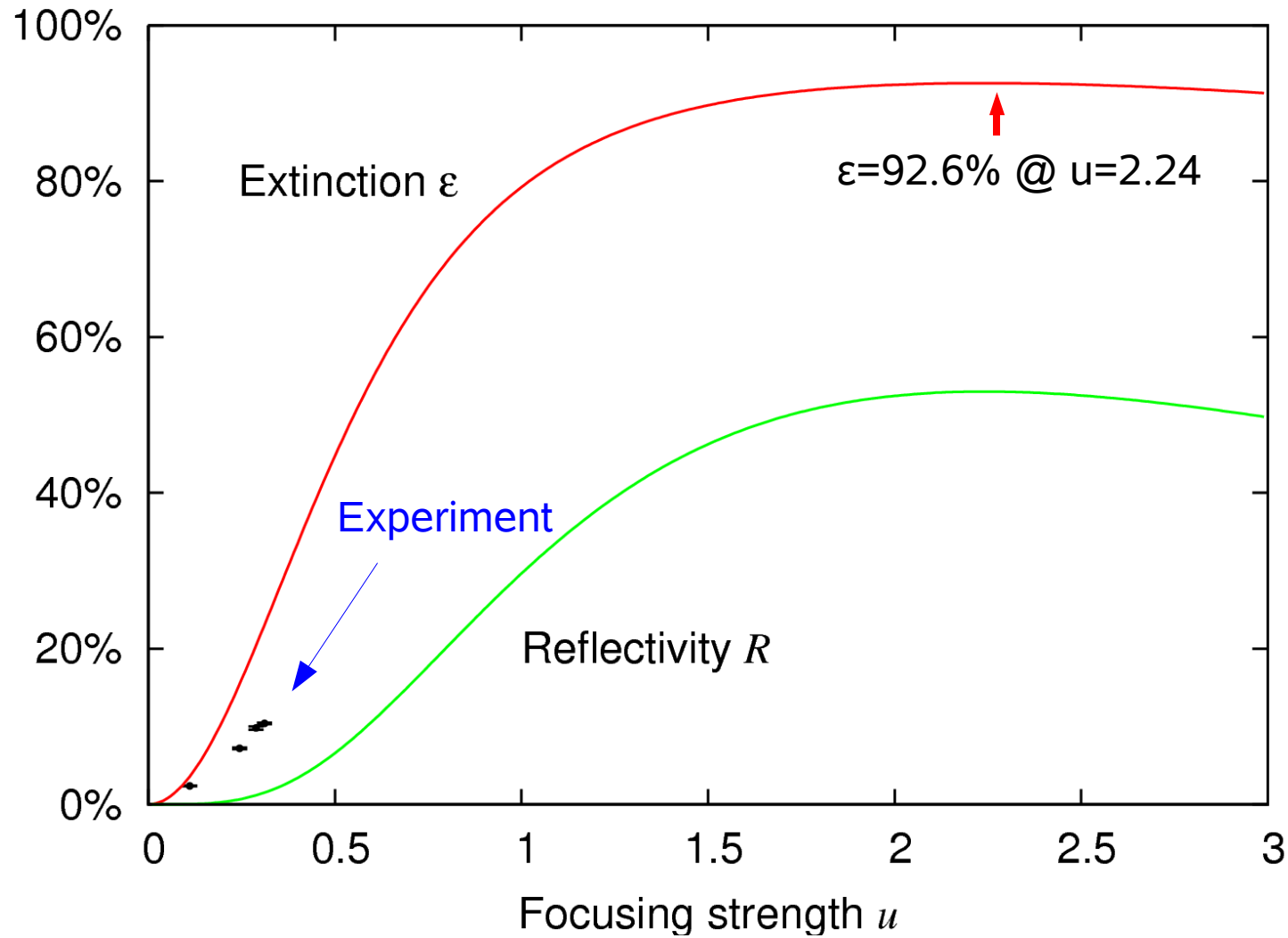
# Scattering vs. focusing



( experimental  $P_{sc}$  extracted out of transmission measurement )

( $f = 4.5$  mm)

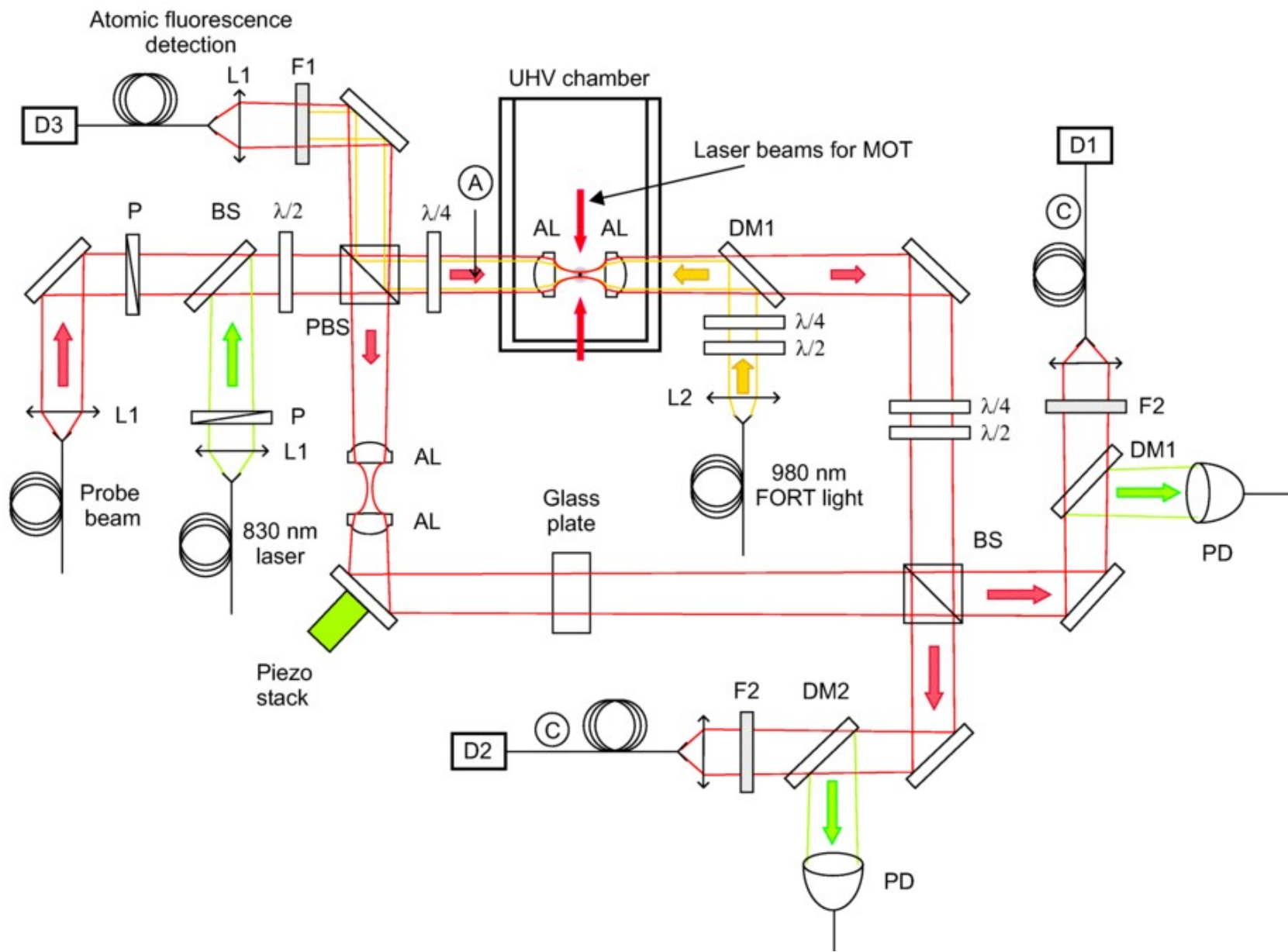
# How far does this go?



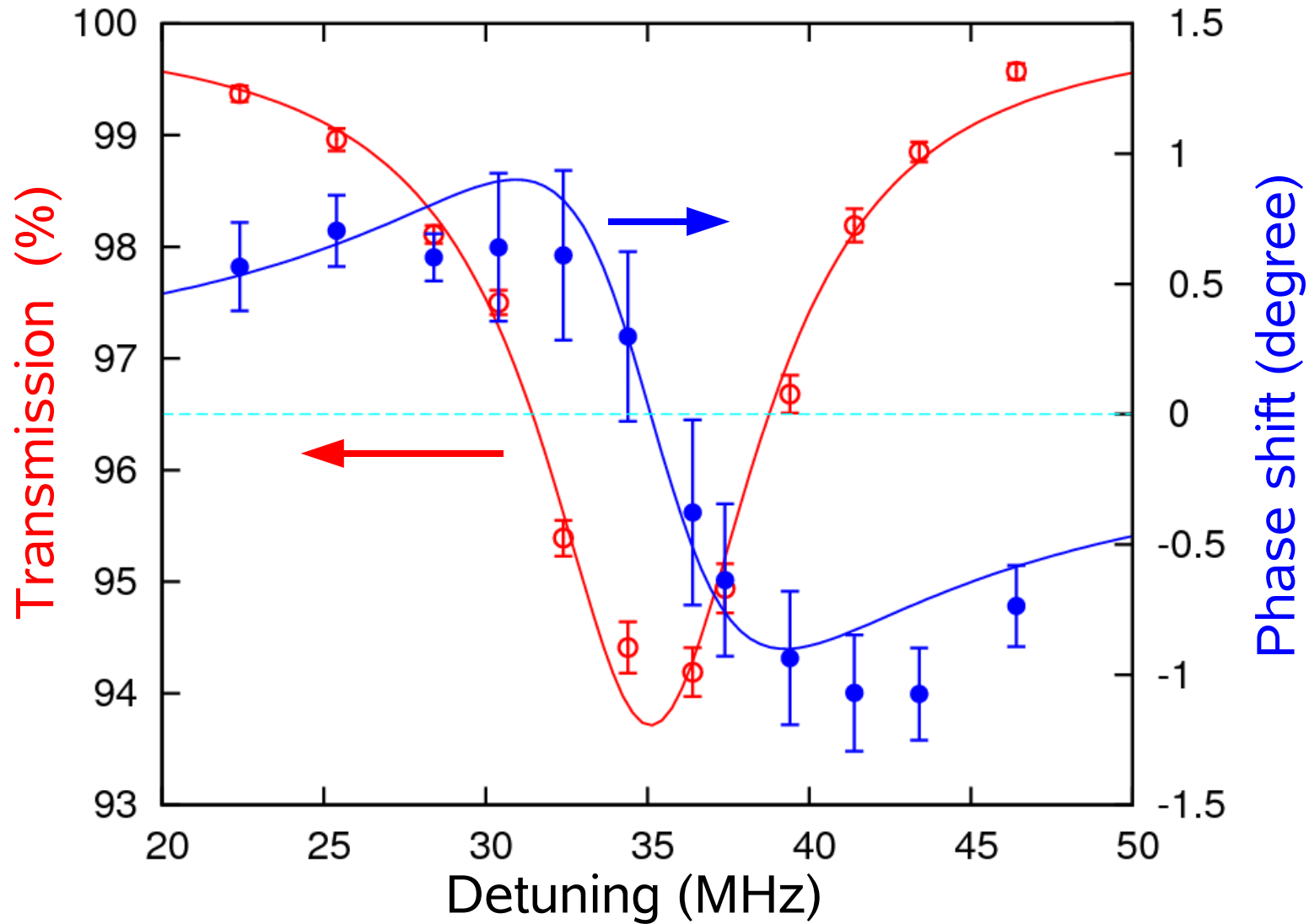
# Phase shift measurement



## Mach-Zehnder interferometer with one atom



# Phase shift / Transmission

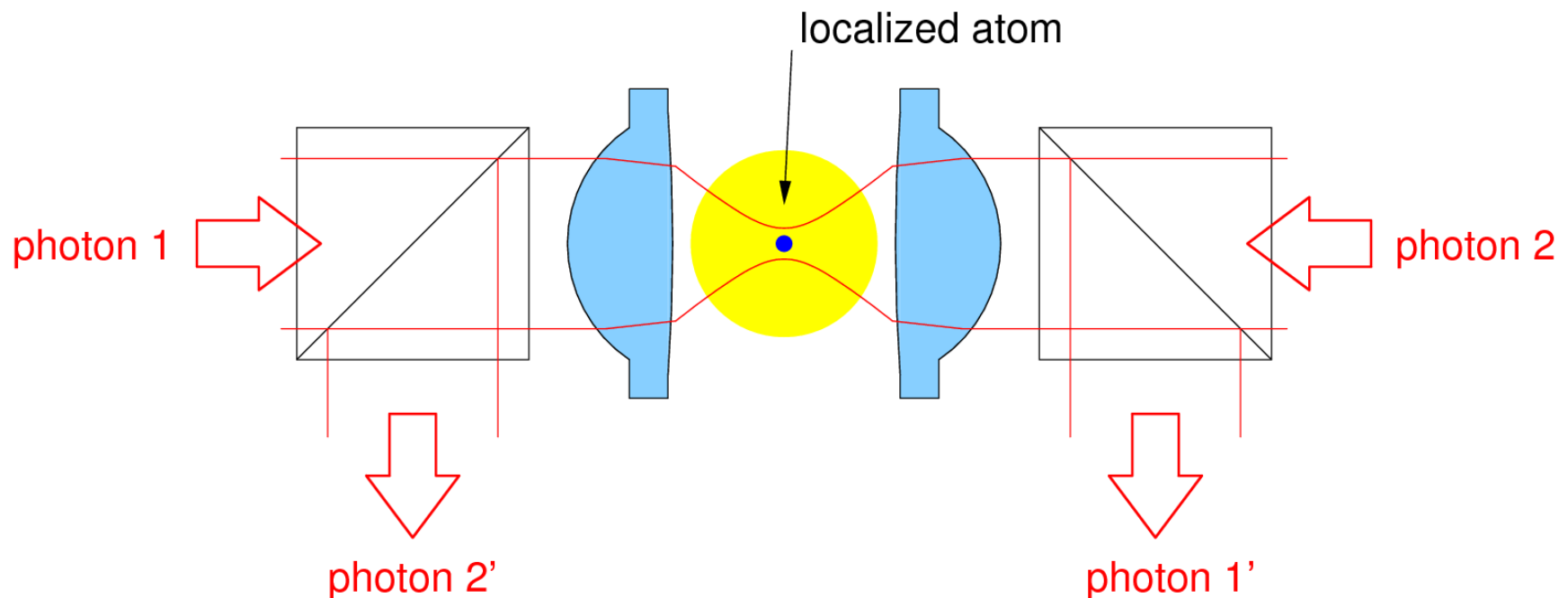


phase shift within factor 2..3 of prediction by stationary atom model!

# Next steps



- Temperature & cooling:  
currently: 20  $\mu\text{K}$  (tested; need to verify)...150  $\mu\text{K}$  (MOT)
- Try larger numerical apertures (lenses are here.. $u \approx 2$ )
- Sci-Fi? conditional phase gate....





# *Comparison to cavity QED*

---



- Could strong focusing replace cavities for strong coupling?

Probably not: imperfect mode match  
Gaussian modes --- atomic dipole modes

- Can strong focusing help in cavity QED experiments?

Probably yes: field enhancement due to focusing  
can lower cavity finesse

- What is the balance of technical problems?

high NA lenses vs. high finesse mirrors (similar effort?)

# *Thank you!*

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*Meng Khoon Tey (now UIBK)*  
*Syed Abdullah Aljunid*  
*Zilong Chen (now JILA)*  
*Florian Huber (now Harvard)*  
*Brenda Chng*  
*Jianwei Lee*

*Timothy Liew*  
*Gleb Maslennikov*

*Valerio Scarani*  
*Christian Kurtsiefer*

<http://www.qolah.org>

# Results (collect full NA)



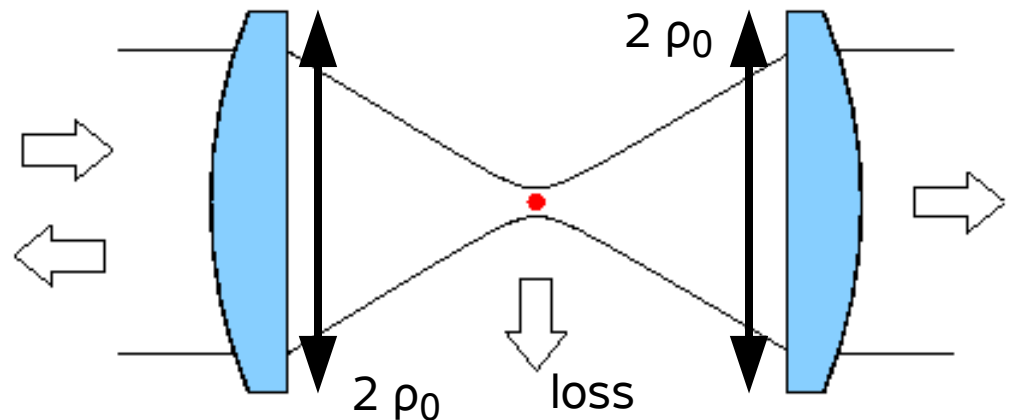
- Extinction

$$\epsilon = \frac{P_{sc}^{\rho_0}}{2 P_{in} (1 - e^{-2\rho_0^2/w_L^2})} \left[ 1 + \frac{4 f^3 + 3 f \rho_0^2}{4 (f^2 + \rho_0^2)^{3/2}} \right]$$

- Reflectivity (backward direction)

$$R = \frac{P_{sc}^{\rho_0}}{2 P_{in}} \left[ 1 - \frac{4 f^3 + 3 f \rho_0^2}{4 (f^2 + \rho_0^2)^{3/2}} \right]$$

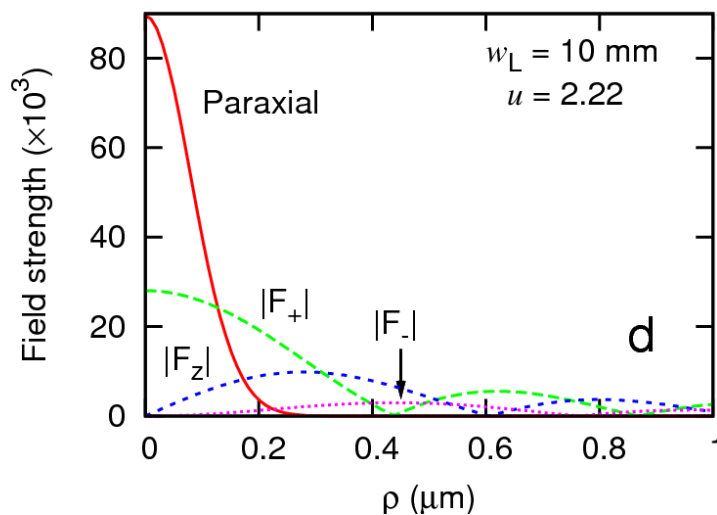
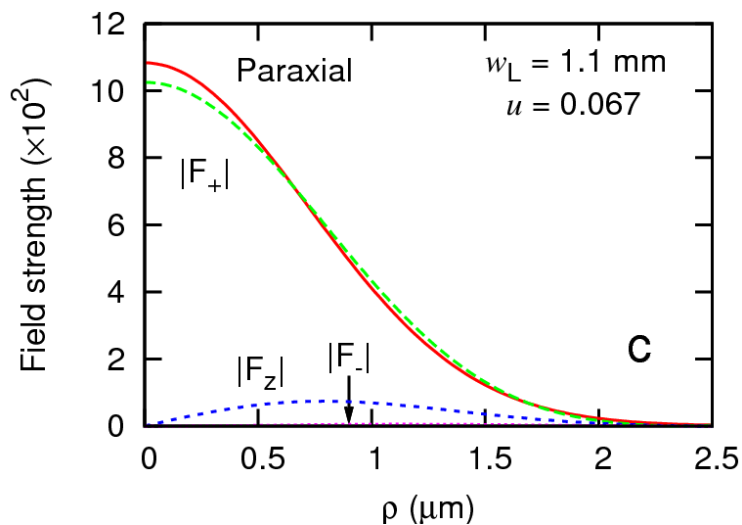
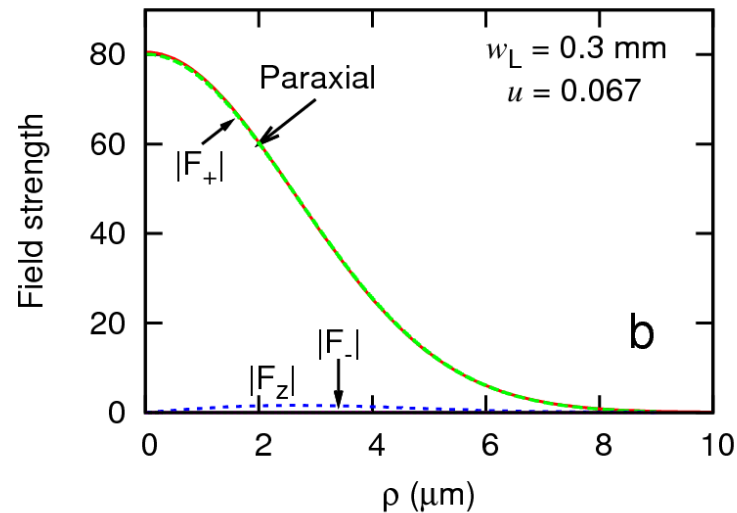
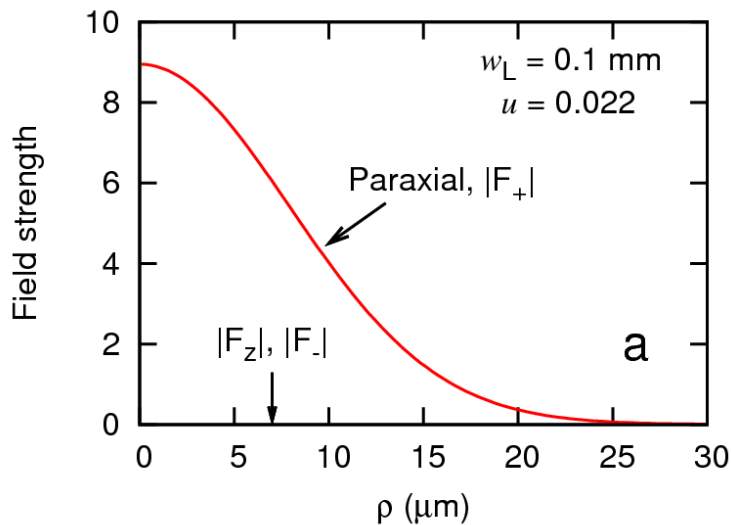
- No energy gets lost



# Focal fields for different $w_L$



- paraxial approximation starts to break down late...

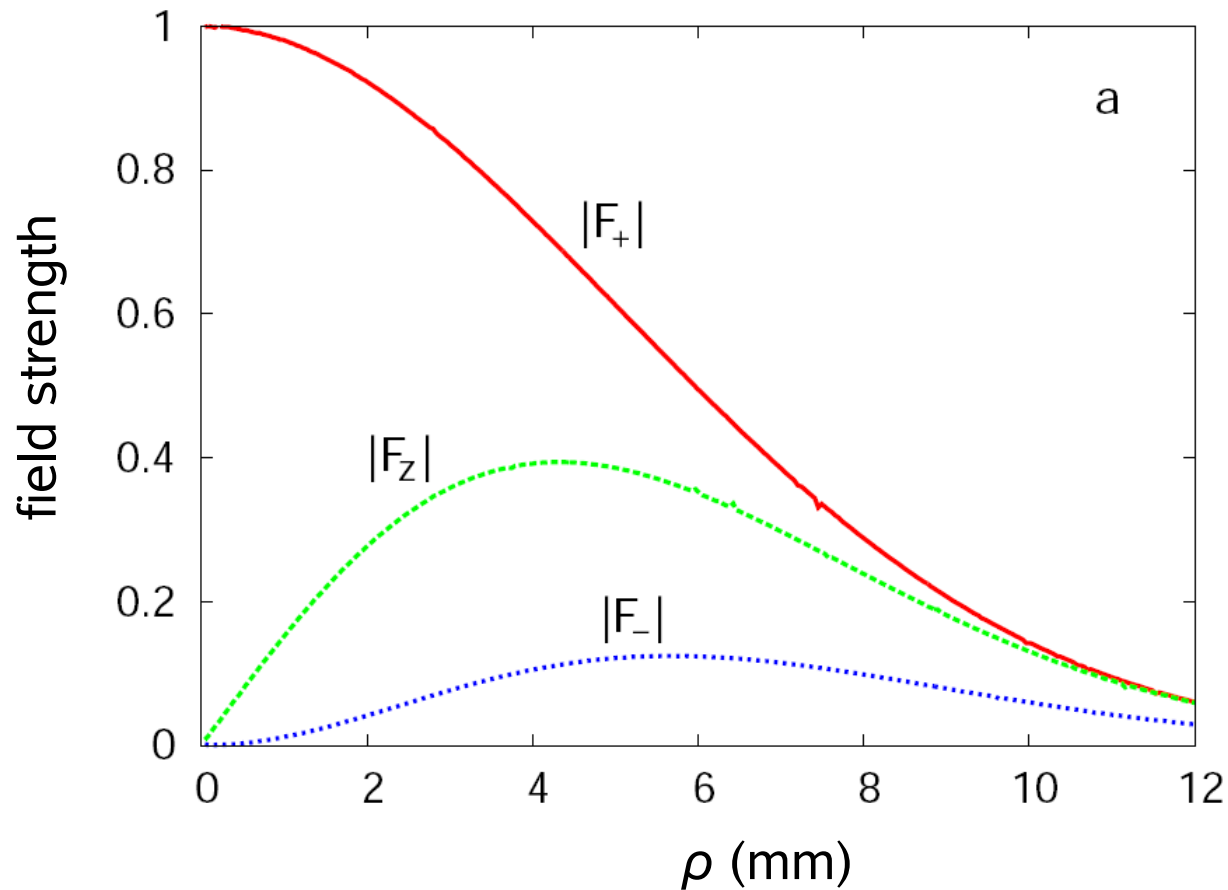


( $f = 4.5 \text{ mm}$ )

# Directly after lens:



$$\mathbf{E}' = E_L e^{-\frac{\rho^2}{w_l^2}} \frac{1}{\sqrt{\cos\theta}} \times e^{-ik\sqrt{\rho^2+f^2}} \times \left( \frac{1+\cos\theta}{2} \hat{\mathbf{e}}_+ + \frac{\sin\theta e^{i\phi}}{\sqrt{2}} \hat{\mathbf{z}} + \frac{\cos\theta-1}{2} e^{2i\phi} \hat{\mathbf{e}}_- \right)$$



- different polarization components appear

beam parameter:  
 $w_l = 7$  mm

focal length:  
 $f = 4.5$  mm

# *A simple scattering model*



- Electrical field in laser beam before lens

$$\mathbf{E} = E_L \frac{1}{\sqrt{2}} e^{-\frac{\rho^2}{w_L^2}} (\hat{x} \cos \omega t + \hat{y} \sin \omega t)$$

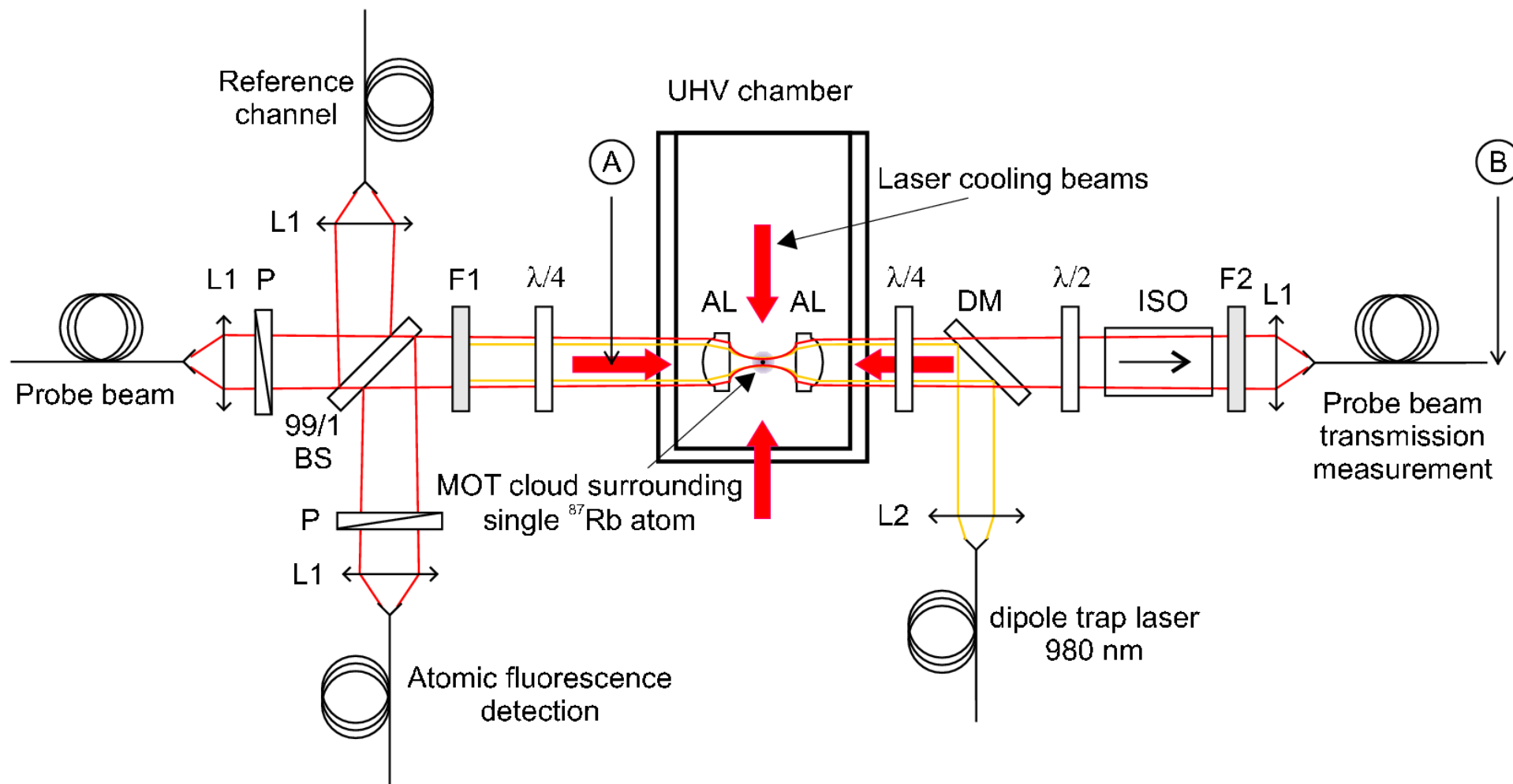
- Total excitation power

$$P_{in} = 1/4 \epsilon \pi c w_L^2 E_L^2$$

- Total power scattered by the atom

$$P_{sc} = 3 \epsilon_0 c \lambda^2 E_A^2 / 4 \pi$$

# Almost the real exp setup



# Single atom evidence



## (almost) Hanbury-Brown—Twiss experiment on atomic fluorescence during cooling

