Passive Stabilisation of Optical Cavity Mounted on 3D Piezo Scanner

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A thesis submitted for the degree
B.Sc. (Hons) in Physics
AY2019/20
April 6, 2020
Abstract

For an atom trapped in an optical cavity, there can be a dominant coherent interaction between the atom and the cavity mode when the interaction strength outweighs the loss mechanisms of the system. The interaction strength is large when the atom experiences a large electric field. Hence, one way to maximise the interaction strength is to focus the mode into a small volume. Amongst the commonly used symmetric cavities, concentric cavities are known to have the smallest mode volume. However, it is necessary to keep the cavity stabilised to allow the atom to interact with the mode. In our experiments, we found that our concentric cavity is too unstable, with a noise of $\sim1$ nm. We require the cavity to be passively stable at a noise of below 1 Å for the atom-cavity interaction to proceed smoothly. To study the noise of a cavity, we use a planar cavity with an expected finesse of 6000. We conduct a systematic study of the noise from the mirrors and the piezo scanner individually. Finally, we build the cavity using a pair of mounts. By adding thin metal strips to contact with the mounts, we are able to improve the passive stability from a noise of $\sim3$ nm to a noise of $0.745(8)$ Å. Stabilising the experimental set-up improves the stability to $0.317(2)$ Å.
Acknowledgements

First, I would like to thank my supervisor Prof Christian Kurtseifer for letting me join QO lab despite not having any experience in a proper lab\textsuperscript{1}. It was the correct decision for me.

Next, I would like to thank Adrian for helping me so much over the course of the past year. Thank you for treating me as an equal and entrusting me with Attocube, the star (and problem child) of this thesis. I still remember fondly the night when we thought Attocube was spoilt just after it got delivered when we were just kultzes who didn’t push that one damn banana plug firmly into the power supply.

To the current and past lab members\textsuperscript{2} whom I share the office with, thank you for making the office warm and fun. Honestly, it was more of a circus than an office. I think the amount of nonsense we shared in the room is probably enough to fill up several lab notebooks.

To the rest of the lab, thank you for the pleasant time working together.

Physics education for four years can only be described as intense. This journey has been made tough yet unforgettable by many people who imparted knowledge and helped shaped my perception of physics. Thank you to the lecturers; I would not have learnt as much if not for your insights. This also includes a particular lecturer\textsuperscript{3} that I believe anyone who passes through the gates of NUS Physics will always remember, fondly or not.

I am grateful that there are companions along the way who made studying (read as arguing) physics much more bearable, enjoyable, and possible. It is liberating to finally meet people who are passionate about physics and actually doing physics! Special thanks goes to Dexter, Han Yu, James, Kian Hwee, Ravin, and Ting You.

Another large part of my university life was spent in the confines of the SPS room and associated facilities. The people I met here have contributed to a large part of my mental well-being. From sassy mentors to toxic batchmates to time-consuming side projects, I have experienced many memorable moments.

Lastly, I thank Jingyin for being a constant pillar of support for me throughout these four years. Thanks for keeping me sane every time I start acting like an idiot.

\textsuperscript{1}The only real lab is level 5 lab’ – Anonymous
\textsuperscript{2}Boon Long, Chang Hoong, Florentin, and Philip
\textsuperscript{3}I sincerely hope you get some official recognition soon...
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1 | Introduction

The cavity is a standard workhorse in modern experimental physics. It can be found in applications over a large variety of fields, such as in gravitational wave detection [1,2], single photon [3] and photon-pair generation [4], and in atom interferometry [5]. Beyond its role as a useful tool, there is much anticipation for using cavities as a key instrument in the domain of Quantum Information Science (QIS).

The rise of QIS in the preceding decades originates from the buzz to harness the weirdness of quantum mechanics, superposition and entanglement, as a novel technological resources that can provide possibilities that are not offered by any classical device. For one, there is the proposal of a quantum computer which has information encoded, not in the classical bits of 0 or 1, but in a coherent superposition of 0 and 1 in each quantum bit. Today, various efforts are coordinated to experimentally realise a quantum computer, mainly through superconducting circuits [6,7] or trapped ions [8,9], with tremendous progress being made in recent years [10-12].

While the definitive quantum computer is still not available, there are proposals to enable quantum communication between quantum computers, thus forming a quantum network [13]. A quantum network consists of stationary nodes which store and process quantum information, and flying qubits which transfer the information from one node to another with high fidelity.

A prime candidate for the role of the flying qubits is a photon. Firstly, photons do not interact with one another unless in the presence of a highly non-linear material. This means that the probability of photon-photon scattering in free space or through an optical fibre is negligible. Moreover, the high frequency of optical photons provide a large bandwidth with which we can store information. There is also the added benefit of being able to tap on the vast infrastructure set up by the telecommunication industry, hence reducing the cost of implementing quantum communications.

On the other hand, what constitutes a suitable quantum node? In order to receive the information from the flying qubit, the node should be able to interact strongly with light at the single-photon level, such that the interaction is ‘deterministic’. This limits the possibilities severely. Most single atom or ion-based ideas are eliminated due to the low absorption probability of a single photon [14-16]. The probability is given by the ratio between
the absorption cross-section, which scales with the square of the wavelength, and the area of the laser beam at the position of the atom. Thus, the more strongly focused the beam is at the position of the atom, the smaller the area of the beam, and probability increases. However, the extent of focusing is restricted by the diffraction limit, which limits the diameter to be on the order of the wavelength. Evidently, the probability of an atom absorbing a single photon in free space is typically lower than desired.

A standard technique to improving the interaction strength between an atom and a single photon is to trap the atom within an optical cavity. Intuitively, the absorption probability is dramatically enhanced because the photon makes many roundtrips between the mirrors before it escapes the cavity. In that case, the probability of absorption is multiplied by the number of roundtrips, which can be made as large as several thousands. A more thorough study of the modifications of atom-light interactions via a cavity is developed in the field of Cavity Quantum Electrodynamics (QED).

### 1.1 A Primer to Cavity QED

A review of Cavity QED can be consulted in [17] [18]. Here, we present the main ideas needed to appreciate the use of a cavity to enhance atom-light interactions. Essentially, a cavity presents a natural scenario in which the modes of the electromagnetic field is quantised. The description of the interaction between an atom and the quantised mode now takes on a quantum spin.

In the most representative model of Cavity QED, an atom is at rest within the mode of a cavity. If the cavity mode is resonant with one of the atomic transition, then the atom can be effectively treated as a two-level system coupled to the cavity mode. The dynamics of this system is governed by the Jaynes-Cummings Hamiltonian [19]:

$$H_{JC} = \hbar \omega_a \sigma^+ \sigma^- + \hbar \omega_c a^\dagger a + \hbar g(r) \left(a^\dagger \sigma^- + \sigma^+ a\right),$$  \hspace{2cm} (1.1.1)

where $\omega_a$ and $\omega_c$ are the resonant frequencies of the atom and the cavity respectively. The pair of operators $a^\dagger$ and $a$, and $\sigma^+$ and $\sigma^-$ are the raising and lowering operators for the cavity and atom respectively. The first two terms in the Hamiltonian refer to the energy levels of the atom and cavity respectively, whereas the third term is describes an exchange of photons.
between the atom and cavity.

The photon in the cavity has a spatial mode given by the dimensionless mode structure $u(r)$. The coupling strength $g(r)$ is proportional to it via

$$g(r) = \left( \frac{\mu^2 \omega_c}{2 \hbar \epsilon_0 V} \right)^{\frac{1}{2}} u(r) \equiv g_0 u(r),$$

(1.1.2)

where $\mu$ is the dipole matrix element of the transition, and $V$ is the mode volume defined by $V = \int |u(r)|^2 d^3r$. The prefactors are collected into a single parameter $g_0$, which is inversely proportional to $\sqrt{V}$. Hence, we can expect to increase the interactions between the atom and cavity mode by decreasing the mode volume. Moreover, $2g_0$ is known as the single-photon Rabi frequency, which is the maximum rate at which a photon is exchanged coherently between the atom and the cavity.

The model above has been overly idealised as it does not account for dissipative processes that naturally accompany actual systems. There are two possible decay channels through which the photon may escape the cavity and disrupt the coherent interaction. Firstly, the mirrors are not perfectly reflective, hence photons may leak out at the cavity decay rate $2\kappa$. Secondly, each time the atom absorbs a photon, it will de-excite through spontaneous emission. The photon may be emitted at the correct frequency but in a direction that does not coincide with the cavity mode. Another possibility is that the photon is of a different frequency entirely. The decay rate of the atom occurs at a rate $2\gamma$.

The three parameters $g_0$, $\gamma$, and $\kappa$ define a characteristic time-scale for the dynamics of the system. If $\gamma$ or $\kappa$ is much larger than $g_0$, then the interaction between the cavity and the atom gets disrupted easily and the system becomes decoherent. This regime is known as weak coupling. On the other hand, we aim to have coherent exchange of photons between
1.2. BRIEF SURVEY OF PREVIOUS WORKS

Figure 1.2: Illustration of the modification of the transmission of a cavity by an atom trapped in the cavity mode. This is still in the weak coupling regime as $C \ll 1$. Image taken from [20].

the atom and cavity for the timescale of the relevant application. Thus, the strong coupling regime is achieved when we have $g_0$ being much larger than the other two rates. Another way of defining the strong coupling regime is to look at the single-atom cooperativity:

$$C = \frac{g_0^2}{2\gamma\kappa}. \quad (1.1.3)$$

The single-atom cooperativity quantifies the extent the transmission of the cavity would be modified by an atom within the cavity mode. In Figure 1.2 there is a shallow dip in the transmission at the resonance of the atom, which indicates that the cavity has interactions with the atom, but at the weak coupling regime. For $C \gg 1$, the transmission is expected to be drastically modified, which is equivalent to strong coupling between the atom and cavity.

1.2 Brief Survey of Previous Works

As a first step to achieving strong coupling, we aim to maximise $C$, either by increasing $g_0$ or by lowering $\kappa$. It is possible to do both simultaneously through the design of a cavity. It is
more straightforward to discuss the minimisation of $\kappa$ as

$$\kappa = \frac{\pi c}{2LF},$$

(1.2.1)

where $L$ is the length of the cavity, and $F$ is the finesse. Thus, a cavity with extremely high finesse can achieve low values of $\kappa$. Indeed, this is the strategy employed in [21, 22], which uses a cavity of finesse on the order of millions. However, this technique cannot be scaled up easily due to the sophisticated manufacturing process of the mirrors to produce such values of finesse.

An alternate approach is to use a relatively larger cavity with a lower finesse, and strongly focus the mode such that the mode volume is small. This is the approach taken by our group, whereby we use a (near-)concentric cavity, which is a geometrical configuration defined by the ratio of the cavity length to the radius of curvature of the mirrors. In the case of concentric cavities, the length is twice the radius of curvature. The advantage of a concentric cavity over the common configurations like planar and confocal cavities is that it focuses the mode the greatest extent, while having the largest length for the same radius of curvature [23]. In other words, the concentric cavity minimises both the mode volume and the cavity decay rate, hence it is possible to achieve strong coupling at a less stringent value of finesse.

However, a flaw of concentric cavities is its sensitivity to misalignment of the mirrors when it approaches the concentric point. For this purpose, it is important to be able to stabilise the mirror positions using a feedback system. To what precision do we need to adjust the position? The transmission of the cavity as one mirror is moved in the transverse direction is plotted in Figure 1.3. At a cavity length of 11 mm, a transverse displacement of merely 60 nm, which is a minuscule ratio of $\sim 10^{-5}$, is sufficient to destroy the cavity mode. At the same time, displacements along the longitudinal direction changes the resonant frequency of the cavity, resulting in a detuning between the atom and cavity mode. For this direction, a displacement of a few nm is sufficient to interrupt the interactions between the cavity and atom. Thus, we need to be able to correct the misalignments with sub-nm resolution with a relatively quick response time. This can be done using actuators such as piezoelectric materials. These materials expand or contract when a voltage is applied to them.

In the first iteration, we use a piezostack that allows movement range of $\pm 5\ \mu m$. An

\footnote{Read as reasonably cost effective}
1.3. RELATION OF THESIS TO PREVIOUS WORK

In upgrading the concentric cavity, we make two changes. The first is to replace the mirrors with higher reflectivity to achieve a finesse of $\sim$600. Next, we find that when baking out the vacuum for experiments, the cavity gets misaligned due to thermal expansion. The correction required exceeds the 5 $\mu$m range of the piezostack. Hence, we replace the piezostack with a piezo scanner with a flexure stage, granting a larger movement range of 50 $\mu$m.

In this new configuration, we find that we are unable to trap the atom long enough to make meaningful measurements. Our analysis reveals after compensating for the low finesse, the length fluctuations in the longitudinal direction, measured to be $\sim$1 nm, is large enough to disrupt the cavity modes. Even with an active control loop, the length of the cavity does not...
not stabilise within a tolerable range for the experiment to proceed smoothly. Hence, we need to improve the passive stability of the cavity. A passively stable cavity is one that has a low length fluctuations as it stands, without relying on the use of any corrective measures.

1.3.1 Goal of Thesis

Our calculations suggest that at the current finesse of 600, the tolerable range for the length fluctuation is less than 1 Å. This means that we have to improve the passive stability of the cavity to around 1 Å, which is an improvement by a factor of 10 compared to the current value of 1 nm. The passive stabilisation should not interfere with the movement range of the piezo scanner as much as possible.

There are several steps taken to conduct the study of the passive stability:
1. Prepare a set-up that can measure length fluctuations to sub-Å resolution.
2. Build a test cavity to study the possible mechanisms to damp the mechanical vibrations.
3. Incorporate the piezo scanner and assess the feasibility of implementing the improvements found in the previous step.

1.3.2 Organisation

The thesis is organised as follows. In Section 2, we review some important concepts regarding cavities. In Section 3, we briefly introduce a key technique for our measurement known as Frequency Modulation (FM) spectroscopy. We provide an overview to the experiment procedure and the various considerations taken in the experiment in Section 4. Finally, we discuss the two main phases of our experiments in Sections 5 and 6. The first phase involves a preliminary investigation into the noise spectrum of a test cavity. We apply the ideas gleaned from the first phase to the cavity mounted to the piezo scanner in the second phase. Finally, we conclude the thesis in Section 7 and make a few comments on extensions to the work presented.
The main subject of this thesis is optical cavities, which comprise two reflective surfaces enclosing an optical medium. In this configuration, light incident on a cavity will be selectively transmitted depending on the frequency, whereas the rest are reflected. A review of the pertinent properties of optical cavities is covered in this section to establish the considerations needed in designing a stable cavity.

2.1 Properties of a Cavity

The simplest cavity model is formed by two plane mirrors separated by a distance $L$, as shown in Figure 2.1. Each mirror has its own reflectivity $r_i$ and transmissivity $t_i$, and is assumed to be lossless. A beam incident on the cavity will give rise to a reflected, transmitted, and circulating part. This simple model is sufficient to deduce the main properties of a cavity.

2.1.1 Cavity Transfer Function

To see how a cavity allows only selected frequencies to transmit, we consider the effect of the mirrors on the incident beam as it propagates from one end to the other. Naturally, the beam can be reflected several times between the two mirrors before it is transmitted. Hence, the net transmitted beam can be viewed as the superposition of beams that have made different numbers of round trip within the cavity. Each round trip would also incur a phase lag of $e^{-2i\phi}$.

![Figure 2.1: An idealised cavity made of two plane mirrors. The mirrors are lossless, and have reflectivity $r_i$ and transmissivity $t_i$. As the incident beam propagates, it will be reflected and transmitted indefinitely. The overall effect is to split the incident beam into three parts: reflected, transmitted and circulating.](image-url)
per round trip. The phase angle $\phi$ is given by

$$\phi(k) = nkL,$$  \hspace{1cm} (2.1.1a)

$$\phi(\omega) = \frac{n\omega L}{c},$$  \hspace{1cm} (2.1.1b)

where $n$ is the refractive index of the optical medium between the mirrors, $k = 2\pi/\lambda$ is the wave number in vacuum, and $\omega = 2\pi\nu$ is the angular frequency. Since the optical medium is typically vacuum or air for the experiments in this thesis, $n$ takes the value 1 henceforth. Note that the phase lag depends on the frequency or wavelength of the incident beam.

For an incident plane wave of amplitude $E_0$, the transmitted beam is

$$E_{\text{trans}} = E_0 t_1 t_2 e^{-i\phi} \left(1 + r_1 r_2 e^{-2i\phi} + (r_1 r_2 e^{-2i\phi})^2 + \ldots\right)$$

$$= E_0 t_1 t_2 \frac{e^{-i\phi}}{1 - r_1 r_2 e^{-2i\phi}}.$$  \hspace{1cm} (2.1.2)

The transfer function for transmission of the cavity is defined as the ratio between the output beam and the input beam:

$$T(\omega) \equiv \frac{E_{\text{trans}}}{E_0} = \frac{e^{-i\phi}}{1 - r_1 r_2 e^{-2i\phi}},$$  \hspace{1cm} (2.1.3)

where the constant prefactors in (2.1.2) have been absorbed into the normalisation constant. In words, the transfer function characterises a cavity’s effect on the incident beam. For a beam of given $\omega$, the transmitted beam has the same $\omega$, but has its amplitude scaled by a factor of $|T(\omega)|$, and a phase shift of $\arg(T(\omega))$ is incurred. The physical situation is analogous to a driven harmonic oscillator, thus justifying the name ‘optical resonator’.

### 2.1.2 Resonance Condition

The transfer function relates the amplitudes of the incident and transmitted beam. However, the intensity of the beam is the measurable quantity, hence it is more useful to examine the modulus square of $T(\omega)$. We find that, after another normalisation,

$$T(\omega) = |T(\omega)|^2 = \frac{1}{1 + \left(\frac{4r_1 r_2}{(1 - r_1 r_2)^2} \sin^2(\phi)\right)}$$

$$= \frac{1}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2(\phi)}.$$  \hspace{1cm} (2.1.4)
2.1. PROPERTIES OF A CA VITY

where the pre-factor that depends only on the mirror reflectivities can be collected into a single parameter

\[ F = \pi \sqrt{r_1 r_2} \frac{1}{1 - r_1 r_2}, \]  \hspace{1cm} (2.1.5)

which is commonly termed the finesse of the cavity.

The function given in (2.1.4) is known as the Airy distribution (Figure 2.2), which achieves a maximum whenever the sine function is zero. This determines the resonance condition

\[ \phi = 2m\pi, \quad m = 1, 2, 3, \ldots \]  \hspace{1cm} (2.1.6)

which can be explicitly written as a function of the frequency of the incident beam as

\[ \nu = m\frac{c}{2L}. \]  \hspace{1cm} (2.1.7)

In fact, resonance is achieved for multiple input frequencies which are equally spaced apart. This frequency spacing is termed the free spectral range:

\[ \Delta \nu_{FSR} = \frac{c}{2L}. \]  \hspace{1cm} (2.1.8)

Only the length of the cavity appears as a free parameter, which means that the resonant frequencies are characterised solely by the length of the cavity and we can engineer the desired resonant frequencies by an appropriate choice of the cavity length. Another way to interpret this is to introduce the wavelength instead, yielding

\[ L = m\frac{\lambda}{2}, \]  \hspace{1cm} (2.1.9)

which states that the incident beam must form a standing wave between the mirrors. In the same manner, we can define a free spectral range in length, defined by

\[ \Delta L_{FSR} = \frac{\lambda}{2}. \]  \hspace{1cm} (2.1.10)

Both pictures are equally useful when analysing a cavity.

From Figure 2.2 we see that the resonance peak becomes sharper as the finesse of the cavity increases. Thus, the finesse can be seen as the parameter that controls the width of the resonance peak, serving as the optical equivalent of the quality factor in mechanical oscillators. We can limit or widen the bandwidth of the transmission frequency by choosing
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Figure 2.2: Plot of the Airy distribution for different values of the finesse. As the finesse increases, the resonance peak becomes sharper and the lowest transmission drops to nearly zero.

the reflectivity of the mirrors. A parameter for quantifying the bandwidth of a cavity will be introduced in the subsequent section.

2.1.3 Off-resonant Transmission

A realistic situation involves a small detuning between the cavity and one of the resonant frequencies. Such a situation often arises because the incident beam fluctuates in frequency or when the cavity length changes slightly due to interactions with the environment. As a result, the phase angle of the transmitted beam changes from the resonant value. Thus, we are interested in analysing the changes in the transmission due to a detuning from resonance.

This is achieved via a Taylor expansion of the sine function in (2.1.4) by a small angle \( \delta \phi \) to leading order:

\[
T(\omega) = \frac{1}{1 + \left( \frac{2F}{\Delta \nu_{\text{FSR}} \frac{\delta \omega}{2\pi}} \right)^2}
\]  

(2.1.11)

where \( \delta \omega \) is the value of detuning from the resonance.

The transmission is now described by the Lorentzian distribution. From this expression, it is clear that the transmission drops to half when \( \delta \omega = 2\pi \times \frac{\Delta \nu_{\text{FSR}}}{2F} \). This value of detuning defines the half-width-at-half-maximum (HWHM)

\[
\kappa = 2\pi \times \frac{\Delta \nu_{\text{FSR}}}{2F} = 2\pi \times \Delta \nu_{\text{HWHM}}.
\]  

(2.1.12)
2.1. PROPERTIES OF A CAVITY

for which we may take as the characteristic bandwidth, or linewidth, of the cavity. It is also common to use the full-width-at-half-maximum (FWHM) as the measure of the linewidth, which is twice the HWHM. Evidently, the linewidth is inversely proportional to the finesse. This offers the alternative view that the finesse can be seen as the number of FWHM that can be fitted within one $\Delta \nu_{\text{FSR}}$. In this thesis, we refer to the HWHM as the linewidth.

In summary, we have two expressions for the transmission of a cavity: the Airy distribution of (2.1.4) and the Lorentzian distribution of (2.1.11). A comparison of the two about the same resonance frequency is shown in Figure 2.3. The Lorentzian distribution rapidly approaches the Airy distribution for modest values of finesse (>10). In the remainder of the thesis, the Lorentzian distribution takes the form

$$T(\omega) = \frac{1}{1 + \left(\frac{\omega - \omega_0}{\kappa}\right)^2},$$

(2.1.13)

where $\omega_0$ is the resonant angular frequency.

Figure 2.3: Comparison of the Airy (solid blue) and the Lorentzian (dashed orange) distribution for increasing values of the finesse. The match between the two increases as the finesse increases.
2.2 Cavity Modes

The essential features of an optical cavity have been identified by considering plane waves and plane mirrors. However, real cavities typically use spherical mirrors, which have a radius of curvature. Moreover, the incident field is rarely a plane wave; it has spatial variations in amplitude and phase. Thus, it is necessary to take these factors into account for a more complete description of the cavity.

2.2.1 Paraxial Approximation

The field pattern of an electromagnetic wave is determined by the wave equation obtained from Maxwell’s equations. In particular, if we assume that the solution is separable into the product of a function solely of time and a second function solely in spatial coordinates $u(x, y, z)$, then $u(x, y, z)$ is a solution to the Helmholtz equation

$$(\nabla^2 + k^2)u(x, y, z) = 0. \quad (2.2.1)$$

Taking the wave to propagate along the $z$ direction, an ansatz is

$$u(x, y, z) = U(x, y, z)e^{ikz}, \quad (2.2.2)$$

where the amplitude $U(x, y, z)$ has spatial variations instead of constant as in plane waves. This ansatz represents a deviation from the idealised plane wave. Certainly, $U(x, y, z)$ must fulfill some conditions in order for the ansatz to be a valid solution to the Helmholtz equation.

As a starting step, we may assume that the spatial variations along the $z$-direction is small as compared to the transverse directions, leading to the paraxial approximation:

$$\frac{\partial U(x, y, z)}{\partial z} \ll kU(x, y, z), \quad \frac{\partial^2 U(x, y, z)}{\partial z^2} \ll k\frac{\partial U(x, y, z)}{\partial z}. \quad (2.2.3)$$

The paraxial approximation allows us to reduce (2.2.1) into the paraxial Helmholtz equation:

$$\nabla^2_{\perp} U(x, y, z) + 2ik\frac{\partial U(x, y, z)}{\partial z} = 0, \quad (2.2.4)$$

where $\nabla^2_{\perp} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian in the transverse directions.
2.2. CAVITY MODES

2.2.2 Optical Stability

Before solving (2.2.4), it is useful to consider the conditions for which we may anticipate the existence of a solution. A light field may only build up within the cavity if it repeats itself after a round trip. Therefore, a valid solution is one that admits an intensity distribution that reproduces itself after a round trip. Such solutions are termed as eigenmodes of the cavity.

In order to support the eigenmodes, the cavity must be stable. Each time the field makes a round trip, the field pattern changes as it propagates from one mirror to the next, and when it is reflected off the mirrors. It may be possible for the field to change significantly that it cannot be contained within the cavity and escapes. In this case, the field has diverged and we say that the cavity is unstable. On the other hand, if the field does not change appreciably, it can be contained within the cavity and the eigenmode is achieved.

Quantitatively, the cavity is represented by a $ABCD$ matrix $M$, in which the matrix elements depend on the geometry of the system. The problem then reduces to finding the numerical conditions for which $M^j$ has finite eigenvectors when $j$, the number of round trips in the cavity, becomes large. For a Fabry-Perot cavity which has mirrors of radii of curvature $R_1$ and $R_2$ separated by a distance $L$, this leads to the stability criterion

$$0 \leq g_1 g_2 \leq 1,$$

(2.2.5)

where the $g_1$ and $g_2$ are the stability parameters of the mirrors:

$$g_i = 1 - \frac{L}{R_i}.$$

(2.2.6)

It is convenient to visualise the stability criterion as points in the $g_1g_2$ plane (Figure 2.4). All stable cavity configurations are given by the region that is bounded by the coordinate axes and the hyperbola $g_1g_2 = 1$.

A common practice is to use symmetric cavities, where the two mirrors have the same $R$. In this regime, there are three extremal cases for which the cavities are close to instability, indicated by the red circles in Figure 2.4. They are the

- Planar: $L \gg R$.
- Confocal: $L = R$.
- Concentric: $L = R/2$. 

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Figure 2.4: The stability diagram for optical cavities formed by two mirrors. The stable region is given by the shaded area (boundaries inclusive). The symmetric configurations are given by the dotted line, and the (near) unstable cases are indicated by the red circle.

Each type has its own strengths and disadvantages, which will not be discussed in details. Instead, we focus on the advantage of concentric cavities in the context of Cavity QED. As mentioned in the introduction, they provide a very strong focusing of the cavity mode, with a beam waist of

$$w_0^2 \approx \frac{L \lambda}{\pi} \sqrt{\frac{2R - L}{8L}}$$

(2.2.7)

which can achieve a few $\mu$m, significantly smaller than the beam waists produced by the other two cavities. As a result, the cavity field is concentrated to a mode volume that is comparable to the absorption cross-section of the atoms trapped in the location of the focused field. This would, in principle, enable strong interactions between the atom and the cavity mode.

2.2.3 Eigenmodes

For a stable cavity, any valid solution to the paraxial Helmholtz equation (2.2.4) can be expanded using the set of basis functions called the Hermite-Gaussian (HG) modes

$$u_{nm}(x, y, z) = U_0 \frac{w_0}{w(z)} H_n \left( \frac{\sqrt{2}x}{w(z)} \right) H_m \left( \frac{\sqrt{2}y}{w(z)} \right) \exp \left( -\frac{x^2 + y^2}{w(z)^2} \right) \times \exp \left( -i \left( k z - \Phi(n, m; z) + k \frac{x^2 + y^2}{2R(z)} \right) \right)$$

(2.2.8)
2.2. CAVITY MODES

(a) Hermite-Gaussian Modes. The indices follow the convention HG_{nm}.

(b) Laguerre-Gaussian Modes. The indices follow the convention LG_{lp}.

Figure 2.5: Intensity distributions of some HG and LG modes. Note that the fundamental modes of the two bases are the same.

where the functions $H_n$ and $H_m$ are the Hermite polynomials of order $n$ and $m$. The spot size $w(z)$ describes the characteristic size of the beam as viewed in the transverse plane, for which the smallest value is denoted as the beam waist $w_0$. $R(z)$ represents the radius of curvature of the wavefronts. Finally, the function $\Phi(n, m; z)$, called the Gouy phase, is a modification to the phase of the beam due to the transverse spatial confinement [25].

Due to the modifications in the phase, the resonance condition now leads to

$$\nu_{qnm} = \frac{c}{2L} \left( q + (n + m + 1) \frac{\arccos \left( \frac{\pm \sqrt{g_1 g_2}}{\pi} \right)}{\pi} \right),$$

(2.2.9)

where $q$ is also an integer. For any given pair of $n$ and $m$, the frequencies differ by $\frac{c}{2L}$, which is essentially the free spectral range. Thus, $q$ indicates the longitudinal modes, which are modes that have the same transverse intensity patterns and are separated by the free spectral range, whereas the $n$ and $m$ refer to transverse modes, where the intensity patterns change.

When the cavity possess cylindrical symmetry, another set of basis functions can be used to describe the solution. They are the Laguerre-Gaussian (LG) modes [26]:

$$u_{lp}(r, \phi, z) = \frac{C_{lp}}{w(z)} \left( \frac{r \sqrt{2}}{w(z)} \right)^{|l|} L^{|l|}_p \left( \frac{2r^2}{w(z)^2} \right) \exp \left( -\frac{r^2}{w(z)^2} \right) \times \exp \left( -i \left( k z - \Phi(l, p; z) + k \frac{r^2}{2R(z)} + i \phi \right) \right),$$

(2.2.10)
where $C_{lp}$ is a mode-dependent normalisation constant, and $L_p^{||}$ represents the generalised Laguerre polynomials. In an analogous manner, the resonant frequencies are given by

$$\nu_{alp} = \frac{c}{2L} \left( q + (|l| + 2p + 1) \arccos \left( \pm \sqrt{g_1g_2} \right) \right).$$

(2.2.11)

Figure 2.5 shows the intensity distribution for some of the HG and LG modes. Despite the seemingly different form for the frequencies of the transverse modes between HG and LG, the two bases are equivalent. This means there is a mapping between the two bases. For example, LG_{10} is a linear combination of HG_{10} and HG_{01}. In particular, the fundamental mode, known as the Gaussian mode, is the same for both.

2.2.4 Mode-Matching

By taking the actual cavity geometry into consideration, the cavity can accommodate several modes at the same time. However, the most common usage of a cavity is to couple only a single mode. The process of maximising the coupling of the preferred mode while simultaneously minimising the coupling to the rest is called mode-matching.

The main step taken to perform mode-matching is to shape the spot size of the incoming beam via a system of lenses. By a judicious choice of focal lengths, the incoming beam can be focused such that the beam waist is located at the centre of the cavity, and the radius of curvature of the wavefronts of the field at the mirrors match with the radius of curvature of the mirrors. In essence, this ensures that the incoming mode satisfies the boundary conditions of the cavity. Subsequently, the frequency of the beam or the length of the cavity can also be tuned to match the mode. Experimentally, mode-matching is achieved when the observed transmission is maximised.
In many experiments, the goal is to keep the frequency of a laser stable at a fixed frequency, with relatively small fluctuations away from this frequency. In such situations, it is critical that one is able to measure the detuning of the laser from the reference frequency and bring the laser back to resonance. This reference frequency is usually set by what is required of the experiment. For example, a laser needs to be in resonance with an atomic transition in order to probe the state of the atom. To keep things general, the sample that provides the reference frequency will be called the frequency reference.

The measurement of the detuning is derived from an error signal. A good error signal is dispersive\(^1\); it is a function of detuning that has a steep linear gradient for small values of detuning, and is zero on resonance. A steep slope is preferred so that small detunings are easily discerned and corrected. To generate the error signal, a common technique used in practice is Frequency-Modulation (FM) spectroscopy\(^{27,28}\). The general principles of FM spectroscopy and its implementations in the thesis are discussed in this section.

### 3.1 General Principles

A frequency modulation can be introduced through a time-dependent phase shift. In the case of a sinusoidal modulation with a modulation index \(\beta\) and modulation frequency \(\omega_m\), the modulated beam, modelled as a plane wave, with a (carrier) frequency \(\omega_0\) is given by

\[
E(t) = E_0 e^{i(\omega_0 t + \beta \sin(\omega_m t))}
= E_0 e^{i\omega_0 t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{i\omega_m t},
\]

where \(J_n(\beta)\) are the Bessel functions of the first kind. The phase-modulated beam can be interpreted as a superposition of a strong beam with a carrier frequency of \(\omega_0\), and many pairs of weaker sidebands centred about the carrier frequency. In the case of optical frequencies (THz), \(\omega_0 \gg \omega_m\) as the modulation frequency is usually chosen to be a radio frequency (MHz). For FM spectroscopy, the modulation index is small, hence \(\beta \ll 1\) and only the

\(^1\)Not to be confused with dispersion of an optical medium, which talks about the frequency dependence of the refractive index.
\( n = \pm 1 \) sidebands contribute significantly. Thus, the modulated beam in this regime is given by

\[
E(t) = E_0 e^{i\omega_0 t} \left( 1 + \frac{\beta}{2} e^{i\omega_m t} - \frac{\beta}{2} e^{-i\omega_m t} \right). \tag{3.1.2}
\]

After the modulation, the modulated beam is sent through the frequency reference, which has its characteristic transfer function \( T(\omega) \) that determines its interaction with the beam, similar to the transmission transfer function of the cavity in (2.1.3). Since the transfer function is a complex number, it can be written as

\[
T(\omega) = |T(\omega)| e^{-i\phi} = e^{-\delta(\omega)-i\phi(\omega)}. \tag{3.1.3}
\]

The real exponent indicates the attenuation of the amplitude, which invites the interpretation of \( \delta(\omega) \) as the absorption spectrum commonly encountered in spectroscopy. On the other hand, the imaginary exponent reflects changes in phase, hence \( \phi(\omega) \) is the dispersion.\(^2\)

The dependency on \( \omega \) states that the carrier and the sidebands would undergo different modifications upon interaction with the frequency reference, which leads to an output of

\[
E_{\text{out}}(t) = E_0 e^{i\omega_0 t} \left( T_0 + T_1 \frac{\beta}{2} e^{i\omega_m t} - T_{-1} \frac{\beta}{2} e^{-i\omega_m t} \right), \tag{3.1.4}
\]

where \( T(\omega_j) \equiv T_j \) with \( j = 0, \pm 1 \) representing the carrier and sideband frequencies respectively. In order to extract the spectroscopic information, the output beam is sent to a photodiode. Once again, only the intensity is measurable. Ignoring terms of \( \beta^2 \) and above, the intensity is

\[
I(t) = I_0 e^{-2\delta_0} \left[ 1 + \left( e^{\delta_0-\delta_1} \sin(\phi_1 - \phi_0) - e^{\delta_0-\delta_{-1}} \sin(\phi_0 - \phi_{-1}) \right) \beta \sin(\omega_m t) \\
+ \left( e^{\delta_0-\delta_1} \cos(\phi_1 - \phi_0) - e^{\delta_0-\delta_{-1}} \cos(\phi_0 - \phi_{-1}) \right) \beta \cos(\omega_m t) \right]. \tag{3.1.5}
\]

The intensity detected by the photodiode has an envelope that oscillates with a beat frequency of \( \omega_m \) due to the interference between the carrier and the sidebands. Thus, the photodiode measures both a DC (stationary in time) and AC current, but the spectroscopic information is encoded only in the AC current. By extracting the AC current and demodulating it, we can retrieve the coefficients of the sine or cosine term. It turns out that either components can be used as the error signal.

\(^2\)Not to be confused with dispersive.
3.1. GENERAL PRINCIPLES

To gain some intuition for how we can extract the error signal from the in-phase or quadrature components, we assume that $|\delta_0 - \delta_1|$, $|\delta_0 - \delta_{-1}|$, $|\phi_0 - \phi_1|$, and $|\phi_0 - \phi_{-1}|$ are all $\ll 1$. Ignoring terms of second order, the intensity is

$$I(t) = I_0 e^{-2\delta_0} \left[ 1 + (\phi_1 + \phi_{-1} - 2\phi_0) \beta \sin(\omega_m t) + (\delta_{-1} - \delta_1) \beta \cos(\omega_m t) \right] \quad (3.1.6)$$

The coefficient of the cosine term is evaluated as

$$\delta_{-1} - \delta_1 = \delta(\omega_0 - \omega_m) - \delta(\omega_0 + \omega_m) = -2\omega_m \frac{\partial \delta(\omega)}{\partial \omega} \bigg|_{\omega=\omega_0} \quad (3.1.7)$$

which is proportional to the first derivative of the absorption spectrum at the carrier frequency. By varying $\omega_0$ while keeping $\omega_m$ fixed, one can sample the derivative at different frequencies. On resonance, the absorption spectrum is stationary, hence the first derivative is zero and the error signal will have a value of zero. This point is also known as the zero crossing. For detunings close to the resonance, the first derivative is indeed linear in the detuning. Repeating the same analysis for the other term demonstrates that it is the second derivative of the dispersion. Either quantity satisfies the aforementioned dispersive quality, hence they are valid error signals.

The assumption that $|\delta_0 - \delta_1|$, $|\delta_0 - \delta_{-1}|$, $|\phi_0 - \phi_1|$, and $|\phi_0 - \phi_{-1}|$ are small is only valid when $\omega_m$ is smaller than the bandwidth $\Gamma$ of the spectral feature of the frequency reference. For example, the linewidth of a cavity would be taken as $\Gamma$. This regime is known as the slow modulation regime. Likewise, there is also a fast modulation regime, where $\omega_m \gg \Gamma$. In that regime, the error signal is no longer the derivatives although they are still dispersive. The exact expression given in (3.1.5) covers all possible range of $\omega_m$, which will be used for simulation and fitting of error signals.

Finally, in the demodulation step, the AC signal from the photodiode is mixed with the modulation signal via a mixer, which simply multiplies the two signals together. Focusing on only the AC signal, the mixer generates

$$I_{AC}(t) \cos(\omega_m t + \Delta) = A \sin(\omega_m t) \cos(\omega_m t + \Delta) + B \cos(\omega_m t) \cos(\omega_m t + \Delta)$$

$$= \frac{A}{2} [\sin(\Delta) + \sin(2\omega_m t + \Delta)] + \frac{B}{2} [\cos(\Delta) + \cos(2\omega_m t + \Delta)],$$

(3.1.8)
where $\Delta$ is a phase shift that can be added to the modulation signal. $A$ and $B$ refer to the coefficients of the sine and cosine term in \((3.1.5)\). After mixing, the DC signal is extracted using a low-pass filter to obtain the error signal

$$\epsilon(\Delta) = \frac{A}{2} \sin(\Delta) + \frac{B}{2} \cos(\Delta).$$

\((3.1.9)\)

where we can extract either the in-phase ($\propto \cos(\omega_m t)$) or quadrature ($\propto \sin(\omega_m t)$) components, or a mixture of both. The inclusion of $\Delta$ allows the optimisation of the amplitude and slope of the error signal. This is partly due to delays between the modulation signal and photodiode signal arising from the different paths taken to reach the mixer, which can be corrected by introducing $\Delta$.

A typical experimental setup for FM spectroscopy is shown in Figure 3.1. The phase modulator is usually an electro-optic modulator (EOM). The local oscillator is a radio frequency (RF) source, such as a direct digital synthesiser, that provides the modulation frequency. The phase shifter and the mixer can be integrated into a single circuit board.
3.2 Implementations

Two types of FM spectroscopy techniques are used in this thesis. The first is the Pound-Drever-Hall (PDH) method [29, 30], which uses an optical cavity as the frequency reference, and the second is the Modulation-Transfer Spectroscopy (MTS) [31], which uses an atomic species as the frequency reference.

3.2.1 Pound-Drever-Hall

In PDH, one may choose to sample the modulated beam that has been transmitted or reflected off the cavity. The usual choice is to work with the reflection due to the ease of preparation and control. One has to carefully mode-match the incoming beam with the cavity to maximise the intensity of the transmission before the signal can be detected; in contrast, most of the power is reflected off the cavity, hence one only needs to limit the intensity sent to the photodiode.

Some examples of the error signal generated through the in-phase and quadrature terms of the transmission and reflection are shown in Figure 3.2. They are divided into the slow ($\frac{\omega_m}{\kappa} = \frac{1}{20}$), intermediate ($\frac{\omega_m}{\kappa} = 1$), and fast ($\frac{\omega_m}{\kappa} = 20$) regimes. While the exact shapes and amplitudes differ, the linear slope about resonance is present as advertised. For those that do not have a linear slope, or the linear slope spans a small range, they can be optimised by tuning $\Delta$, as demonstrated by the comparison between the in-phase and quadrature components.

The error signals shown are ideal. In practice, the reflection introduces background noise in the off-resonant regions due to imbalances between the $\delta$-s and $\phi$-s of the carrier and the sidebands. In more severe cases, the background noise can obscure the position of the zero crossing and prevent us from locking to the correct frequency. Using the transmission eliminates this problem because there is close to no transmission in the off-resonant regions, hence there is no signal.

Usually the PDH method is used to lock a laser’s frequency so that it remains stable over a long period of time. In the context of this thesis, it is used to measure the detuning between the cavity and the laser induced via changes in the length of the cavity. This will be discussed greater details in the next chapter.
3.2.2 Modulation-Transfer Spectroscopy

In standard FM spectroscopy on atoms, an intense pump beam is used to saturate the absorption of the atoms, while a much weaker probe beam is used to probe the Doppler-free absorption spectrum of the atom. By modulating the probe beam, an error signal is produced.

In the case of MTS, the reverse occurs. The pump beam is modulated instead. Using the atomic vapour as a non-linear medium, the pump beam, either sideband, and the probe beam undergo a non-linear process known as four-wave-mixing (FWM) to transfer the modulation to the probe beam \[32\]. Since FWM occur only when the pump beam is resonant with an atomic transition, MTS produces error signals that is almost free of background, since there is no modulation away from the atomic transition.

The AC signal for MTS is given by

\[
I_{AC}(t) = \frac{C}{\sqrt{\Gamma^2 + \omega_m^2}} \left[ (D_1 - D_{-\frac{1}{2}} - D_{-\frac{1}{2}} - D_{-1}) \sin(\omega_m t) + \left( L_{\frac{1}{2}} + L_{-\frac{1}{2}} - L_{-1} \right) \cos(\omega_m t) \right],
\]

(3.2.1)

where

\[
L_n = \frac{\Gamma^2}{\Gamma^2 + (\omega - n\omega_m)^2},
\]

(3.2.2)

and

\[
D_n = \frac{\Gamma(\omega - n\omega_m)}{\Gamma^2 + (\omega - n\omega_m)^2}.
\]

(3.2.3)

The constant \(\Gamma\) is the natural linewidth of the atomic transition, and \(C\) represents all other properties of the medium and probe beam. Once again, the in-phase and quadrature terms are related to the absorption and dispersion of the atom. Indeed, the error signals obtained are dispersive too.

The expression given in (3.2.1) allows us to simulate and fit the error signal obtained from MTS. In this thesis, it is used to characterise the inherent noise of the laser, independent of the noise due to the cavity.
3.2. IMPLEMENTATIONS

Figure 3.2: Ideal error signals obtained from the PDH method using the transmission (blue solid) and reflection (orange dotted) in the slow, intermediate, and fast modulation regimes. The in-phase components are grouped in the left column, whereas the right column contains the quadrature components.
Coaxing a Cavity

As mentioned in the introduction, the goal of this thesis is to improve upon the passive stability of the cavity when it is mounted to the piezo scanner. To quantify the passive stability, we first need to establish a quantitative value for the noise in the cavity. Here, noise refers solely to the length fluctuations arising from the mechanical motion of the mirrors. There are other possible sources, such as thermal expansion and changes in the refractive index of the optical medium. However, we will assume they are negligible because these are changes that occur on a timescale much longer than the mechanical vibrations.

This chapter establishes the experimental procedure to characterise the noise in a cavity. The goal is (1) to measure the noise spectrum, which tells us the contribution to the total noise per frequency interval, and (2) to quantify the total noise. The two allows us to characterise the stability of the cavity. We first generate the transmission and error signal spectra to provide a conversion recipe from detunings in frequency to changes in length. Next, we record a time trace of the transmission and error signal. By taking a periodogram of the time trace, we can find the noise spectrum.

4.1 Noise Measurement

The noise in a cavity has a complicated origin. It results from the cavity forming a mechanical structure that has internal degrees of freedom. One can imagine each mirror to rotate and vibrate about its centre-of-mass. These motions cause a misalignment between the two mirrors, effectively modifying the geometrical structure of the cavity, while also changing the cavity length. Moreover, the cavity’s internal motion can also be affected by the coupling to the environment, thus forming an even more complex object that is difficult to model mathematically.

As the modelling would be a difficult task, we take the heuristic approach that the net effect of all the complicated motion is equivalent to an overall change in length of the cavity with the mirrors maintaining their alignment. This is a relatively sensible approach because the effect of length changes is generally more significant than the transverse misalignment, even for a concentric cavity. We can use the noise spectrum to guess at the more complicated
4.1. NOISE MEASUREMENT

motion because the frequencies at which they occur serve as a general guide.

4.1.1 Frequency-to-Length Conversion

Using the heuristic model above, the detuning induced by a change of length can be found easily. From (2.1.7) and (2.1.10), we find that

\[
\delta \omega = - \frac{2 \pi \Delta \nu_{FSR}}{\lambda/2} \delta L;
\]

(4.1.1a)

\[
= - \frac{2 F \kappa}{\lambda/2} \delta L.
\]

(4.1.1b)

We see that the detuning is proportional to the length fluctuation, with the scaling factor determined by the optical properties of the cavity and the wavelength of the input beam. Thus, we can convert the detuning in frequency into length fluctuations by designing the cavity to have a certain finesse and linewidth. The next step would be to combine the contents of (3.1.9) with (4.1.1) so that we can measure the noise from the cavity.

A quick analysis of (4.1.1) illustrates some guiding principles for the design of a cavity. For atomic experiments, the transitions are typically in the optical domain. Hence, \( \lambda \sim 100 \) nm, and for our goal of \( \delta L \sim 1 \) Å, then \( \delta \omega / \kappa \sim 10^{-3} F \). The cavity is relatively insensitive to length fluctuations when the finesse is very low. However, this is far too impractical for atomic experiments because a low finesse results in a high loss rate for the cavity, which prevents us from achieving the strong coupling regime. On the other hand, by using very high finesse, the cavity eventually becomes very sensitive to the noise. For our purpose, it is better to build a high-finesse cavity so that the noise can be measured accurately.

4.1.2 Analysis Algorithm

In principle, (4.1.1) immediately gives us a mapping of length fluctuation to frequency detuning. However, we can only measure the values of detuning directly. Actually, to be pedantic, we can only measure the error signal via the output of an oscilloscope, which we must calibrate in order to get the correct conversion from the oscilloscope scale to frequency. A more severe problem is that the error signals are generally not one-to-one functions of detuning (refer to Figure 3.2), hence we cannot simply invert the error signal to get the detuning. In other words, if we were to only use the error signal, the same numerical value may correspond to
several detunings. This is truly a problem when the detuning is large enough that the error signal goes beyond the linear region, which is common for a cavity with low passive stability.

We can resolve the problem by incorporating the transmission signal. Wherever there may be confusion to the true value of the detuning given the measured value of the error signal, the transmission can be used to determine it. In fact, it might be better to invert the Lorentzian distribution in (2.1.13) immediately to find the magnitude of the detuning and use the error signal only to determine the sign. The problem with this approach is the resolution of the transmission signal is very low near resonance as the transmission at resonance is a maximum. The error signal is more precise due to the steep slope.

Since it is inadequate to rely on solely one signal or the other, the resolution is to obtain both the transmission and error signal spectra (Figure 4.1a), which will be collectively known as the calibration spectra. Next, we can combine them into a single ‘detune map’ (Figure 4.1b). We choose a threshold for the transmission, above which the detuning falls within the steep linear slope of the error signal. In this regime, the error signal is one-to-one and has high resolution, which is the better choice for us to determine the detuning. Below the threshold, the error signal is no longer one-to-one but the transmission is, up to the sign difference which can be accounted by the error signal. We can perform the inversion to find the detuning in this regime. We set the threshold to 0.5 of the transmission as it is close to the end of the linear slope; the maximum and minimum points of the error signal occur at approximately 0.48, but the gradient is not constant there.

Thus, the algorithm is as follows:

1. Obtain the calibration spectra by scanning the laser. With the spectrum, we can determine the conversion from oscilloscope time to frequency units via fitting. Here, $\omega_m$ serves as a reference for the calibration. All the cavity parameters, such as the linewidth and finesse, are determined from the fit and (4.1.1) can be evaluated. We refer to this as the calibration step.

2. Separate the signals according to those with (normalised) transmission of at least 0.5 (A) and less than 0.5 (B).

3. Since the error signal is one-to-one for A, we invert the error signal to get the detuning as a function of the error signal value. This is done using interpolation numerically.

4. For B, we directly invert (2.1.13) to calculate the detuning and account for the sign.
4.1. NOISE MEASUREMENT

(a) Example of the transmission and error signal spectra.
(b) Detune map generated by plotting the transmission against the error signal.

Figure 4.1: Generating a detune mapping for the noise measurement of the cavity. The transmission and error signal spectra in (a) shows what is obtained on an oscilloscope. By plotting the transmission against the error signal, we get the detune map, which allows us to determine the detuning values either via the transmission or the error signal.

There are also several offsets to be corrected due to experimental conditions, but the main gist of the analysis follows the steps above. The algorithm is summarised in Figure 4.2.

4.1.3 Experimental Implementation

A schematics of the setup for measuring the noise of the cavity is shown in Figure 4.3. The external cavity diode laser (ECDL) has its output modulated by the EOM at a frequency of 20 MHz and we monitor the transmission and error signal using two photodiodes. The polarising beam splitter (PBS) and quarter waveplate (QWP) pair is used to control the intensity of the reflected signal. We include a flip mirror and a camera so that we are able to check the cavity mode visually.

A typical experimental sequence for the noise measurement is split into two parts. Firstly, we perform the calibration step, which we have described above. Next, we fit the measured spectra to the theoretical expressions to obtain the cavity parameters. Secondly, we lock the laser to the cavity to obtain time traces of the transmission and error signal, which records the variation in the transmission and error signal over time. For an ideal cavity with no noise, the transmission will be constant over time at the peak, whereas the error signal is constant at zero. Here, we assume that the laser does not contribute any noise. Thus, any changes in
CHAPTER 4. COAXING A CA VITY

Figure 4.2: A visual representation of steps 2 to 4 of the algorithm.

Figure 4.3: Schematics of the experimental setup for measuring the noise of the cavity. ECDL: External cavity diode laser. PBS: Polarisng beam splitter. QWP: Quarter waveplate. PD: Photodiode.
4.1. NOISE MEASUREMENT

(a) Measurement of the transmission and error signal

(b) Subset of the time traces of the transmission and error signal.

Figure 4.4: Data obtained for noise measurement. The time trace has been restricted to a small portion of the full 5 seconds in order to show the variations.

the transmission is due to the detuning induced by length changes.

One measurement cycle consists of one set of calibration spectra and five sets of time traces of 5 seconds each. A sample of the calibration spectra and time traces as seen on the oscilloscope are displayed in Figure 4.4. There are offsets to the measured calibration spectra because of the background light in the transmission and the background noise in the off-resonant regions of the error signal. The fit to the theoretical signals give an estimate of the background offset, which we can subtract from the time traces. In order to get a visual gauge of the noise in the cavity, we plot a scatter plot of the measured transmission against error signal at each time point and compare it against the analytical detune map in Figure 4.5. We see that the scatter plot roughly follows the shape of the analytical expression and forms a finite band about the theoretical line. The finite band results from a variety of reason, such as the resolution of the photodiodes and the oscilloscope.

Extra care has to be taken to properly perform the mapping. We used two different methods: one uses a modified version of the algorithm to estimate the detuning using the measured values as they are, and another that maps the experimental point to its nearest location on the theoretical line. Either method agrees with one another to within errors, hence we opt to use the first method as it is computationally less expensive.

Finally, we generate a time trace of the length fluctuation using the detune map at each time step. We can compute the noise spectrum from the time trace through a Fourier transform of the autocorrelation function. The use of the noise spectrum is twofold. Firstly,
we can identify the frequency of the mechanical resonances and deduce the type of motion that leads to that resonance. Secondly, we can explore the dominate frequencies that contribute to the noise, as well as the total noise. A time trace for $\delta L$ and a histogram of it is shown in Figure 4.6. We see that the histogram resembles a normal distribution, hence we can use the standard deviation as a measure of the noise and stability. A small standard deviation compared to the tolerable noise would indicate a high passive stability. In our case, this tolerance is 1 Å.

The noise spectrum of the time trace is given in Figure 4.7. We see that for frequencies below 100 Hz, the noise contribution is nearly flat, which shows that this region is mainly due to white noise. In contrast, the dominant noises occur mainly in the range 200–1000 Hz. Sharp resonances are typically not mechanical, hence they cannot be easily eliminated. On the other hand, the broader resonances are usually mechanical, and we can proceed to identify their mode and attempt to damp them. Beyond 1000 Hz, the contribution is very small, as those frequencies are too high to couple to mechanical modes. The integration of the noise spectrum gives a total noise of 0.66(2) Å, which is comparable to the standard deviation of the time trace. In this example, we consider this cavity has having achieved the desired level of passive stability. Although the total noise obtained through either method is
4.2 Preparing a cavity

In this section, we prescribe a recipe to build an optical cavity. A cavity needs to be carefully aligned in order to ensure maximal coupling to any of the eigenmodes. We also describe our choice of mirrors to construct the cavity.

4.2.1 Mirror Design

According to the considerations in Section 4.1.1, we want to use a cavity with high finesse such that the effect of length fluctuations on the detuning is significant and detectable. As we are using a laser diode meant to target the D2 transition of rubidium, we are operating at a wavelength of 780 nm. Thus, we need a finesse of at least 2000 for noises on the order of 1 Å to be a significant fraction of the linewidth, which leads to a minimum reflectivity of 99.93% at 780 nm for the mirrors. We use a set of 99.975% reflective plano-concave mirrors with $R = 75$ cm. This gives us a theoretical finesse of $\sim 6000$, although this value is expected to be slightly lower due to losses in the mirrors. The large value of $R$ indicates that we are using a planar cavity, but this does not have much impact on the study of passive stability and we can easily apply the results to a concentric cavity.
Figure 4.7: Noise spectrum for the time trace, plotted on a log-log scale. The noise spectrum indicates the frequencies at which the noise contribution is high, as well as the bandwidth of these resonances.

The second consideration is the choice of the free spectral range, which we can specify by choosing the length of the cavity. Due to the high finesse, we need a longer cavity in order for the linewidth to be sufficiently large that we can restrict the dips in transmission to a comfortable range. This translates into a short cavity. For the first phase of this thesis, this does not present much difficulty in terms of alignment and measurement of the noise. However, in the second phase, we aim to work under similar conditions with the concentric cavity, hence the cavity is longer and the high finesse leads to us working under more stringent conditions. At the same time, this also increases the difficulty of aligning the mirrors, as the optical axes must coincide over the larger distance.

4.2.2 Aligning a Cavity

To ensure that the cavity mode is readily found, we need to align the cavity mirrors such that their optical axes coincide. This is a crucial step for the concentric cavities, although it is applicable to all cavity types, and it is beneficial to be able to align the cavity systematically. The schematics of the alignment setup is shown in Figure 4.8.

Firstly, we couple the same laser beam into two single mode fibre couplers (SM) and
4.2. PREPARING A CAVITY

Figure 4.8: Schematic of the set-up used to align a cavity. SM: Single mode fibre coupler. TS: Translation stage. TM: Tip-tilt mount. FM: Flip mirror.

Counterpropagate their outputs. By ensuring that the two outputs overlap spatially, they form a reference for the optical axes of the cavity mirrors. The two pairs of mirrors right after the fibre couplers are used to adjust the beams. Once the reference beam is ready, we first align the right cavity mirror such that the optical its optical axis coincide with the reference beam. We can make a coarse adjustment by using the xyz translation stage (TS) and make fine adjustment using the tip-tilt mount (TM). This combination allows us to adjust the mirrors with three degrees of freedom in translation and two in rotation. The alignment is achieved when the reflection from the mirror overlaps with the reference beam. We repeat the same process with the left cavity mirror.

When the individual cavity mirrors are well-aligned with the reference, the left cavity mirror is slowly moved towards the right, and positioned into the holder that will house the mirrors. Once the cavity mirrors are close enough, we switch to the camera to check that a cavity mode has formed. Typically, minor adjustments has to be made because the left cavity mirror gets slightly misaligned during the moving process.

Visually, the alignment process resembles the sequence of camera images shown in Figure 4.9. Initially, the misalignment is severe, hence a very high transverse mode is seen. By
4.3 Characterisation of Cavity

With the alignment process completed, we can now measure the noise of the cavity. However, we still need to characterise the properties of a cavity, the most important is the free spectral range as it determines most of the other properties once it has been specified. Obtaining the free spectral range also serves as a measurement of the cavity length.

4.3.1 Measuring the FSR

The easiest method to measure the free spectral range is to find the frequency difference between two consecutive longitudinal modes, as defined in (2.2.9) or in (2.2.11). We first couple the cavity to a fundamental mode and lock the laser to this mode, which specifies some $q$. Then, we measure the frequency using a wavemeter. Next, we move to the next fundamental mode, either $q + 1$ or $q - 1$, by changing the laser current and temperature, which causes the frequency of the output to change. The free spectral range is then given by the frequency difference measured by the wavemeter. We repeat the measurement process by climbing up and down several consecutive values of $q$ and taking the difference between each value. Finally, we take the average to compensate for any drifts in the frequency that occur over the duration.
4.3. CHARACTERISATION OF CAVITY

4.3.2 Finesse or Linewidth

Since the free spectral range is a product of the finesse and linewidth, we have to measure one in order to determine the other. A technique used to measure the linewidth is the cavity ringdown [33], which measures the exponential decay of the transmission of the cavity when the incident beam is suddenly cut off. The time constant of the decay is the reciprocal of the FWHM of the cavity, hence a measurement of the time constant allows us to deduce the linewidth. Unfortunately, this method requires more instrumentation.

Here, we measure the transmission spectrum instead. This is advantageous because this only requires the probing laser to have a narrower linewidth than the cavity, which is easily achievable by the ECDL. Moreover, this can be done easily with the set-up in Figure 4.3. After obtaining the transmission, we can fit it to find the linewidth and the finesse is obtained accordingly.
5 | Phase 1: Test Cavity

We begin a preliminary investigation into the noise of a cavity by constructing a geometrically simple cavity. This cavity consists of simply two mirrors glued to a common platform. The motivation of this phase is to study the noises arising solely from the motion of the mirrors alone, without involving any complications from the structure. Hence, we expect the test cavity to be very stable as it stands. At the same time, it is easy to modify the cavity length and to test out ideas for damping the vibrations with such a simple geometry.

5.1 Baseline Measurements

The test cavity is shown in Figure 5.1. The two mirrors are affixed to a metal bar using a UV cure adhesive (Dymax OP-67-LS), colloquially called glue, and the metal bar is fastened to the platform using a pair of screws. The length of the cavity was chosen to be around 4 mm so that it has an expected free spectral range of 37.5 GHz. As mentioned, the mirrors have a reflectivity of 99.975\%, leading to a finesse of \(~\sim\)6000. Hence, the expected conversion as calculated from (4.1.1) is 1 Å to 9.6 MHz, which is much larger than the expected linewidth of 3 MHz for the free spectral range and finesse given above.

The experimental parameters are summarised in Table 5.1. We see that a noise of 1 Å is nearly three times the linewidth of the cavity, hence a change of 1 Å leads to a significant dip in the transmission to only 0.1 of the peak. Indeed, the cavity is capable of measuring length changes with the desired sub-Å resolution.

We measure the noise of the cavity using the procedure described in the Section 4 and the noise spectrum is plotted in Figure 5.2. The noise spectrum shows that the main sources

![Figure 5.1: Photo of the test cavity. The two mirrors are glued to a metal bar. The cavity length was measured to be \(3.38\) mm.](image-url)
of noise are the resonances in the region of 100–1000 Hz. There are three broad resonances which we believe to correspond to the rotation of the mirrors about the point of contact with the metal bar and the vibration of the mirrors along the cavity axis. A schematic for the possible degrees of motion is illustrated in 5.3. Unfortunately, it difficult to deduce the exact mapping of the frequency to the resonance mode. We may anticipate that the rotational modes to have a lower frequency compared to the vibrational mode along the cavity axis.

We measure the total noise of this cavity to be 0.17(1) Å, which may seem very low. However, this value is not meaningful for us at this stage, but merely reflective of the fact that this configuration inherently has low noise due to the simple structure of the cavity. This low value is also reassurance that most instrumental cavities, such as etalons, have sufficient passive stability as they are.

5.2 Explorations

From the noise spectrum in Figure 5.2 we suggest that there are three types of motion that lead to the main resonances. In this section, we discuss the successful attempts to eliminate the noises as far as possible. The two main ideas here are to stiffen the point of contact between the mirrors and the platform, and to clamp the mirrors.

5.2.1 Stiffening the Point of Contact

The glue that we use to affix the mirrors to the metal bar can be treated as an elastic material. It has a corresponding stiffness, depending on its elastic modulus and geometrical dimensions, with which it resists the deformations caused by the mechanical motions of the mirrors. This stiffness parameter is analogous to the spring constant of a harmonic oscillator, hence we draw
CHAPTER 5. PHASE 1: TEST CAVITY

Figure 5.2: Noise spectrum for the test cavity. The noise covers a very broad band of 100-1000 Hz.

Inspiration from the archetypal model of a driven harmonic oscillator and recognise that an increase in the spring constant decreases the amplitude of vibrations.

We increase the stiffness by increasing the amount of glue and the area of contact with the mirrors. We compare the noise spectrum before and after stiffening in Figure 5.4. Indeed, we see that the peaks have all reduced in amplitude. Moreover, the frequencies have also shifted, which we take as an indication that the harmonic oscillator model is applicable to the structure. The total noise is now 0.053(1) Å, which is an improvement of a factor of 3 compared to the total noise of 0.17 Å before stiffening.
5.2. EXPLORATIONS

5.2.2 Clamping

Although we are able to reduce the noise significantly by stiffening the ‘joints’, it does not stop the mirrors from vibrating. Once again, drawing inspiration from the harmonic oscillator, we wish to introduce a dissipative mechanism into the structure. Here, we take a more extreme approach of damping by intentionally clamping the mirrors. This is done by gluing the top of the mirrors to a small metal piece, effectively restricting motion about two points of contact, as well as coupling the motion of the two mirrors together. We can model this structure as a spring of infinitely large $k$ that couples the two mirrors together, hence we expect it to eliminate the rotational motions and to severely restrict the vibrational motion as the mirrors. The implementation of the clamp is shown in Figure 5.5.

The noise of the clamped cavity is measured and compared against the bare (unclamped) cavity, which has been stiffened, in Figure 5.6. With the addition of the clamp, all mechanical vibrations have been eliminated, as evidenced by the lack of a broad resonance. The total noise is now $1.53(3)$ pm, which is an improvement of a factor of 10.
Figure 5.5: Photo of the clamped cavity. A small metal piece is placed above the mirrors and glued.

Figure 5.6: Comparison of the noise spectra between the bare and clamped cavity. All mechanical resonances are gone.

5.3 Laser Noise

The noise spectrum of the clamped cavity showed that there are still noise on the cavity, but they are no longer mechanical resonances. As such, we investigated the possible source of the noise. In our analysis above, we made the assumption that the laser does not contribute to the detuning at all. Certainly, that is not true in practice. Here, we measure the noise from the laser and compare with the noise of the clamped cavity.

For the noise measurement, we use MTS to lock the laser and observe the time trace of the error signal. Unlike the case of a cavity, we do not have a transmission spectrum to
5.3. LASER NOISE

![Noise Spectra: Cavity vs Laser](image)

Figure 5.7: Comparison of the noise spectra of the cavity and the laser. The two spectra match very well, hence we conclude that the noise we find on the cavity is due to the noise of the laser.

Comparing the noise spectra, we see that the noise seen on the cavity matches with the noise from the laser. The total noise of the laser is higher, at a value of $2.40(4) \text{ pm}$, due to the different values of current and temperature of the laser diode when locking the laser with PDH or with MTS. Thus, we can conclude that the clamp has effectively stopped all mechanical motion such that the noise on the cavity is actually not detectable even at the relatively high finesse of 5000. Moreover, the noise from the laser corresponds to about 450 kHz, which is sufficiently small that we may treat the laser as essentially ideal for our purposes in the subsequent experiments.
6 | Phase 2: Attocube

Having explored two successful techniques for reducing the mechanical vibrations of a cavity in a simple geometry, we now turn to incorporating the 3D piezo scanner (Attocube ANSxyz100), known as Attocube hereafter, into the structure and look at how we can apply the lessons from the test cavity to improve the passive stability of the cavity with attocube. The addition of Attocube into the system increases the total noise drastically due to its construction that allows it to move in all three translational directions. Also, the manoeuvrability of Attocube presents a huge challenge in determining the steps we can take to limit the noise. Nonetheless, we use the ideas from Phase 1 as guiding principles for stabilising the cavity with Attocube.

This section of the thesis is split into two stages: in the first stage, we build the cavity such that one mirror is mounted directly onto Attocube; in the second stage, we use a set of mounts to construct the cavity in a compact configuration that can be fit inside a cuvette. Once again, we do a systematic study of the noise due to Attocube alone to gain some intuition on the modes of the mechanical motion before proceeding to work with a more complicated geometry.

6.1 Attocube Characteristics

Attocube is a scanner that can move in all three coordinate directions. It uses PZT (lead zirconate titanate) ceramics as the actuators to move the platform. The PZT ceramics expand when a DC voltage is applied, resulting in a movement range of 50x50x24 µm in the xyz direction defined in Figure 6.1. There is a set of PZT ceramics for each direction, making the internal structure responsible for the movement rather complicated. Moreover, it is capable of working even at a cryogenic temperature and in an ultra low vacuum. Attocube has a compact size of 24x24x10 mm and supports a maximum load of 100 g.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Value (µm V⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0.29(1)</td>
</tr>
<tr>
<td>y</td>
<td>0.29(1)</td>
</tr>
<tr>
<td>z</td>
<td>0.86(3)</td>
</tr>
</tbody>
</table>

Table 6.1: Calibration of Attocube movement range.
6.2. STAGE 1: TEST ATTOCUBE

![Attocube Diagram](image)

Figure 6.1: Attocube as viewed from the top. Units are in mm. The circles on the right side are tapered holes for adding screws. The extra piece at the bottom of the image is the wire connection through which we supply the DC voltage. Not shown in the diagram is the height, which is 10 mm.

6.2 Stage 1: Test Attocube

In this stage, we explore a similar geometry to the test cavity in order to look at the noises that are introduced by Attocube into the system. We use the same pair of mirrors used in the test cavity. Unlike the test cavity, the two mirrors cannot be placed together on the same platform. This is because Attocube has to be mounted onto a base plate designed to hold it securely. We fashion a makeshift platform to prop the second mirror using two blocks of metal. The overall structure is shown in Figure 6.2. We use a generous amount of glue to stiffen the mechanical motion of the mirrors. Hence, we expect that the noise contribution would be dominated by Attocube.

6.2.1 Baseline Measurements

Before we can make any measurements of the noise, we need to establish the free spectral range of the cavity. To do so, we need to lock the laser to the cavity. However, initial attempts to lock the laser to the cavity failed because the laser runs out of lock in a few seconds. By scanning the laser, we can observe the behaviour of the error signal spectrum over time. We see that the zero crossing of the spectrum 'jiggles' wildly. As the location of the zero crossing indicates the resonance of the cavity, this indicates that the laser is detuned over several linewidths of the cavity. Correspondingly, the length fluctuation is severe and we need to stabilise the cavity before we can make any measurements. The problem is illustrated
Despite the issue, we can make a rough estimate of the noise by taking snapshots of the error signal spectrum and trace the changes in the position of the zero crossing. This is done by fitting the linear slope of the error signal and use the fits to estimate the position of the zero crossing. We estimate that the position of the resonance is given by the mean value of the positions and the noise is given by their standard deviation. Admittedly, this is a crude estimate, for which we obtain a value of \( \sim 2 \) nm for the noise of the cavity. Despite the roughness of this estimation, this value corroborates with the noise of 1 nm observed in the concentric cavity. Due to the high finesse of the mirrors, this level of noise is sufficient to disrupt us from using the cavity for any meaningful experiments.

6.2.2 Tracking the Resonances

As we are unable to lock the laser to the cavity, we are unable to produce the noise spectrum. However, we can circumvent this situation by using an interferometer. The idea is to drive Attocube at a given frequency and look at the intensity variations of the interference pattern.
6.2. STAGE 1: TEST ATTOCUBE

(a) First 10 snapshots of the error signal spectrum

(b) Plot of the linear slopes of the error signal spectra. There are 111 sets of data shown here.

Figure 6.3: Fluctuation of the zero crossing of the cavity with Attocube. The fluctuations occur over several linewidths, which prevents us from locking the laser to the cavity. The standard deviation of the position of the zero crossing gives an estimate for the length fluctuation as \(\sim 2 \text{ nm}\).

Figure 6.4: Schematic of the set-up for the interferometric analysis. ECDL: External-cavity diode laser. M: Mirror. BS: Beam splitter. PD: Photodiode. NA: Network analyser.

on a photodiode. Since the intensity variations of the fringe pattern results from the path difference between the two beams, this method allows us to measure the path difference as a function of the driving frequency. At a resonance, the path difference is expected to be large, hence the change in intensity is large and vice versa. For the method to be accurate, we need to choose the correct set point and ensure that the voltage of the signal sent to Attocube is not too large so that the intensity variations is linear in the path difference.

The schematic for the interferometry set-up is shown in Figure 6.4. We use a network analyser to output a sinusoidal signal at a given frequency, and sweep the frequency through a range of 0–2000 Hz. An attenuator is used to limit the power sent by the network analyser.
to Attocube to maintain the linearity of the response. The network analyser monitors the gain of the power on the photodiode with respect to the power fed into Attocube at each frequency. Effectively, we measure the voltage-to-motion frequency response of Attocube through interferometry and the peaks of the response correspond to the resonances.

The gain curve in Figure 6.5 shows that the mechanical motion of Attocube is far more complicated than the test cavity. In particular, there are multiple strong resonances below 1000 Hz. The main resonances that we need to target are the ones at 400 and 700 Hz, which are one or two orders of magnitude larger than the rest. The curve also indicates that Attocube does not contribute much noise at higher frequencies.

### 6.2.3 Improving the Situation

Owing to the design of the cavity with Attocube, we cannot simply apply a clamp to the two mirrors as that runs contrary to the intended purpose of using Attocube. However, since the resonances are mechanical in nature, we may substitute the infinitely stiff clamp with something that has a relatively high stiffness and also allows for dissipation of energy.
6.2. STAGE 1: TEST ATTOCUBE

A possibility we consider is to use a metal block, which would have a high stiffness, of a (sensibly) large weight to apply a frictional force on the mirrors to damp the vibrations.

We use an aluminium block of approximately 7 g as the weight and placed it on top of the two mirrors as in Figure 6.6. This time, we find that the error signal is sufficiently stable for locking and we are able to measure the cavity parameters. They are summarised in Table 6.2. We find that the finesse of the cavity has degraded.

Of greater interest is the noise spectrum of the cavity after we employ the simple stabilisation process. We find that the total noise has decreased from the estimated 1.5–2 nm to 1.10(2) Å. This is a drastic improvement given the simplicity of the method, and already places us very near the goal. Furthermore, we compare the noise spectra of the test cavity

![Image of the cavity with a mass added to stabilise the cavity.](image)

**Figure 6.6:** Photo of the cavity with a mass added to stabilise the cavity.

**Table 6.2:** Measured values of the test Attocube parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \nu_{FSR}$</td>
<td>18.5(2) GHz</td>
</tr>
<tr>
<td>Finesse</td>
<td>2130 (30)</td>
</tr>
<tr>
<td>Linewidth</td>
<td>4.33(6) MHz</td>
</tr>
<tr>
<td>MHz/Å</td>
<td>4.70(5)</td>
</tr>
</tbody>
</table>
in Section 5.2.1, without clamp, against that of Attocube in Figure 6.7. The effect of the multiple resonances of Attocube on the cavity noise is the broad spectrum about 300–1000 Hz. Moreover, the noise at about 250 Hz, which is due to the vibrations of the mirror, has been amplified.

In order to push the stability further, we experimented with the mass of the block. Going back to the proposed damping mechanism, we believe that the friction between the block and the mirrors provide the damping. Hence, a first guess would be to increase the friction by increasing the weight of the block. We use another two blocks of 15 and 21 g and test this hypothesis. The measurements for the total noise are summarised in Table 6.3. There seems to be an improvement in the stability as the weight increases.

However, if we look at the noise spectra for each of the masses, we suspect that the material of the block may have an effect on the noise. Between the copper 15 and steel 21 g masses, the noise spectra are very similar, except for a strong resonance at around 150 Hz for steel. By integrating the noise about this peak, we find that it contributes about 23% of the total noise of 0.70 Å, which is a rather significant ratio. This certainly suggests that we need to be mindful of the choice of materials, otherwise we might introduce a substantial amount of noise.
Table 6.3: Measured values of the total noise for different masses.

<table>
<thead>
<tr>
<th>Mass/g</th>
<th>Noise/Å</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1.10(2)</td>
<td>Aluminium</td>
</tr>
<tr>
<td>15</td>
<td>0.67(2)</td>
<td>Copper</td>
</tr>
<tr>
<td>21</td>
<td>0.70(1)</td>
<td>Steel</td>
</tr>
<tr>
<td></td>
<td>0.54 (After correction)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.8: Comparison of noise spectra of the different masses. The total noise decreases as the mass increases, except the sudden appearance of the peak at 150 Hz for 21 g.

of noise. Assuming that there is no contribution from this resonance, the noise would be 0.54 Å, and the noise exhibits a downwards trend as we increase the mass.

Despite the improvement in the stability, there are a few problems that make this method undesirable for us. Firstly, we have to apply about twice the DC voltage than expected to change the cavity length by one free spectral range of 390 nm. This means that the mirrors have their movement restricted and we lose the flexibility to change the length of the cavity freely. In addition, the cavity and its mounts has to be compact enough to enter a cuvette with a cross-sectional area of 25x25 mm. The metal block is simply too large to be of practical use.
6.2.4 Other Ventures

We briefly discuss some of the other modifications we explored:

- Using a spring in place of the weight.
  We made a spring out of copper wire, with a spring constant of \( \sim 0.2 \text{ mN} \text{µm}^{-1} \), and placed it between the two mirrors. Unfortunately, it barely has any effect on the noise and we could not lock the laser. Even when a weight was added, there was no further improvement to the stability, which suggests the spring constant might be too low. In addition, we can only increase the spring constant at most by one order of magnitude in order to stay within the maximum load of Attocube. This is not sufficient to match the effect of the weight. Hence, add a spring would not be a viable method for us.

- Increasing the mass of the mirrors.
  We add small masses to the individual mirrors in order to decrease the amplitude of the vibrations. However, the effect is too miniscule to stabilise the cavity. There is a slight shift in the resonance frequencies but it does not suppress any of them.

6.3 Stage 2: Mounted Cavity

In this stage, we build the cavity in the configuration designed for the atomic experiment. This configuration is shown in Figure 6.9. The configuration consists of a cavity mount and a mirror mount. The cavity mount has a solid base plate to which we mount Attocube. Due to space constraints, we use the mirror mount to connect the mirror to Attocube instead of directly mounting it. As it turns out, this structure introduces more noise than having the mirror directly mounted to Attocube. The total height of the set-up is 24 mm, leaving us with only a minuscule leeway of 1 mm for any stabilisation mechanism.

6.3.1 Baseline Measurements

Once again, we encounter the problem that the noise is too high for us to make the preliminary measurements for the cavity parameters. We estimate the noise in the same manner as before and estimate it to be on the order of 3 nm. After stabilisation with a mass, the parameters are measured and shown in Table 6.4.
6.3. STAGE 2: MOUNTED CAVITY

Figure 6.9: 3D diagram of the configuration for experiments. Adapted from [20].

Table 6.4: Measured values of the mounted cavity parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \nu_{\text{FSR}}$</td>
<td>10.52(4) GHz</td>
</tr>
<tr>
<td>Finesse</td>
<td>3100 (80)</td>
</tr>
<tr>
<td>Linewidth</td>
<td>1.70(4) MHz</td>
</tr>
<tr>
<td>MHz/Å</td>
<td>4.70(5)</td>
</tr>
</tbody>
</table>
Table 6.5: Measured values of the total noise for different configuration of contact points.

<table>
<thead>
<tr>
<th># of strips</th>
<th>Noise/Å</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.95(3)</td>
</tr>
<tr>
<td>2, both above mirrors</td>
<td>2.6(1)</td>
</tr>
<tr>
<td>2, one contacting the side</td>
<td>1.17(3)</td>
</tr>
<tr>
<td>3</td>
<td>0.745(8)</td>
</tr>
</tbody>
</table>

6.3.2 Downsizing the Weight

Due to the space constraints, we need to dramatically downsize the metal block used for stabilisation. Unfortunately, there is no other way but to use a thin strip of metal such that we can maintain the stiffness of the clamp. We cut strips of 24x5x0.5mm out of a sheet of aluminium and place them across the mounts.

To our pleasant surprise, the aluminium strips managed to stabilise the cavity down to 2.6(1) Å. This is unexpected as the mass of the strip is only about 0.2 g, lighter than the 7 g block by a factor of 30. This result also suggests that the mechanism involved in stabilising the cavity is more complex than the arguments described previously. A careful modelling of the dynamics of the structure will be needed to conduct a proper analysis, but this is beyond the scope of the thesis. Instead, we focus on the possibilities opened up by this rather unsophisticated method.

To get an overview of how effective this method is, we try several configurations of strips placed at strategic contact points. The results are listed in Table 6.5. We see that as the number of contact point increases, the stability also increases. In particular, when one strip is placed at the side of the mirror mount, there is a greater stabilisation effect on the cavity than when both strips are placed above the mirrors. With three points of contact, the cavity is passively stabilised well below the goal of 1 Å.

Further attempts to bring the noise down by adding more points of contact do not help. In fact, it is detrimental to us as it increases the size of the set-up beyond the constraints. Thus, we believe this is the best that can be done using the materials we have readily available. While there is the possibility of using fancier materials designed for reducing vibrations, we are ready to apply the current design to the concentric cavity.
6.4 Meta-stage

Over the course of the experimentation, we discover that some of the resonances do not change as when we add more contact points to the cavity. This suggests that these noises do not originate from the mechanical motion of either the mirrors or Attocube. Rather the cavity acts as a pick-up for these noises. We are lead to explore the possibility that we have to go beyond merely stabilising the cavity. This section covers the excursion into stabilising the platform that houses the cavity.

The cuvette is attached to a vacuum flange, which is supported by a holder. As the
flange is much shorter than the cuvette, the structure is unbalanced about the pivot point.

By attaching an extension (vacuum) tube as a counter balance, we find that the noise is lowered from 0.745 to 0.456 Å. This is yet another drastic improvement to the overall noise.

Further action taken are:

1. Adding a damping material to the top of the cuvette.
2. Adding a damping material to the extension tube.
3. Attaching a small post at the edge of the cuvette and immersing the post in a mixture of glass and sillica beads. The idea is to provide a resistance to the vibrations as the post moves through the beads.

We study the effect of each modification individually before looking at the combined effect. The results are shown in Table 6.11. Each method are roughly equal in its effectiveness, and it gets increasingly difficult to suppress the noise. We find that this is the best results we can achieve: we have stabilised the cavity by reducing the noise from 2–3 nm to 0.3 Å, which is a factor of 100.

Table 6.6: Measured values of the total noise for different methods of stabilising the cuvette.

<table>
<thead>
<tr>
<th>Method</th>
<th>Noise/Å</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dampener on cuvette</td>
<td>0.377(3)</td>
</tr>
<tr>
<td>Dampener on extension</td>
<td>0.381(4)</td>
</tr>
<tr>
<td>tube</td>
<td></td>
</tr>
<tr>
<td>Glass beads</td>
<td>0.376(5)</td>
</tr>
<tr>
<td>Combined</td>
<td>0.317(2)</td>
</tr>
</tbody>
</table>

The inspiration for this came from Christian.

Please add your preferred adjective to qualify this factor.
In our previous work on concentric cavities, we find that we need to upgrade the current cavity in order to progress towards the strong coupling regime. After improving the finesse of the cavity, the noise from the piezo scanner (Attocube) is now significant enough to interfere with the interactions between the atom and the cavity mode. Thus, the goal is to passive stabilise the cavity such that we reduce the noise from $\sim1$ nm to sub-Å.

To that end, we need to measure the noise of a cavity with sub-Å resolution. This is done by using a high finesse cavity on $\sim$2000–5000 to amplify the effect of the length fluctuations on the detuning. We construct an experimental set-up that measures the transmission and error signal of the cavity in order to extract the length fluctuations.

Subsequently, we build a cavity in a simple geometrical configuration to study the contribution of the noise from the mirrors, as well as to test the ideas for reducing the noise. We find that the effective methods are to stiffen the points of contact between the mirror and the platform, and to clamp the mirrors. The latter is very effective as the noise measured from the cavity comes entirely from the laser.

Next, we incorporate Attocube into the cavity and study the noise contribution from it. The bare cavity, without any stabilisation, has a noise of $\sim2$ nm, which is similar to the noise found on the concentric cavity. The stabilisation method we adopt here is to use a metal block as a stiff spring with some friction for dissipation.

Finally, we prepare the cavity in the configuration for experiments in the cuvette. The space constraint prevents the usage of the large metal block. Hence, we opt to replace them with metal strips, which would still be relatively stiff. Despite the large reduction in mass, we find that the metal strips can stabilise the cavity substantially. By experimenting with the location of the strips, we find that only three points of contact is needed to achieve a passive stability below 1 Å. Further reductions in the noise is achieved by stabilising the set-up housing the cavity instead.

The improvements in the passive stability are summarised in Table 7.1. With the best-case of 0.745(8) Å for the passive stability of the cavity, and further stabilisation on the structure, we are more than ready to apply the lessons to the concentric cavity.
Table 7.1: Summary of the improvement of the passive stability across the phases.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Cavity Configuration</th>
<th>Noise/Å</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bare Clamped</td>
<td>0.17(1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.024(4)</td>
</tr>
<tr>
<td>2-1</td>
<td>Bare</td>
<td>~20</td>
</tr>
<tr>
<td></td>
<td>15 g</td>
<td>0.67(2)</td>
</tr>
<tr>
<td></td>
<td>21 g corrected</td>
<td>0.54</td>
</tr>
<tr>
<td>2-2</td>
<td>Bare</td>
<td>~30</td>
</tr>
<tr>
<td></td>
<td>3 point</td>
<td>0.745(8)</td>
</tr>
<tr>
<td>2-M</td>
<td>Combined</td>
<td>0.317(2)</td>
</tr>
</tbody>
</table>

7.1 Future Work

7.1.1 Resurrecting the Concentric Cavity

The most pertinent, and pressing extension of the passively stabilised cavity is to modify the concentric cavity to use this design. As we have gone beyond the goal of 1 ångstrom, we can consider pushing the finesse to slightly higher values, maybe to 1000. This would lead to an improvement in $C$ by a factor of 10. Having a more stable cavity opens up many possibilities, such as looking at the use of higher order transverse modes or coupling atoms to multiple cavity modes.

7.1.2 Exploring Other Materials

In this thesis, we used aluminium for the metal strip since it was readily available. However, there are more complex materials which have higher damping capacity than aluminium. The damping capacity of a material measures its ability to dissipate elastic strain energy from mechanical vibrations [34]. We can consider making the mounts out of materials with high damping capacity and are vacuum compatible. This is another way of increasing the passive stability of the cavity.

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1Because I spent my first half of my FYP trying to resurrect the concentric cavity but to no avail. Finally, it is time to trap the atoms!
A | Appendix

A.1 Guide to Elements used in Experiments

This section collates a brief description of the elements used in the various set-ups.

Waveplate

A waveplates are used to rotate the polarisation of light. It is made from a birefringent material, which has different refractive indices for light linearly polarised along the fast and slow axes. As such, a phase difference between polarisation components can be applied. By choosing the correct length of the material, a phase different of $\frac{\pi}{2}$ or $\pi$ can be applied.

- $\frac{\pi}{2}$: This is a quarter waveplate. The primary function is convert the polarisation of light from linear to circular and vice versa.
- $\pi$: This is a half waveplate. The primary function is to rotate the polarisation of linearly polarised light.

Beam Splitter

A beam splitter splits the incident beam into two beams in a fixed ratio; typically 50:50. The polarisation of the incident beam is maintained after the split.

Polarising Beam Splitter

Similar to a beam splitter, the polarising beam splitter split an incoming beam into two according to the polarisation components. The transmitted light is horizontally polarised whereas the reflected light is vertically polarised.

Electro-Optic Modulator

The EOM contains a crystal that changes its refractive index upon the application of an electric field. The crystal has a capacitance which can be exploited to make a LC resonator. The resonance frequency of this resonator serves as the modulation frequency.

Single Mode Fibre

A single mode fibre is a waveguide that accepts only one spatial mode. It is commonly used as a mode filter and to transport the beam from one location to another.

Wavemeter

A wavemeter is an interferometer used to make precise measurements of laser wavelengths. We use a wavemeter that has a precision of 10 MHz.
External Cavity Diode Laser

The ECDL is a laser module that consists of a laser and a blazed grating used in the Littrow configuration. When the laser is incident on the grating, there will be a zeroth order reflected beam and a first order diffracted beam. The grating is aligned such that the diffracted beam is sent back to the laser diode as an optical feedback. The grating and the laser diode behaves collectively like an external cavity, in contrast to the cavity serving as the gain medium inside the laser diode.

Usually, a piezo actuator is attached to the grating. By moving the piezo, one can change the frequency of the laser output. This is the standard way we use to scan the laser.
Bibliography


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