Interaction of Photons with Single Atoms - a complementary approach to cavity QED

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Motivation:

- Advanced quantum communication schemes (q repeater etc) will require universal gates between photonic qubits

- Bulk optical materials (LiNbO3 etc) have too low nonlinearity

- Atoms and photons are good for different quantum information tasks – allow an exchange of quantum information

- Explore possibilities of controlled phase gates & friends for photonic qubits
Photonic Phase Gate Concept

- universal 2-qubit operations, require large optical nonlinearity
- hopeless with typical bulk nonlinearities
- possible with atoms close to resonance:
  - S. Harris & team, Stanford: atomic clouds
  - M. Lukin & team, Harvard: atoms in fibers

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<tr>
<th>A, B</th>
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<td>(1,1) e^{i\phi}</td>
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Atom-Photon interface

- e.g. transfer of information from flying qubits into a quantum memory

\[ \Psi_L = \alpha |L\rangle + \beta |R\rangle \]

\[ \Psi_A = \alpha |m = -1\rangle + \beta |m = +1\rangle \]

- requires internal states of atom and an **absorption process**
The basic problem

- Get strong coupling between an atom and a light field on the single photon level

electromagnetic field / photon  2-level atom
One solution: Use a cavity

- High electrical field strength even for a single photon
- Preferred spontaneous emission into the cavity mode
- A cavity can enhance the interaction between a propagating external mode and an atom
Why cavities are nice

- discrete mode spectrum
- 'textbook' field energy eigenstates

\[ \hat{H}_{\text{field}} = \frac{\epsilon_0}{2} \int (\hat{E}^2 + c^2 \hat{B}^2) dV = \hbar \omega (\hat{n} + \frac{1}{2}) \]

Electrical field operator (single freq):

\[ \hat{E}(x, y, z) = i \sqrt{\frac{\hbar \omega}{2\pi \epsilon_0 V}} \left( g(x, y, z) \hat{a}^\dagger - g^*(x, y, z) \hat{a} \right) \]

mode function, e.g.

\[ g(x, y, z) = e^{\frac{-x^2 + y^2}{w^2}} \sin kz \]
Atom in a cavity

- atom Hamiltonian
  \[ \hat{H}_{\text{atom}} = E_g |g\rangle \langle g| + E_e |e\rangle \langle e| \]

- electric dipole interaction
  \[ \hat{H}_I = \hat{E} \cdot \hat{d} \quad \text{with} \quad \hat{d} = e d_{\text{eff}} (|e\rangle \langle g| + |g\rangle \langle e|) \]

- (treat other field mode as losses)...

...Jaynes-Cummings model with all its aspects

- treat external fields as perturbation/spectator of internal field
External view of cavity + atom

- continuous mode spectrum with enhanced/reduced field mode function:

\[ g(z; \omega) \]
An alternative approach

- use a **focusing lens pair** to enhance center mode function:

\[ g(z;\omega) \]
Experiment

One atom in an optical dipole trap, loaded from a MOT

- use Rubidium-87 atom because it is convenient

Focusing geometry...

...as seen by a CCTV camera at high Rb pressure
Step 1: Scattering from an atom

two-level atom in external driving field (quick & dirty)

- stationary excited state population:
  \[ \rho_{ee} = \frac{\Omega^2/4}{\delta^2 + \Omega^2/2 + \Gamma^2/4} \]
  \[ \Omega = E_A |d_{12}| / \hbar \] Rabi frequency
  \[ \Gamma = \frac{\omega_{12}^3 d_{12}^2}{3 \pi \varepsilon_0 \hbar c^3} \] excited state decay rate

- photon scattering rate \( \rho_e \Gamma \) leads to
  \[ P_{sc} = 3 \varepsilon_0 c \lambda^2 E_A^2 / 4 \pi \] scattered power
Field at focus (simple)

\[ R_{sc} = \frac{P_{sc}}{P_{in}} = \frac{3 \lambda^2}{\pi w_L^2} \left( \frac{E_A}{E_L} \right)^2 \approx \frac{3 \lambda^2}{\pi w_f^2} = 3u^2 \approx \sigma_{max} / A \]

focal area

\[ A \approx \pi w_f^2 / 2 \]

atomic scattering cross section

\[ \sigma_{max} = \frac{3 \lambda^2}{2 \pi} \]

paraxial approximation

focusing strength \[ u : = w_L / f \]

\[ E_A \text{ diverges, use full expression for field} \]
Field to focus (exact)

Exact propagation to focus:

$$E_A(z = f, \rho = 0) = \sqrt{\frac{\pi P_{in}}{\epsilon_0 c \lambda^2}} \cdot \frac{1}{u} e^{1/u^2} \left[ \sqrt{\frac{1}{u}} \Gamma\left(-\frac{1}{4}, \frac{1}{u^2}\right) + \sqrt{u} \Gamma\left(\frac{1}{4}, \frac{1}{u^2}\right) \right] \hat{e}_+, \]$$

leads to “scattering ratio”

$$R_{sc} := \frac{P_{sc}}{P_{in}} = \frac{3}{4u^3} e^{-2/u^2} \times$$

$$\times \left[ \Gamma\left(-\frac{1}{4}, \frac{1}{u^2}\right) + u \Gamma\left(\frac{1}{4}, \frac{1}{u^2}\right) \right]^2$$

1.456 @ $u = 2.24$
Collection into Gaussian mode

- Project total field onto Gaussian mode of collection fiber

\[ P_{out} = \left| \langle \tilde{g}, \tilde{E}_{Tot} \rangle \right|^2 \]

\[ \langle \tilde{g}, \tilde{E} \rangle := \int_{\tilde{x} \in S} E_{Tot}(\tilde{x}) \cdot \tilde{g}(\tilde{x})(\hat{k}_g \cdot \vec{n}) dA \]

- Forward transmission:

\[ 1 - \epsilon = \frac{P_{out}}{P_{in}} = \left| 1 - \frac{P_{sc}}{P_{in}} \right|^2 \]

- Reflectivity (backward direction)

\[ R = \left( \frac{P_{sc}}{P_{in}} \right)^2 - \frac{1}{4} \]
Reflection & Transmission

(σ-probe)

Transmission $T$ (%) vs. detuning $\Delta/2\pi$ (MHz) for reflection $R$ (%).
How far does this go?

Earlier Experiments

Custom lens

Extinction $\varepsilon$

Reflectivity $R$

$\varepsilon = 92.6\%$ @ $u = 2.24$

Phase shift measurement

Mach-Zehnder interferometer with one atom
Phase shift / Transmission

phase shift within factor 2..3 of prediction by stationary atom model!

S.A. Aljunid et al. PRL 103, 153601 (2009)
Dreams...

- Try to see conditional phase gate....

need photons with compatible bandwidth
Next steps in the real world

- Atom does not sit nicely in our trap:

For $T = 35 \, \mu K$:

$\Delta x = 160 \, \text{nm}$

$\Delta z = 1.3 \, \mu m$
Raman Sideband Cooling

- Reduce vibrational quanta directly:
First steps: Raman transitions

Atom state manipulation: Raman Rabi oscillations

Decoherence time $\sim 100 \mu s$
Combine focusing & cavity

- Get easier into “strong coupling” regime

Electrical field operator (single freq):

$$\hat{E}(x, y, z) = i \sqrt{\frac{\hbar \omega \pi}{\epsilon_0 \lambda}} \left( \frac{1}{L} \right)^2 R_{sc} \left( g(x, y, z) \hat{a}^+ - g^*(x, y, z) \hat{a} \right)$$

- Scattering ratio, 0...2
- Mode function, $g=1$ at focus

Effective mode volume: $V = L \lambda^2 / R_{sc}$
Weak cavity – strong coupling?

Example: Rb
$L = 10 \text{ mm}$
$\lambda = 780 \text{ nm}$
$\tau = 26 \text{ ns}$

Coupling strength:

$$g_0 = \hbar \sqrt{\frac{\pi c R_{sc}}{\tau L}}$$

Not exactly a new idea...

- Ibn Sahl, ~ 984: optimal focusing
- Today's version of an anaclastic lens
Comparison to cavity QED

• Could strong focusing replace cavities for strong coupling?
  
  Probably not: imperfect mode match
  Gaussian modes --- atomic dipole modes

• Can strong focusing help in cavity QED experiments?

  Probably yes: field enhancement by focusing
  can lower cavity finesse for a given coupling strength

• What is the balance of technical problems?

  high NA lenses vs. high finesse mirrors (similar effort?)
Thank you!

Postdoc Positions Available!

Syed Abdullah Aljunid
Brenda Chng
Jianwei Lee
Dao Hoang Lan
Kadir Durak
Gleb Maslennikov, C.K.

Theory Support:
Wang Yimin
Valerio Scarani

Former members:
Meng Khoon Tey (now UIBK)
Zilong Chen (now JILA)
Florian Huber (now Harvard)
Martin Paesold (now ETHZ)
Timothy Liew (now Lausanne)

http://www.qolah.org
Experimental setup


nature physics 4, 924 (2008)
Atomic levels in a dipole trap

- optically pump with the probe beam into 2-level system
Step 2: Get exact field in focus

Circularly polarized Gaussian beam.....

\[ E = E_L \hat{e}_+ e^{-\frac{\rho^2}{w_0^2}} \]

....transformed by an ideal lens:

- spherical wave front
- locally transverse
- conserve power through each small area

(Richardson/Wolf criteria, ~1950)

\[
E' = E_L e^{\frac{-\rho^2}{w_0^2}} \frac{1}{\sqrt{\cos \theta}} \times e^{-ik \sqrt{\rho^2 + f^2}} \times \left( \frac{1+\cos \theta}{2} \hat{e}_+ + \frac{\sin \theta e^{i \phi}}{\sqrt{2}} \hat{z} + \frac{\cos \theta - 1}{2} e^{2i \phi} \hat{e}_- \right)
\]
Step 3: Combine with probe

scattered field for $\sigma^+$ transition: \[ E_{sc}(r) = E_A \frac{3}{2} \frac{e^{ikr + \pi/2}}{kr} \hat{\epsilon}_+ - (\hat{\epsilon}_+ \cdot \hat{r}) \hat{r} \]

\[ E_{Tot} = E_{in} + E_{sc} \]
(almost) Hanbury-Brown—Twiss experiment on atomic fluorescence during cooling

Single atom evidence

Photon antibunching

Rabi oscillation

\[ g^{(2)}(\tau) \]