Interferometry of the intensity fluctuations in light
II. An experimental test of the theory for partially coherent light

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A theoretical analysis is given of the correlation to be expected between the fluctuations in the outputs of two photoelectric detectors when these detectors are illuminated with partially coherent light. It is shown how this correlation depends upon the parameters of the equipment and upon the geometry of the experiment.

The correlation may be detected either by linear multiplication of the fluctuations in the two outputs or by a coincidence counter which counts the simultaneous arrival of photons at the detectors. The theory is given for both these techniques and it is shown that they are closely equivalent.

A laboratory test is described in which two photomultipliers were illuminated with partially coherent light and the correlation between the fluctuations in their outputs measured as a function of the degree of coherence. The results of this experiment are compared with the theory and it is shown that they agree within the limits of accuracy of the test; it is concluded that if there is any systematic error in the theory it is unlikely to exceed a few parts per cent.

1. Introduction

The basic principles of intensity interferometry, that is interferometry based on correlating the fluctuations of intensity in two beams of radiation, were presented in part I of this paper (Hanbury Brown & Twiss 1957). We showed there that the intensity fluctuations in two coherent beams are correlated and that, in the optical case, this correlation should be preserved in the process of photoemission so that the fluctuations in the anode currents of two phototubes should be partially coherent when they are illuminated by coherent beams of light.

We have already described (Hanbury Brown & Twiss 1956*a*) a preliminary test which established the existence of this correlation; however, this test was not designed to yield a precise quantitative result. It seemed desirable, especially in view of the considerable controversy about the principles involved, that a more exact check of the theory should be carried out, and so we have recently repeated the experiment with improved apparatus operating under more carefully controlled conditions. The results are reported below together with the necessary theoretical treatment of partially coherent light.

The principles of an intensity interferometer can be presented either in the classical terms of wave interference, or in terms of the relative times of arrival of photons. These two ways of looking at the phenomenon suggest two different experimental techniques for testing the correlation. The first technique, which may conveniently be regarded as illustrating the wave picture, consists in finding the correlation between the fluctuations in the anode currents of two light detectors by means of a linear multiplier which takes the time average of their cross-product. The second technique, which illustrates the corpuscular nature of light, makes use of a coincidence counter to detect individual events in which photoelectrons are emitted simultaneously from the cathodes of two light detectors.

At first sight, the second of these techniques appears more attractive since, by the use of coincidence counters, it should be possible to circumvent the major technical difficulty in the design of an extremely sensitive correlator, namely the elimination of random drifts in the apparatus. However, it is shown below that, with present-day counters and photomultipliers, the use of a coincidence counter demands a highly monochromatic and brilliant light source if significant results are to be obtained in a reasonable observing time. The apparatus used in the present experiment was also intended for use in a stellar interferometer, and it was not considered practicable to restrict the spectrum of the starlight to an extremely narrow band. It was therefore decided to employ a system based on a linear multiplier, since such a system can be used with light of arbitrarily broad bandwidth and significant correlation can be obtained in periods of a few minutes using a standard arc lamp as a source of light.

Although the present paper deals only with an experiment using a linear multiplier we also give in an appendix a short theoretical treatment of the results to be expected with a coincidence counter. This treatment demonstrates the equivalence of the two techniques and the results can be used to interpret the experimental data of other experimenters who have attempted to detect correlation using coincidence counters.

2. The correlation expected between the outputs of two photoelectric light detectors illuminated by partially coherent light

In this section we shall consider theoretically the behaviour of the system shown in simple outline in figure 1. Two phototubes $P_1 P_2$ are illuminated from a single source of light. The fluctuations in their anode currents are amplified by the amplifiers $B_1 B_2$ and are multiplied together by the linear multiplier C. The time average of the multiplier output is proportional to the correlation and is measured by the integrating device M_1.

The expressions given in part I of this paper for the average value of the correlation between the fluctuations in the anode currents of two separate phototubes were calculated for the idealized case in which the incident radiation was a plane wave. However, we also showed that the extension to the general case, where the light source and the apertures of the photocathodes are of arbitrary size, could be made entirely within the framework of a classical theory in which the photocathodes are regarded as square law detectors of suitable conversion efficiency.

2·1. *Light detectors with small aperture*

Under the conditions where the apertures of the individual photocathodes are too small to resolve the source appreciably, we can apply to the optical problem a similar analysis to that we have given elsewhere for a radio interferometer based on the same principle (Hanbury Brown & Twiss 1954), and this is given in the course of the general discussion in appendix A.

When the incident radiation is approximately monochromatic we show in appendix A that $\overline{C(d)}$, the average value of the correlation when the apertures of the radiation detectors are separated by a distance d, is related to $\overline{C(0)}$ the correlation observed with zero separation, by the equation

$$\overline{C(d)} = \Gamma^2(\nu_0, d)\,\overline{C(0)}, \tag{2·1}$$

where ν_0 is the midband frequency of the incident radiation and where $\Gamma^2(\nu_0, d)$, the normalized *correlation factor*, is equal to the square of the modulus of the Fourier transform of the intensity across the equivalent line source. As we have pointed out in analyzing the radio case, this normalized *correlation factor* is related to the square

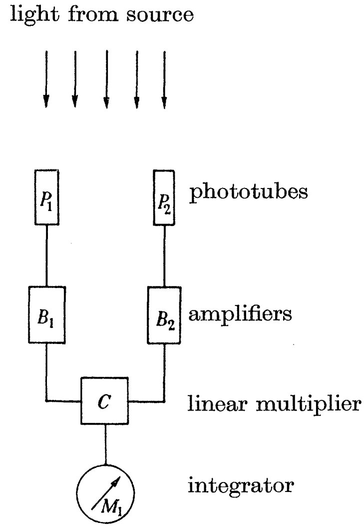

FIGURE 1. A simplified outline of an intensity interferometer.

of the visibility of the fringes that would be observed by a Michelson interferometer with the same baseline; it is also related to the square of the so-called *degree of coherence* (Zernike 1938) and the modulus of the *coherence phase factor* (Hopkins 1951).

Equation (2·1) is only valid when the fractional light bandwidth is very small and, when this is not the case, one must integrate over the light spectrum to find $\overline{C(d)}$. Thus, it was shown in part I that, if the incident light is a linearly polarized plane wave,

$$\overline{C(0)} = 2e^2 A_1 A_2 \int_0^\infty \alpha^2(\nu)\, n_0^2(\nu)\, \mathrm{d}\nu \int_0^\infty |F(f)|^2\, \mathrm{d}f, \tag{2·2}$$

where A_1, A_2 are the areas of the photocathode apertures; $\alpha(\nu)$ is the cathode quantum efficiency; $n_0(\nu)$ is the number of quanta incident on unit area in unit bandwidth; $F(f)$ is the combined frequency response of the photomultipliers and of the amplifiers. This equation is equally valid when the light source is of finite angular size, provided that the photocathode apertures are themselves small, and hence we may write

$$\overline{C(d)} = 2e^2 A_1 A_2 \int_0^\infty \Gamma^2(\nu, d)\,\alpha^2(\nu)\,n_0^2(\nu)\,d\nu \int_0^\infty |F(f)|^2\,df. \tag{2·3}$$

The assumptions underlying equation (2·3) are, however, too idealized for it to be applicable to a practical case even when the apertures of the light detectors are very small. It is necessary to take account of the fact that the quantum efficiency and spectral response may be different at the two photocathodes, and that the light may not be linearly polarized; furthermore, the electrical frequency response of the photomultipliers and amplifiers may not be identical in the two channels and there will inevitably be a slight loss of correlation in the correlator system. To take account of these factors equation (2·3) may be written in the convenient form

$$\overline{C(d)} = e^2 \epsilon A_1 A_2 \beta_0 \overline{\Gamma^2(d)}\,\alpha^2(\nu_0)\,n_0^2(\nu_0)\,\sigma B_0 b_v\,|F_{\text{max.}}|^2, \tag{2·4}$$

where $(1-\epsilon)$ is the fraction of the correlation lost in the correlator circuits; B_0 is the effective bandwidth of the light defined by

$$B_0 = \left[\int_0^\infty \alpha_1(\nu)\,n_1(\nu)\,d\nu \int_0^\infty \alpha_2(\nu)\,n_2(\nu)\,d\nu\right]^{\frac{1}{2}} \Big/ \alpha(\nu_0)\,n_0(\nu_0) \tag{2·5}$$

and

$$\left.\begin{aligned} n_1(\nu) &= n_{1a}(\nu) + n_{1b}(\nu), \quad n_2(\nu) = n_{2a}(\nu) + n_{2b}(\nu), \\ \alpha^2(\nu_0)\,n_0^2(\nu_0) &= \alpha_1(\nu_0)\,\alpha_2(\nu_0)\,n_1(\nu_0)\,n_2(\nu_0) \end{aligned}\right\} \tag{2·6}$$

and the subscripts a, b refer to two orthogonal directions of polarization; where σ, the spectral density, is defined by

$$\sigma = \int_0^\infty \alpha_1(\nu)\,\alpha_2(\nu)\,n_1(\nu)\,n_2(\nu)\,d\nu / B_0\,\alpha^2(\nu_0)\,n_0^2(\nu_0) \tag{2·7}$$

and the polarization factor β_0 is defined by

$$\beta_0 = 2[n_{1a}(\nu)\,n_{2a}(\nu) + n_{1b}(\nu)\,n_{2b}(\nu)]/n_1(\nu)\,n_2(\nu), \tag{2·8}$$

so that $\beta_0 = 1$ when $n_a(\nu) = n_b(\nu)$ as in the case of randomly polarized light. The mean value $\overline{\Gamma^2(d)}$ of the normalized correlation factor is defined by

$$\overline{\Gamma^2(d)} = \frac{\int_0^\infty \Gamma^2(\nu, d)\,\alpha_1(\nu)\,\alpha_2(\nu)\,[n_{1a}(\nu)\,n_{2a}(\nu) + n_{1b}(\nu)\,n_{2b}(\nu)]\,d\nu}{\int_0^\infty \alpha_1(\nu)\,\alpha_2(\nu)\,[n_{1a}(\nu)\,n_{2a}(\nu) + n_{1b}(\nu)\,n_{2b}(\nu)]\,d\nu} \tag{2·9}$$

and b_v the effective cross-correlation bandwidth of the amplifiers, is defined by

$$|F_{\text{max.}}|^2 b_v = \frac{1}{2}\int_0^\infty [F_1(f)\,F_2^*(f) + F_1^*(f)\,F_2(f)]\,df, \tag{2·10}$$

where $|F_{\text{max.}}|$ is the maximum value of $\frac{1}{2}[F_1(f)\,F_2^*(f) + F_1^*(f)\,F_2(f)]$.

We shall use equation (2·4) in a later part of this paper to interpret some observations on Sirius which have been briefly reported elsewhere (Hanbury Brown & Twiss 1956*b*). However, it cannot be applied to the experiment analyzed in the present paper in which the apertures of the light detectors were so large that they partially resolved the source. In this case the light cannot be regarded as coherent over the whole aperture of the light detectors and the correlation given by equation (2·4) must be reduced by the *partial coherence factor* $\bar{\Delta}$, where the bar denotes that the factor has been found by averaging over the frequency spectrum. The value of $\overline{\Gamma^2(d)}$, the normalized *correlation factor*, is now a function of both the angular size of the source and aperture of the light detectors.

2·2. *Light detectors with large apertures*

If no restrictions are placed upon the spectrum of the light or the intensity distribution over the source or the shape of the photocathodes, the general expression for the expected correlation is impracticably complex. However, for the purposes of the present paper we have greatly simplified the analysis by making the assumptions: (1) the intensity is uniform over the source of light, which is taken to be either circular or rectangular; (ii) the two photocathodes have identical rectangular apertures and the quantum efficiency is constant over each cathode; (iii) the quantum efficiency and the number of quanta received per unit bandwidth do not vary significantly

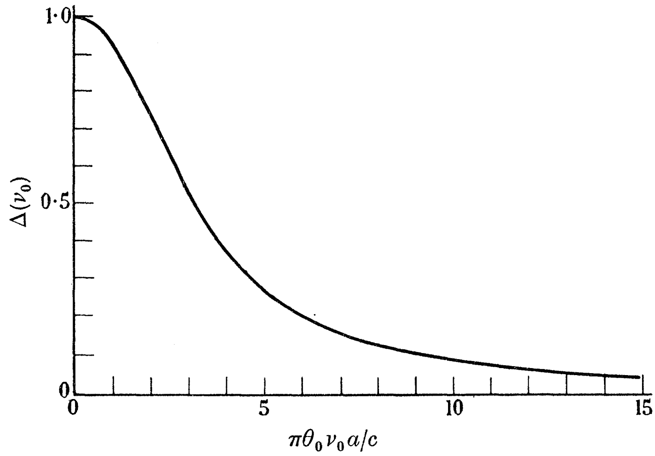

FIGURE 2. The variation of the partial coherence factor, $\Delta(\nu_0)$, with the parameter $\pi\theta_0\nu_0 a/c$, calculated for a circular source of angular diameter θ_0 viewed by two identical light detectors with square apertures $a \times a$.

over a frequency range equal to the bandwidth of the correlator. On the other hand, it will be assumed that the spectral width of the light is so narrow that both $\Gamma^2(\nu_0, d)$ and $\Delta(\nu_0)$ may be taken as constant over the light bandwidth, and that we may put

$$\overline{\Delta\Gamma^2(d)} \simeq \Delta(\nu_0)\,\Gamma^2(\nu_0, d).$$

The expressions for $\Gamma^2(\nu_0, d)$ and $\Delta(\nu_0)$, based on the assumptions stated above, have been derived in appendix A. The numerical values for the case of a circular source viewed by two square cathodes of identical size have been calculated from equations (A 25) and (A 26) by means of the Manchester University Electronic Computing Machine. Figure 2 shows the partial coherence factor $\Delta(\nu_0)$ as a function of $\pi\theta_0\nu_0 a/c$, where θ_0 is the apparent angular diameter of the source and a is the width of the photocathode apertures. Figure 3 shows the normalized correlation factor $\Gamma^2(\nu_0, d)$ as a function of $\pi\theta_0\nu_0 d/c$, where d is the cathode separation and $\pi\theta_0\nu_0 a/c$ has the value 3·06 which is appropriate to the present experiment.

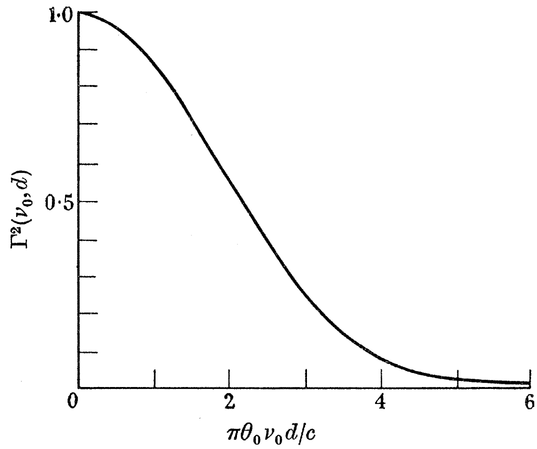

FIGURE 3. The variation of the normalized correlation factor $\Gamma^2(\nu_0, d)$ with the parameter $\pi\theta_0\nu_0 d/c$, calculated for a circular source of angular diameter θ_0 viewed by two identical light detectors with square apertures $a \times a$, where $\pi\theta_0\nu_0 a/c = 3{\cdot}06$.

2·3. *The expected signal to noise ratio*

We proved in part I that the r.m.s. uncertainty $N(T_0)$ in the correlation measured over an observing time T_0 depends, in a practical case, only upon the magnitude of the photoemission current, since the excess photon noise is negligible with any known light source. This enables us to write $N(T_0)$ immediately in the form

$$N(T_0) = \frac{e^2\mu}{\mu-1}(1+a)(1+\delta)(A_1A_2)^{\frac{1}{2}}\left(\frac{2b_v\eta}{T_0}\right)^{\frac{1}{2}} |F_{\text{max.}}|^2 \alpha(\nu_0)\, n_0(\nu_0)\, B_0, \qquad (2{\cdot}11)$$

which differs from equation (3·60) in part I by the two factors $(1+a)$ and $(1+\delta)$. The first factor has been introduced to allow for the presence of stray light, so that $(1+a)$ is the ratio of the total incident light to that from the source alone. The second factor has been introduced to allow for the presence of excess noise produced in the correlator, so that $(1+\delta)$ is the ratio of the total r.m.s. uncertainty in the reading of the integrating motor M_1 to the r.m.s. uncertainty due to fluctuations in the output of the photomultiplier tubes. In addition the quantity η, which is the normalized spectral density of the cross-correlation frequency response of the amplifiers following the phototubes, must be defined by

$$\eta = \int_0^\infty |F_1^2(f)\, F_2^2(f)|\, \mathrm{d}f/b_v\, |F_{\text{max.}}|^4 \qquad (2{\cdot}12)$$

in the practical case where the individual frequency responses of the two amplifiers are *not* identical, rather than by equation (3·58) of part I.

The only other quantity which has not previously been defined in the present paper is the factor $\mu/(\mu-1)$ which represents the excess noise introduced by the photomultiplier chain. It is known (Shockley & Pierce 1938) that μ is equal to the multiplication factor of the first stage, at least as long as this is not too great.

If we compare equations (2·4) and (2·11) it will be seen that both the correlation $\overline{C(d)}$ and the uncertainty, or noise, in the output are linearly proportional to $|F_{\text{max.}}|^2$. It follows that their ratio is independent of the gain of the equipment. We have therefore adopted the practice in the present papers of expressing both the observed and theoretical values of correlation as *signal to noise ratios S/N*. The theoretical value of this ratio, from equations (2·4) and (2·11) is

$$\frac{S}{N} = \frac{\overline{C(d)}}{N(T_0)} = \epsilon\beta_0 \frac{\mu-1}{\mu} \frac{(A_1A_2)^{\frac{1}{2}}}{(1+a)(1+\delta)} \alpha(\nu_0)\, n_0(\nu_0) \left(\frac{b_v T_0}{2\eta}\right)^{\frac{1}{2}} \sigma\Delta(\nu_0)\, \Gamma^2(\nu_0, d), \tag{2·13}$$

where the factor $\Delta(\nu_0)\,\Gamma^2(\nu_0, d)$ has been substituted for $\Gamma^2(d)$ to allow for the finite size of the photocathode apertures.

To compare the theory with experiment we need to develop equation (2·13) to give the signal to noise ratio in the practical case where successive measurements may be made with different values of light flux. Let us suppose that, in the rth measurement, of duration T_r, the number of quanta incident in unit bandwidth at frequency ν_0 is $n_r(\nu_0)$. If the gain of the equipment has been kept the same for all the observations, the average value of the final reading of the integrating motor is proportional to $\Sigma n_r^2(\nu_0)T_r$; while the r.m.s. uncertainty in the value of a given reading can be found by adding the individual fluctuations incoherently, so that it is proportional to $(\Sigma n_r^2(\nu_0)\, T_r)^{\frac{1}{2}}$. The theoretical value of the signal to noise ratio, in a convenient form for comparison with experiment, may therefore be written,

$$\frac{S}{N} = \frac{\overline{C(d)}}{N(\Sigma T_r)} = \epsilon\beta_0 \frac{\mu-1}{\mu} \frac{(A_1A_2)^{\frac{1}{2}}}{(1+a)(1+\delta)} \alpha(\nu_0) \left(\frac{b_v}{2\eta}\right)^{\frac{1}{2}} \sigma\Delta(\nu_0, d) \frac{\sum_{r=1}^{M} n_r^2(\nu_0)\, T_r}{\left[\sum_{r=1}^{M} n_r^2(\nu_0)\, T_r\right]^{\frac{1}{2}}}. \tag{2·14}$$

2·4. *The limiting signal to noise ratio for an arbitrarily large source*

In the limiting case where the source is completely resolved by an individual photocathode, the signal to noise ratio, with the cathodes optically superimposed, tends monotonically to a value determined simply by the effective black-body temperature of the source at the received wavelength, and this limit is independent of the source shape. To show this we consider first the case where the source is of arbitrary rectangular shape with an apparent angular width θ_1, θ_2. Then, in the limit,

$$A\theta_1\theta_2\nu_0^2/c^2 \to \infty \tag{2·15}$$

and we have from equations (2·13, A 23) that the signal to noise ratio tends to the limiting value

$$\lim_{\theta\to\infty} \left(\frac{S}{N}\right) = K_1 \frac{\alpha(\nu_0)\, n_0(\nu_0)\, c^2}{\nu_0^2\theta_1\theta_2}, \tag{2·16}$$

where K_1 is a constant of proportionality given by

$$K_1 = \frac{\epsilon\beta_0\sigma}{(1+a)(1+\delta)} \frac{\mu-1}{\mu} \left(\frac{b_v T_0}{2\eta}\right)^{\frac{1}{2}}, \tag{2·17}$$

which depends solely on the parameters of the electronic system and upon the spectral density of the light.

It may be noted that
$$Q_0 = \frac{2\gamma_0 n_0(\nu_0)}{\theta_1 \theta_2} \tag{2·18}$$

is the number of quanta of both polarizations emitted from an area of the source in unit bandwidth and unit solid angle. The factor 2 is included to allow for the fact that in practice the light beam must be split in order to superimpose the photocathodes, and the factor γ_0 takes account of any loss of light in the optical system. If the black-body temperature of the source at a frequency ν_0 is Θ_0, then

$$Q_0 = \frac{2\nu_0^2}{c^2}[\exp(h\nu_0/k\Theta_0) - 1]^{-1} \tag{2·19}$$

and substituting in equation (2·16) the maximum signal to noise ratio from a rectangular source, however large, is given by

$$\left(\frac{S}{N}\right)_{\text{max.}} = K_1 \frac{\alpha(\nu_0)}{\gamma_0}[\exp(h\nu_0/k\Theta_0) - 1]^{-1}. \tag{2·20}$$

This limit will clearly apply whatever the shape of the source, so long as it can be approximated by a series of rectangles or its area can be defined by a Riemann integral. It is interesting to note that under these conditions the signal to noise ratio depends upon the temperature but not upon the shape of the source; thus, effectively the equipment operates as pyrometer.

3. Description of the apparatus

3·1. *The optical equipment*

A simplified outline of the optical equipment is shown in figure 4. A secondary light source was formed by a circular pinhole 0·19 mm in diameter on which the image of a mercury arc was focused by a lens. The image of the arc in the plane of the pinhole was approximately 5 cm in length and its position was adjusted so that the pinhole lay in the relatively bright part of the arc close to one of the electrodes. The arc lamp, Mazda type ME/D 250 W was supplied by a direct current of approximately 4 A and the 4358 Å line of the mercury spectrum was isolated by means of a liquid filter with a transmission of 82 % at 4358 Å. The beam of light from the pinhole was divided by a semi-transparent mirror to illuminate the cathode of the photomultipliers P_1, P_2. The mirror surface was formed by evaporating pure aluminium on to glass, the reverse side being bloomed with cryolite to reduce unwanted internal reflexions. The area of each cathode exposed to the light was limited by a square aperture of 5 × 5 mm, and the distance from the pinhole to each cathode was adjusted to be 2·24 m with an accuracy of about 2 mm.

The photomultipliers were a matched pair, R.C.A. type 6342, with flat end-on cathodes and ten stages of multiplication. The photocathode surfaces had a maximum response at about 4000 Å. Tests at the National Physical Laboratory showed that the quantum efficiencies of the two cathodes, measured at 4000 Å, were 16·9 and 14·6 % and that the shapes of the spectral response curves were almost identical. The type 6342 photomultiplier has a small spread in electron transit time, particularly when the photocathode aperture is limited, and the effective bandwidth of the secondary emission amplification considerably exceeds the limit of 45 Mc/s set by the amplifiers in the correlator.

In order that the degree of coherence between the light on the two cathodes might be varied at will, one of the photomultipliers (P_2) was mounted on a horizontal slide which could be traversed normal to the incident light. Thus the cathode apertures, as viewed from the pinhole, could be superimposed or separated by any amount up to several times their width.

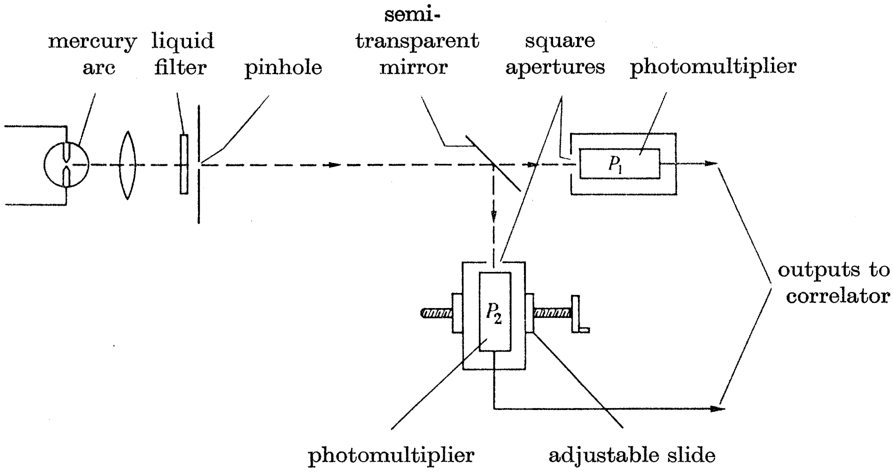

FIGURE 4. A simplified outline of the optical system.

The fluctuations in the anode currents of the photomultipliers were transmitted to the correlator through coaxial cables of equal length. In each case a simple high-pass filter was inserted between the anode and the input to the cable to remove the direct current component.

3·2. *The correlator*

A simplified diagram of the correlator is shown in figure 5. The cable from each of the photomultipliers was terminated in a matched load and the voltage fluctuations across this load were applied to one of the two input channels of the correlator. Both channels consisted of a phase-reversing switch followed by a wide-band amplifier. The switch (S_1) in channel 1 was electronic and reversed the phase of the input voltage 10 000 times per second in response to a 5 kc/s square wave from the generator G_1. It is essential to reduce amplitude modulation of the signal by this switch to an extremely low level in order to prevent spurious drifts in the equipment; for this reason the gain of the switch was equalized in both positions by means of an automatic balancing circuit comprising a detector, a selective 5 kc/s amplifier B_3 and a synchronous rectifier R_1. The phase-reversing switch S_2 in channel 2 consisted of a relay-operated coaxial switch which reversed the phase of the input every 10 s in response to a 0·05 c/s square wave from the generator G_2.

The wide-band amplifiers B_1, B_2 were identical in construction, their gain was substantially constant ($\pm$ 1db) from about 5 to 45 Mc/s and decreased rapidly outside this band. The outputs of these amplifiers were multiplied together in the multiplier C, which consisted of a balanced arrangement of two pentode valves with their anodes in push-pull. The output of the multiplier was then amplified by a high-gain selective amplifier B_4 tuned to 5 kc/s with a bandwidth of 70 c/s. The output of B_4 was applied to the synchronous rectifier R_2 which was of the conventional

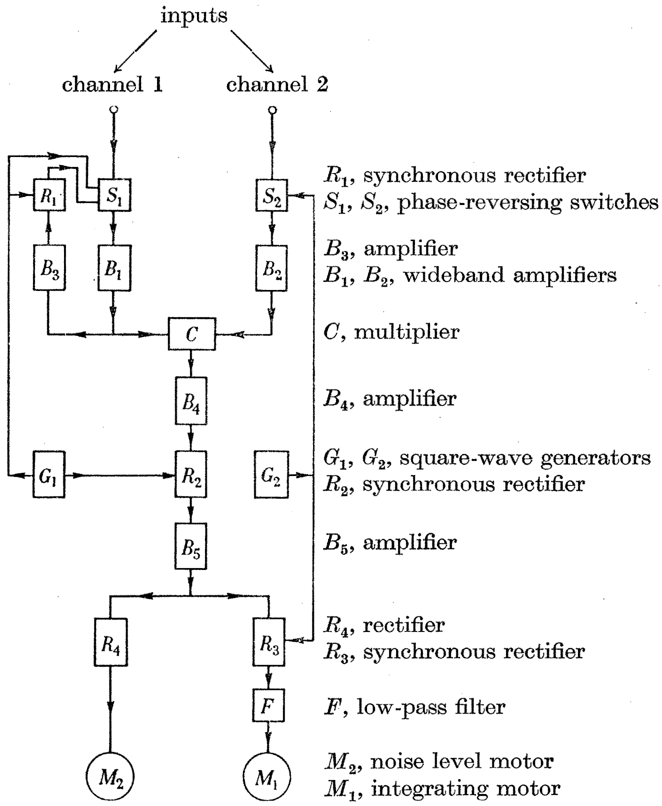

FIGURE 5. An outline of the correlator.

type using a ring of four diodes synchronized by the 5 kc/s switching wave generated by G_1. The rectifier R_2 was followed by the 0·05 c/s amplifier B_5 which was relatively broadband and passed frequencies from about 0·01 to 0·25 c/s. The final synchronous rectifier R_3 consists of a relay-operated switch which, in response to the 0·05 c/s square wave from G_2, periodically reversed the connexions between the output of the amplifier B_5 and the integrating motor M_1. A low-pass filter, containing only passive elements, was inserted between the output of R_3 and the motor to restrict the bandwidth of the signal to the range 0 to 0·01 c/s. The motor itself was a miniature integrating motor coupled through a reduction gear to a revolution counter; it was capable of rotation in either direction and tests showed the relation between speed and input voltage to be linear to better than 1 %. An additional integrating motor M_2 was provided to monitor the r.m.s. level of the output voltage from the amplifier B_5.

If the photomultipliers are illuminated with uncorrelated beams of light then the inputs to the correlator are mutually incoherent random noise voltages. Under these conditions the output of the multiplier is random noise with a spectral density which has a maximum around zero frequency and which decreases to zero at about 40 Mc/s. The corresponding output from the 5 kc/s amplifier B_4 is random noise centred about 5 kc/s with a bandwidth of 70 c/s. After passing through the synchronous rectifier R_2 the spectrum extends from 0 to 35 c/s, and after passing through amplifier B_5 and the second synchronous rectifier R_3 it is reduced to a band extending from about 0 to 0·25 c/s. The low-pass filter following the rectifier R_3 finally restricts the bandwidth to the range 0 to 0·01 c/s. Under the influence of this noise the motor spins in either direction at random and the reading of the revolution counter remains close to zero. However, if there is any correlation between the output voltage of the photomultipliers a 5 kc/s component appears at the anode of the multiplier; this component is coherent with the 5 kc/s switching wave and reverses in phase every 10 s in synchronism with the 0·05 c/s switching wave. After amplification by the selective amplifier B_4 the 5 kc/s component produces a 0·05 c/s square wave in the output of the synchronous rectifier R_2, which in turn is amplified by B_5 and rectified synchronously by R_3 to produce a direct current component in the voltage applied to the integrating motor M_1. Thus, when there is correlation between the input voltages, the integrating motor revolves more in one direction than the other and the reading on the revolution counter increases with time.

The principal difficulty in designing the correlator was to reduce the random drifts in the output to an acceptable value. It was desirable that any drift should be less than the r.m.s. deviation of the integrating motor M_1, due to noise alone, in a period of several hours. This requirement sets an unusually stringent limit to the tolerable level of any spurious signals in the correlator or to any drift in the synchronous rectifiers. For example a 5 kc/s signal, coherent with the switching wave frequency, will produce an output equal to the r.m.s. deviation of the output counter in 1 h if it is greater than 120 db below noise at the output of the multiplier. In a simple system employing a phase-reversing switch in only one channel, it is difficult to reduce random drifts to an acceptable value; however, by the use of two reversing switches and two synchronous rectifiers in cascade it was found possible to reduce the drift by several orders of magnitude without the use of precisely balanced circuits. It was also necessary to ensure that there was no electrical coupling between the inputs to the correlator, and that these circuits did not pick up signals from external sources. For this reason the photomultipliers were heavily screened and all their supply leads were thoroughly decoupled. Any coupling between the two channels which takes place after the phase-reversing switches does not give rise to spurious correlation, and therefore the switches were put as close as possible to the input terminals of the equipment. Apart from these precautions the equipment was mounted in enclosed racks to improve the screening and to help in stabilizing the temperature; all supplies to the equipment were stabilized.

Extensive tests of the correlator, using independent light sources to illuminate the photomultipliers, have shown that over a period of several hours the drift in the output is less than the r.m.s. uncertainty in the counter readings due to noise alone. However, a more detailed examination of the counter readings shows that over periods of a few minutes there are occasional deviations which are unexpectedly large, and it is believed that this effect is due to short-term drifts in the correlator. For the purposes of the present experiment these short-term drifts are unimportant since tests show that their average effect on the counter readings is not significant when readings are taken over periods of $\frac{1}{2}$ h or more.

4. Experimental procedure and results

4·1 *Calibration of the equipment*

The first step in calibrating the equipment was to measure the various parameters which are involved in the theoretical expression for the correlation in equation (2·14).

The combined spectral response of the arc lamp, lens, liquid filter and photocathodes was measured with a spectrograph with a resolving power of about 5 Å, and the result is shown in figure 6. The response has been plotted in terms of the frequency of the light and corresponds to the quantity $\alpha(\nu)\, n_0(\nu)/\alpha(\nu_0)\, n_0(\nu_0)$. From this curve the effective bandwidth B_0 and the normalized spectral density σ were found to be

$$\left.\begin{aligned} B_0 &= 0{\cdot}85\times 10^{13}\,\mathrm{c/s},\\ \sigma &= 0{\cdot}451, \end{aligned}\right\} \tag{4·1}$$

where B_0 is defined by equation (2·5), $\lambda_0 = c/\nu_0 = 4358$ Å, and σ is defined by equation (2·7).

The frequency response curves $|\,F_1^2(f)\,|$, $|\,F_2^2(f)\,|$ of the two amplifiers, B_1 and B_2 in figure 5 were measured directly with a signal generator. The cross-correlation frequency response $\frac{1}{2}[F_1(f)\,F_2^*(f) + F_1^*(f)\,F_2(f)]$ was measured by feeding a signal of variable frequency and constant amplitude into the inputs of the correlator in parallel and observing the output of the multiplier. From these results the bandwidth of the correlator b_v (equation (2·10)), and the spectral density factor η (equation (2·12)) were found to be

$$\left.\begin{aligned} b_v &= 38\,\mathrm{Mc/s},\\ \eta &= 0{\cdot}98. \end{aligned}\right\} \tag{4·2}$$

The excess noise introduced by the correlator and by the stray light reaching the photocathodes is represented in equation (2·14) by the factors $(1+a)$ and $(1+\delta)$, respectively. Measurements showed that there was a small noise contribution from the correlator, mainly due to shot noise in the multiplier which was about 6 % of the total noise, but that stray light was negligible; the two factors therefore have the values

$$\left.\begin{aligned} 1+\delta &= 1{\cdot}06,\\ 1+a &= 1. \end{aligned}\right\} \tag{4·3}$$

It is also necessary, in order to evaluate equation (2·14) to know the value of $G\mu/(\mu-1)$ for each photomultiplier, where G is the overall current gain and μ is the gain of the first stage. As a preliminary test $G\mu/(\mu-1)$ was measured in two different ways. In the first method the current gains G and μ were measured directly by observing the ratio of the respective currents. In the second method the noise voltage across the anode load of each photomultiplier was compared with the noise generated across the same load by a temperature-limited tungsten-filament diode, this comparison being made at the output of the amplifiers B_1 and B_2 to ensure that the noise bandwidth was the same as that used in the actual tests. The quantity $G\mu/(\mu-1)$ for each photomultiplier was then calculated from the simple relation

$$G\mu/(\mu-1) = I_D/I_A, \tag{4·4}$$

where I_D, I_A are the anode currents of the diode and the photomultiplier, respectively, when their noise outputs are adjusted to be equal. The values obtained by the two methods described above agreed within the limits of experimental error.

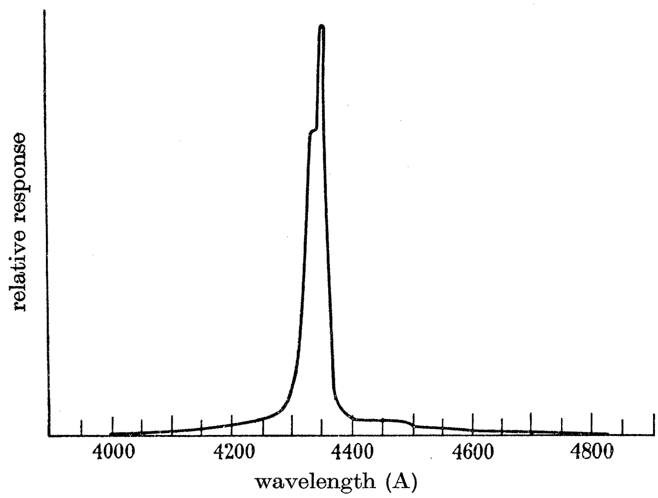

FIGURE 6. Combined spectral response of the arc lamp, optical system, filter and photocathodes.

It was therefore decided to use the second method, employing a noise diode, because it yields the value of $G\mu/(\mu-1)$ directly from a simple observation of the ratio of two currents, and the final theoretical value of the correlation is obtained without an independent measurement of the excess noise introduced by the multiplication process in the phototube. The measured values of $G\mu/(\mu-1)$, under the conditions of the experiment, were found to be $3{\cdot}43\times10^5$ and $4{\cdot}63\times10^5$ for the two photomultipliers, respectively.

Loss of correlation can occur both in the optical and in the electronic part of the equipment. In the present equipment the loss in the optical equipment arises because of polarization produced by the semi-transparent mirror and this is allowed for, in equation (2·14) by the factor β_0 defined by equation (2·8). For the optical system described here it was found that

$$\beta_0 = 0{\cdot}96. \tag{4·5}$$

The loss of correlation in the electronic equipment occurs almost entirely at the synchronous rectifiers. Ideally, the amplifiers before these rectifiers should pass all the harmonics of the square-wave switching waveform if there is to be no loss of sensitivity. However, in the present equipment it was necessary, in order to reduce the level at the input to the synchronous rectifier R_2 (figure 5) to a convenient value, to restrict the bandwidth of the amplifier B_4 to the fundamental of the phase-switching frequency of 5 kc/s. Under these conditions it can be shown (Dicke 1946) that the signal to noise ratio is reduced by the factor $\sqrt{8}/\pi \simeq 0{\cdot}90$. The frequency of the second phase-switch was only 0·05 c/s and it was possible to reduce the loss at the second synchronous rectifier R_3 to a factor 0·955. The overall loss of correlation in the electronic part of the equipment therefore reduced the signal to noise ratio by the factor

$$\epsilon = 0{\cdot}90 \times 0{\cdot}955 = 0{\cdot}86. \tag{4·6}$$

The final step in calibrating the equipment was to relate the r.m.s. fluctuations in the readings of the signal motor M_1 (figure 5) to the rate of revolution of the noise level motor M_2. This calibration was performed as follows. (*a*) Noise from two independent generators was fed into the inputs of the correlator and the output N_1, N_2 of the amplifiers B_1 and B_2 were adjusted so that the noise motor M_2 was turning at the arbitrarily chosen rate of 14 rev in 5 min. (*b*) The two noise generators were replaced by a single generator which was fed into both inputs in parallel, and the power output of this generator was adjusted to give levels N_1, N_2, as before, at the output of the amplifiers, the signal motor M_1 being disconnected to avoid overloading. (*c*) The power output of the single source was then reduced by a factor ρ^2 by means of a precision attenutator. The signal motor was reconnected and the change in output reading C_0 in a time of 5 min was recorded for various values of ρ^2.

It can be shown that the r.m.s. uncertainty $N(T)$ in the reading of the signal motor after a time T is

$$N(T) = \frac{C_0}{\rho^2}\left(\frac{1+\delta}{\epsilon}\right)\left(\frac{\eta}{2b_v T}\right)^{\frac{1}{2}}, \tag{4·7}$$

where δ, ϵ, η, b_v are parameters of the equipment defined previously. In the present case it was found that $C_0 = 131$ rev, $T = 5$ min, $\rho^2 = 1{\cdot}34 \times 10^4$, and substituting the appropriate numerical values for the other parameters, we get that,

$$N(5\,\text{min}) = N_0 = 14{\cdot}7\ \text{rev}, \tag{4·8}$$

where N_0 is defined as the r.m.s. uncertainty in the signal motor reading in a period of 5 min when the noise level motor is revolving at 14 rev in 5 min.

It is interesting to note that the factor $\dfrac{1+\delta}{\epsilon}\left(\dfrac{\eta}{2b_v T}\right)^{\frac{1}{2}}$ in equation (4·7) also appears in the theoretical expression for the signal to noise ratio equation (2·14). Therefore provided the measurement of N_0 is carried out by the method described here, a comparison of the theoretical and experimental signal to noise ratios is independent of the constants ϵ, δ, η and b_v.

4·2. *Experimental procedure*

The measurements were carried out as follows. The two photocathodes, as viewed from the light source, were superimposed by adjusting the position of the photomultiplier P_2 (figure 4). Readings were then taken every 5 min, for a total period of 4 h, of the revolution counters on the integrating motors M_1 and M_2 and also of the anode currents of the photomultipliers. The centres of the two photocathodes, as seen from the light source, were then separated by 1·25, 2·50, 3·75, 5·0 and 10·0 mm. In each of these positions readings were taken at 5 min intervals for about 30 min, the readings were then repeated with the cathodes separated by the same distances but in the opposition direction.

Throughout the experiment the gain of the amplifier B_4 (figure 5) was controlled to keep the output noise from the correlator approximately constant at a level such that the noise motor M_2 was recording 14 rev in 5 min. The gains of the two photomultipliers were measured before and after every run by comparison with a noise diode, as described in §4·1. In practice the gains of the two photomultipliers, which were operated at an anode current of about 100 μA, remained constant throughout the experiment.

4·3. *Experimental results*

A marked correlation was observed in the first run with the cathodes superimposed; the total change in the reading of the integrating motor M_1 after 4 h was 1832 rev which, taking the value of N_0 given in equation (4·8), corresponds to an r.m.s. signal to noise ratio of about 18/1. This correlation was progressively reduced as the cathodes were separated until, when their centres were 10 mm apart, no significant correlation was observed.

TABLE 1. THE EXPERIMENTAL AND THEORETICAL CORRELATION BETWEEN THE FLUCTUATIONS IN THE OUTPUTS OF TWO PHOTOELECTRIC DETECTORS ILLUMINATED WITH PARTIALLY COHERENT LIGHT

run no.	duration (h)	cathode separation (mm) d	observed correlation (r.m.s. signal to noise ratio) (S/N)	theoretical correlation (r.m.s. signal to noise ratio) (S/N)
1	4	0	+17·55	+17·10
2	1	1·25	+ 8·25	+ 8·51
3	1	2·50	+ 5·75	+ 6·33
4	1	3·75	+ 3·59	+ 4·19
5	1	5·00	+ 2·97	+ 2·22
6	1	10·00	+ 0·90	+ 0·13

The actual readings of the counters and the associated anode currents, etc., taken every 5 min, have not been reproduced here; instead the experimental results have been given in the more convenient form of r.m.s. signal to noise ratios, which are shown for each separation of the photocathodes in column 4 of table 1.

The experimental signal to noise ratios shown in table 1 were calculated from the original readings of the counters by the following method. Each 5 min interval was characterized by readings of the integrated correlation C_r recorded by the signal motor M_1, the noise level N_r recorded by the noise motor M_2, and the anode currents I_{1r}, I_{2r} of the photomultipliers. As they stood these results could not be added to give the final signal to noise ratios, because the gain of the correlator had been frequently altered during each run in an attempt to keep the noise level roughly constant and independent of the inevitable small changes in light flux from the arc lamp. However, it can be shown simply that the final signal to noise ratios, formed by combining the observations from M equal intervals, is independent of the correlator gain in each interval if the readings are weighted and added according to the formula

$$\frac{S}{N} = \frac{1}{N_0} \frac{\sum_{r=1}^{M} \frac{I_{1r} I_{2r}}{\bar{I}_1 \bar{I}_2} C_r \frac{\bar{N}}{N_r}}{\left[\sum_{r=1}^{M} \left(\frac{I_{1r} I_{2r}}{\bar{I}_1 \bar{I}_2}\right)\right]^{\frac{1}{2}}}, \tag{4·9}$$

where N_0 is the r.m.s. uncertainty in the correlation recorded in one interval for a standard noise level $\bar{N}$, and $\bar{I}_1$, $\bar{I}_2$ are averaged over all the intervals. The experimental signal to noise ratios shown in table 1 were therefore calculated for each position of the photocathodes by summing the individual readings taken every 5 min according to equation (4·9) using the experimental value of $N_0 = 14{\cdot}7$ rev given in equation (4·8).

5. Comparison between theory and experiment

The theoretical values of the expected correlation for each cathode separation were calculated as follows. For every 5 min interval the quantity

$$\frac{\mu - 1}{\mu} \alpha(\nu_0) \, n_r(\nu_0) \, (A_1 A_2)^{\frac{1}{2}}$$

was derived from the observed anode currents I_{1r}, I_{2r} of the photomultipliers by the relation

$$\frac{(I_{1r} I_{2r})^{\frac{1}{2}}}{e B_0 \{G_1 G_2 \mu_1 \mu_2 / (\mu_1 - 1)(\mu_2 - 1)\}^{\frac{1}{2}}} = \frac{\mu - 1}{\mu} \alpha(\nu_0) \, n_r(\nu_0) \, (A_1 A_2)^{\frac{1}{2}}, \tag{5·1}$$

using the values of B_0 and $G\mu/(\mu - 1)$ given in § 4·3. In a typical 5 min interval

$$(I_{1r} I_{2r})^{\frac{1}{2}} = 104 \times 10^{-6}\,\mathrm{A} \quad \text{and} \quad \frac{\mu - 1}{\mu} \alpha(\nu_0) \, n_r(\nu_0) \, (A_1 A_2)^{\frac{1}{2}} = 1{\cdot}92 \times 10^{-4}.$$

Since $(1 - 1/\mu)\,\alpha(\nu_0)$ had a value of about 0·12, a typical value for the number of quanta per second incident on each photocathode in unit bandwidth at the centre of the emission line was $1{\cdot}6 \times 10^{-3}$. Following equation (2·14), the results for each interval were then added together to give the theoretical signal to noise ratio, taking $T_r = 300$ s and assuming the values for the various parameters of the equipment given in § 4. The partial coherence factor $\Delta(\nu_0)$ for the present equipment, where $\pi\theta_0 a\nu_0/c = \pi\theta_0 b\nu_0/c = 3{\cdot}06$, was calculated from equation (A 25) to be 0·52; the normalized correlation factor $\Gamma^2(\nu_0, d)$ was computed from equation (A 26) and the values are shown in column 6 of table 2. The final theoretical values for the correlation are shown in column 5 of table 1 where they may be compared with the experimental results in column 4.

The results have also been displayed in table 2 and in figure 7 in a form which is intended to show clearly how the correlation decreased with cathode spacing. To allow for the fact that the photomultiplier currents and the observation times were not the same for every cathode spacing, the observed value of the signal to noise ratio at each spacing has been normalized by the corresponding theoretical value calculated for zero cathode spacing, and for the appropriate values of incident light flux and observing time. Effectively, this procedure yields experimental values for the normalized correlation factor, and the results are shown in column 5 of table 2 where they can be compared directly with the theoretical values in column 6. The

TABLE 2. THE EXPERIMENTAL AND THEORETICAL VALUES FOR THE NORMALIZED CORRELATION FACTOR FOR DIFFERENT CATHODE SPACINGS

run no.	cathode separation (mm) d	observed correlation (r.m.s. signal to noise ratio) (S/N)	theoretical correlation assuming cathodes superimposed (r.m.s. signal to noise ratio) $(S/N)'$	experimental value of normalized correlation factor $\Gamma^2(\nu_0, d) = \frac{(S/N)}{(S/N)'}$	theoretical value of the normalized correlation factor $\Gamma^2(\nu_0, d)$
1	0	+17·55	+17·10	$1·03 \pm 0·04$ (p.e.)	1·00
2	1·25	+ 8·25	+ 9·27	$0·89 \pm 0·07$	0·928
3	2·50	+ 5·75	+ 8·85	$0·65 \pm 0·08$	0·713
4	3·75	+ 3·59	+ 8·99	$0·40 \pm 0·07$	0·461
5	5·00	+ 2·97	+ 9·00	$0·33 \pm 0·07$	0·244
6	10·00	+ 0·90	+ 8·17	$0·11 \pm 0·08$	0·015

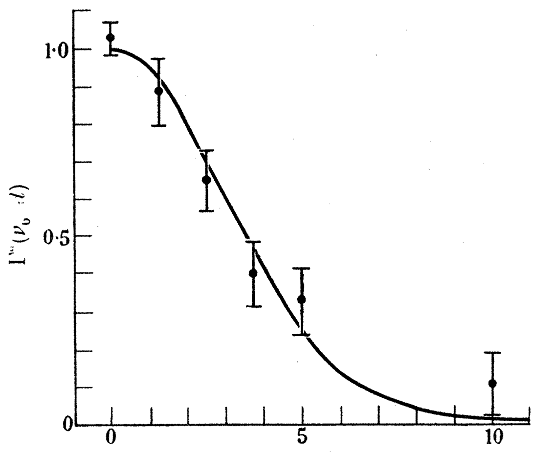

FIGURE 7. The experimental and theoretical values of the normalized correlation factor $\Gamma^2(\nu_0, d)$ for different values of separation between the photocathodes. The full line is the theoretical curve and the experimental results are plotted as points with their associated probable errors.

experimental results have also been plotted in figure 7 for comparison with the theoretical values shown as a solid curve.

A comparison between the theoretical and the experimental values of the correlation given in table 1 and also of the normalized correlation factor given in table 2 and figure 7 shows that, for all cathode spacings, the experimental results are in agreement with theory within the accuracy set by the statistical fluctuations in the measurements. It is true that the difference between theory and experiment is a little greater than the probable error in the case of the widest spacing, but this difference is not significant. The probable error is itself so small a fraction of the observed correlation that the probability that the agreement is due to chance fluctuation and that no real effect is being measured is negligibly small. The probability that the effect is due to some quite different cause, such as fluctuations in the source intensity, is rendered extremely unlikely by the closeness of the agreement between theory and experiment not only with the cathodes superimposed but also with five different cathode spacings.

There remains the possibility that the effect is real, but that its magnitude is not accurately predicted by our theory. This suggestion has been advanced by Fellgett (1957) who has argued on thermodynamical grounds that the excess photon noise, and therefore also the cross-correlation between the fluctuations in different phototubes, should be larger by a factor $\alpha^{-1}(\nu_0)$ than that which we have calculated. We have stated our theoretical objections to Fellgett's analysis in part I of this paper; in addition, his formula is quite incompatible with experimental results reported above, since it would require the correlation to have been about six times greater than that actually observed. In fact, significant disagreement with experiment would arise if the theoretical magnitude of the correlation at every cathode spacing, were increased or decreased† by more than about 12 or 10 %, respectively. However, it must be noted that a systematic error smaller than these limits could not be detected by the present measurements.

In principle, the accuracy of the experiment could be increased indefinitely by increasing the time of observation, since this would reduce the errors due to statistical fluctuations in the correlator output. However, in practice, it is to be expected that the accuracy would soon prove to be limited by errors in the calibration of the equipment or by errors in measurements of such parameters as the spectral density of the light, the cross-correlation frequency bandwidth and the incident light flux, which are probably of the order of 2 or 3 %; furthermore, the theoretical treatment was based on a number of simplifying assumptions, for example, that the quantum efficiency is constant over the photocathodes, and these approximations would also prevent any substantial increase in the precision of the comparison between theory and experiment.

6. Discussion

The experimental results given in the present paper confirm the results of our earlier test (Hanbury Brown & Twiss 1956*a*) and show that the fluctuations of intensity in two coherent beams of light are correlated. They also show that, for the general case of partially coherent light, the observed value of the correlation agrees, within the limits of accuracy of the measurements, with that calculated from a simple classical theory. If there is any systematic error in our calculation of the correlation, for example, due to some quantum effect which has been ignored, it is

† If the theoretical values of the correlation are all increased by the factor a, it can be shown that the best fit with the experimental data, found by minimizing the sum of the weighted squares of the residuals, occurs when $a = 1{\cdot}0137$. The standard deviation of a can be shown to be $0{\cdot}037$ and, if a significant disagreement is defined as three times the standard deviation, we get the result quoted above.

less than 10 %, since any greater error would have produced a significant disagreement with experiment.

Our earlier test has been criticized (Brannen & Ferguson 1956) on the grounds that the observed correlation might have been due to some effect which was modulating the intensity of the light source at frequencies within the passband of the correlator. This suggestion cannot, however, explain the observed decrease of correlation as the separation between the photocathodes is increased, which we have shown here to be in accordance with theory. Admittedly, it has also been suggested, by the same authors, that the decrease in correlation might have been due to light reaching the two photocathodes from different parts of the source as their separation was increased. However, in designing and adjusting the equipment used in both experiments we have taken considerable care to exclude this possibility, and even at the maximum separation of the photocathodes at least 95 % of the incident light came from regions of the source visible to both photocathodes.

The results of our preliminary demonstrations have also been criticized on the grounds that they do not agree with two other experiments which have failed to detect correlation. These experiments were carried out by Ádám, Jánossy & Varga (1955) and by Brannen & Ferguson (1956). In both cases an attempt was made to detect the correlation between the arrival times of photons in two coherent beams of light by means of a coincidence counter. Analyses of these experiments, which have been published elsewhere (Purcell 1956; Hanbury Brown & Twiss 1956*c*), show that they were both too insensitive, by several orders of magnitude, to detect any correlation. In appendix B of the present paper we show the equivalence of the techniques using a coincidence counter and a linear multiplier, but we have also shown that for practical reasons the use of a coincidence counter demands a highly monochromatic, as well as brilliant, source of light. Calculations show that, while it is not feasible to use a standard high-pressure mercury arc, it should be quite practicable to detect correlation with a coincidence counter using a low-pressure mercury isotope lamp. (Since the present paper was written a successful measurement of the correlation between photons using a low-pressure isotope lamp and a coincidence counter has been reported by Twiss, Little & Hanbury Brown (1957).)

It is possible that the principles described here will find practical application in the laboratory. For example, an intensity interferometer can be made to give an extremely high angular resolving power; alternatively, it might perhaps be applied to the measurement of the width and profile of extremely narrow spectral lines. An interesting property, which might have some practical use, was described in § 2·4, where it was shown that, when the source of light is completely resolved, the correlation is a function of the blackbody temperature of the source and effectively the equipment behaves as a pyrometer.

Although we have not considered any of these laboratory applications in detail, we have made a fairly thorough analysis of the application of an intensity interferometer to the measurement of the apparent angular sizes of the visible stars which is given in a later part of this paper. We have also reported briefly a test of the method on Sirius (Hanbury Brown & Twiss 1956*b*). A more detailed account of this work will also be given in a later part of this paper.

We thank the Director of Jodrell Bank Experimental Station for making available the necessary facilities, the Superintendent of the Services Electronic Research Laboratory for the loan of much of the equipment, Dr J. G. Davies for making the numerical computations shown in figures 2 and 3, and Dr A. Burawoy for developing the liquid optical filter.

Appendix A. The theoretical value of the correlation factor and partial coherence factor for partially coherent light

A 1. *The general formula for the correlation between partially coherent fields*

To calculate the correlation, in the general case, where the two light detectors are illuminated with partially coherent light, we shall follow a similar method to that used previously in analyzing a radio interferometer which operated on the same principle (Hanbury Brown & Twiss 1954).

To obtain a quantitative expression for the correlation factor $\Gamma^2(\nu_0, d)$ and the partial coherence factor $\Delta(\nu_0)$ in the case of the simple arrangement shown in figure 1, we shall consider a system of rectangular Cartesian co-ordinates such that the origin lies midway between the centres of the two light detectors, both of which lie in the x axis, and such that the z axis passes through the centre of the light source. We shall assume that the surface of the light source, distant R_0 from the plane containing the photocathodes, can be divided up into elementary areas $\mathrm{d}\boldsymbol{\xi} = \mathrm{d}\xi\,\mathrm{d}\eta$ centred on the points (ξ, η, R_0). It has been shown by Kahn (1957) that the elementary area must be greater than $\lambda^2/2\pi$, where λ is the wavelength of the emitted radiation, but we shall assume here that the area of the source is so large that the error involved in replacing finite summations by integrals is negligible.

Consider the light emitted with a specific polarization from a particular elementary area in a time T. The vector potential at a point distant from the area may be represented by a Fourier series of the form

$$\sum_{r=0}^{\infty} h_r(\boldsymbol{\xi}) \cos\left[\frac{2\pi r}{T}\left(t - \frac{R(\boldsymbol{\xi}, \mathbf{x})}{c}\right) - \chi_r(\boldsymbol{\xi})\right], \tag{A 1}$$

where $\chi_r(\boldsymbol{\xi})$ is a random phase variable distributed with uniform probability between 0 and 2π such that

$$\chi_r(\boldsymbol{\xi})\, \chi_s(\boldsymbol{\xi}') = \delta_{rs}\delta(\boldsymbol{\xi} - \boldsymbol{\xi}')$$

and $h_r^2(\boldsymbol{\xi})\,\mathrm{d}\boldsymbol{\xi}$ is proportional to the number of quanta, incident on unit area at distance R_0, which are emitted with energy h_r/T from an elementary area $\mathrm{d}\boldsymbol{\xi}$ of the source. In what follows we shall assume that the source is so far distant that its apparent angular size at the photocathodes is very small compared with unity.

If we assume that the photocathodes behave like square-law detectors with a conversion efficiency proportional to the quantum efficiency, it follows that the low-frequency fluctuations in one of the photoemission currents due to the intensity fluctuations in the incident light are proportional to

$$\int \mathrm{d}\boldsymbol{\xi} . \mathrm{d}\boldsymbol{\xi}' \sum_{r>s} \sum_{s=1}^{\infty} \int \mathrm{d}\mathbf{x}\, \frac{2e(\alpha_{1r}\alpha_{1s}n_{1r}n_{1s})^{\frac{1}{2}}}{T}$$
$$\times \cos\left[\frac{2\pi(r-s)\,t}{T} - \frac{2\pi}{cT}\{rR(\boldsymbol{\xi}, \mathbf{x}) - sR(\boldsymbol{\xi}', \mathbf{x})\} - \{\chi_r(\boldsymbol{\xi}) - \chi_s(\boldsymbol{\xi}')\}\right],$$

where $n_r(\boldsymbol{\xi}) \sim h_r^2(\boldsymbol{\xi})$ is the number of quanta, emitted by unit area of the source, incident on unit area of the photocathode; e is the charge on the electron; $\boldsymbol{\xi}, \boldsymbol{\xi}'$ are the co-ordinates of arbitrary points on the surface of the source; $\mathbf{x} = (x, y, 0)$ are the co-ordinates of an arbitrary point on one of the photocathodes which we shall take to be rectangular in shape and defined by the inequalities

$$\left.\begin{aligned} -\tfrac{1}{2}b < y < \tfrac{1}{2}b, \quad & -\tfrac{1}{2}d - \tfrac{1}{2}a < x < -\tfrac{1}{2}d + \tfrac{1}{2}a, \\ -\tfrac{1}{2}b < y < \tfrac{1}{2}b, \quad & \tfrac{1}{2}d - \tfrac{1}{2}a < x < \tfrac{1}{2}d + \tfrac{1}{2}a, \end{aligned}\right\} \tag{A 2}$$

respectively.

These fluctuations are amplified by a secondary emission multiplier and an amplifier with a combined complex frequency response $F_1(f)$, and at the output of the amplifier they are proportional to

$$\iint \mathrm{d}\xi\, \mathrm{d}\xi' \sum_{r>s} \sum_{s=1}^{\infty} \int \mathrm{d}\mathbf{x}\, \frac{2e(\alpha_{1r}\alpha_{1s}n_{1r}n_{1s})^{\frac{1}{2}}}{T} \\ \times \mathscr{R}\left\{F_1\left(\frac{r-s}{T}\right) \exp \mathrm{i}\left[\frac{2\pi}{T}\left((r-s)\,t - \frac{rR(\boldsymbol{\xi}, \mathbf{x}) - sR(\boldsymbol{\xi}', \mathbf{x})}{c}\right) - (\chi_r(\boldsymbol{\xi}) - \chi_s(\boldsymbol{\xi}'))\right]\right\}.$$

To the second order in R_0^{-1} we may write

$$\frac{R(\boldsymbol{\xi}, \mathbf{x})}{R_0} = 1 - \frac{\mathbf{x}\,.\,\boldsymbol{\xi}}{R_0^2} + \frac{\boldsymbol{\xi}^2 + \mathbf{x}^2}{2R_0^2},$$

so that

$$\frac{rR(\boldsymbol{\xi}, \mathbf{x}) - sR(\boldsymbol{\xi}', \mathbf{x})}{cT} = \frac{1}{cT}\left\{(r-s)\,R_0 - \mathbf{x}(r\boldsymbol{\xi} - s\boldsymbol{\xi}') + \frac{r\boldsymbol{\xi}^2 - s\boldsymbol{\xi}'^2}{2R_0} - \frac{(r-s)\,\mathbf{x}^2}{2R_0}\right\}. \tag{A 3}$$

In a practical case the last term in equation (A 3) is quite negligible for all values of r, s for which $F_1((r-s)/T)$ differs significantly from zero. Accordingly, the ensemble averaged correlation between the fluctuations in the outputs of the two amplifiers with complex response $F_1(f)$, $F_2(f)$ respectively is proportional to

$$\iiiint \mathrm{d}\boldsymbol{\xi}\, \mathrm{d}\boldsymbol{\xi}'\, \mathrm{d}\mathbf{x}\, \mathrm{d}\mathbf{x}' \sum_{r>s} \sum_{s=1}^{\infty} \frac{2e^2}{T} (\alpha_{1r}\alpha_{1s}n_{1r}n_{1s}\alpha_{2r}\alpha_{2s}n_{2r}n_{2s})^{\frac{1}{2}} \frac{1}{2}\left\{F_1\left(\frac{r-s}{T}\right)\right. \\ \left. \times F_2^*\left(\frac{r-s}{T}\right) + F_1^*\left(\frac{r-s}{T}\right) F_2\left(\frac{r-s}{T}\right)\right\} \cos\left[\frac{2\pi}{cTR_0}\{r(\mathbf{x}-\mathbf{x}')\,\boldsymbol{\xi} - s(\mathbf{x}-\mathbf{x}')\,\boldsymbol{\xi}'\}\right],$$

where $\mathbf{x}$, $\mathbf{x}'$ are the co-ordinates of typical points on the first and second photocathodes, respectively. We shall assume that $\alpha_{1r}n_{1r} \simeq \alpha_{1s}n_{1s}$ and $\alpha_{2r}n_{2r} \simeq \alpha_{2s}n_{2s}$ for all values of r, s for which $F((r-s)/T)$ differs significantly from zero, and also that the angular size of the source is sufficiently small to ensure that,

$$\cos\left\{\frac{2\pi}{cTR_0}\left(r(\mathbf{x}-\mathbf{x}')\,\boldsymbol{\xi} - s(\mathbf{x}-\mathbf{x}')\,\boldsymbol{\xi}'\right)\right\} \simeq \cos\left\{\frac{2\pi(r+s)}{2cTR_0}\,(\mathbf{x}-\mathbf{x}')\,(\boldsymbol{\xi}-\boldsymbol{\xi}')\right\}. \tag{A 4}$$

In this case we can introduce new frequency variables f, ν such that

$$f = (r-s)/T, \quad \nu = (r+s)/2T. \tag{A 5}$$

Then in the limit as $T \to \infty$ we have that, with the cathodes at a spacing d, the ensemble average $\overline{C(d)}$ of the correlation is given by

$$\overline{C(d)} = 2e^2 \iiiint \mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{\xi}'\,\mathrm{d}\mathbf{x}\,\mathrm{d}\mathbf{x}' \int_0^\infty \mathrm{d}\nu\,\alpha_1(\nu)\,\alpha_2(\nu)\,[n_1(\nu,\boldsymbol{\xi})\,n_2(\nu,\boldsymbol{\xi})\,n_1(\nu,\boldsymbol{\xi}')\,n_2(\nu,\boldsymbol{\xi}')]^{\frac{1}{2}}$$
$$\times \cos\left\{\frac{2\pi\nu}{cR_0}(\mathbf{x}-\mathbf{x}')(\boldsymbol{\xi}-\boldsymbol{\xi}')\right\} \int_0^\infty \mathrm{d}f\,\tfrac{1}{2}(F_1(f)\,F_2^*(f) + F_1^*(f)\,F_2(f)), \quad \text{(A 6)}$$

where the integrals are taken over the surfaces of both photocathodes, twice over the surface of the source, over the frequency spectrum of the incident light and over the cross-correlation frequency response of the amplifiers following the photomultiplier tubes.

A 2. *The correlation when the apertures of the light detectors are small*

In the simple case where the apertures of the two photocathodes are too small to resolve the source appreciably and where the bandwidth of the light is small, we may replace $(\mathbf{x}-\mathbf{x}')(\boldsymbol{\xi}-\boldsymbol{\xi}')$ in equation (A 6) by $d(\xi-\xi')$ and write

$$\overline{C(d)} = \Gamma^2(\nu_0, d)\,\overline{C(0)}, \quad \text{(A 7)}$$

where ν_0 is the midband frequency of the light; $\overline{C(0)}$ is the correlation with zero spacing between the photocathodes; $\Gamma^2(\nu_0, d)$ is the normalized correlation factor. It follows from equation (A 6) that

$$\Gamma^2(\nu_0, d) = \frac{\iint \mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{\xi}'\,h^2(\boldsymbol{\xi})\,h^2(\boldsymbol{\xi}')\cos\left\{\dfrac{2\pi\nu_0}{cR_0}d(\xi-\xi')\right\}}{\iint \mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{\xi}'\,h^2(\boldsymbol{\xi})\,h^2(\boldsymbol{\xi}')}, \quad \text{(A 8)}$$

where
$$h^2(\boldsymbol{\xi}) \sim [n_1(\nu_0, \boldsymbol{\xi})\,n_2(\nu_0, \boldsymbol{\xi}')]^{\frac{1}{2}}.$$

Alternatively, we may write

$$\Gamma^2(\nu_0, d) = H(\nu_0, d)\,H^*(\nu_0, d), \quad \text{(A 9)}$$

where $H(\nu_0, d)$ is defined by

$$H(\nu_0, d) = \int_{-\infty}^{\infty} \mathrm{d}\xi \exp\left(\frac{-2\pi\mathrm{i}\xi\nu_0 d}{cR_0}\right) \int_{-\infty}^{\infty} \mathrm{d}\eta\,h^2(\xi,\eta) \Big/ \int_{-\infty}^{\infty} \mathrm{d}\xi \int_{-\infty}^{\infty} \mathrm{d}\eta\,h^2(\xi,\eta), \quad \text{(A 10)}$$

Now
$$\overline{h^2(\xi)} = \int_{-\infty}^{\infty} \mathrm{d}\eta\,h^2(\xi,\eta)$$

is the intensity distribution over the equivalent line source projected parallel to the x axis and $H(\nu_0, d)$ is the normalized Fourier transform of this quantity. It follows, as stated in the text, that $\Gamma^2(\nu_0, d)$ is the square of the amplitude of this normalized Fourier transform.

A 3. *The correlation when the apertures of the light detectors are large*

A 3·1. *The general formula*

In the case where the aperture of the two photocathodes are so large that they appreciably resolve the source, the correlation with zero spacing is reduced by the *partial coherence factor* $\Delta(\nu_0)$ while $\Gamma^2(\nu_0, d)$, the *correlation factor* is now a function of the apertures of the photocathodes as well as of the angular size of the source.

If we assume (see § 2·2) that the light intensity may be taken as uniform over the source and that the quantum efficiency is constant over each photocathode, then from equation (A 6) the correlation is

$$\overline{C(d)} = 2e^2 \iiiint \mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{\xi}'\mathrm{d}\mathbf{x}\,\mathrm{d}\mathbf{x}' \cos\left[\frac{2\pi\nu_0}{cR_0}(\mathbf{x}-\mathbf{x}')(\boldsymbol{\xi}-\boldsymbol{\xi}')\right]$$
$$\times \int_0^\infty \alpha_1(\nu)\,\alpha_2(\nu)\,n_1(\nu)\,n_2(\nu)\,\mathrm{d}\nu \int_0^\infty \tfrac{1}{2}\{F_1(f)\,F_2^*(f) + F_1^*(f)\,F_2(f)\}\,\mathrm{d}f \quad \text{(A 11)}$$

and it has been assumed that the light bandwidth is so narrow that

$$\cos\left[\frac{2\pi\nu}{cR_0}(\mathbf{x}-\mathbf{x}')(\boldsymbol{\xi}-\boldsymbol{\xi}')\right]$$

does not change significantly over the bandwidth for which $\alpha_1(\nu)\,\alpha_2(\nu)\,n_1(\nu)\,n_2(\nu)$ differs appreciably from zero.

From the definition of $\Delta(\nu_0)$ and $\Gamma^2(\nu_0, d)$ in § 2·2, it follows simply from equation (A 11) that

$$\Delta(\nu_0)\,\Gamma^2(\nu_0, d) = \frac{1}{\Omega_0^2 A_1 A_2} \iiiint \frac{\mathrm{d}\boldsymbol{\xi}\,\mathrm{d}\boldsymbol{\xi}'\,\mathrm{d}\mathbf{x}\,\mathrm{d}\mathbf{x}'}{R_0^4} \cos\left\{\frac{2\pi\nu_0(\mathbf{x}-\mathbf{x}')(\boldsymbol{\xi}-\boldsymbol{\xi}')}{cR_0}\right\}, \quad \text{(A 12)}$$

where the areas A_1, A_2 of the photocathodes and the solid angle subtended by the source at the photocathodes are given by

$$A_1 = \int \mathrm{d}\mathbf{x}, \quad A_2 = \int \mathrm{d}\mathbf{x}', \quad \Omega_0 = \int \frac{\mathrm{d}\boldsymbol{\xi}}{R_0^2} = \int \frac{\mathrm{d}\boldsymbol{\xi}'}{R_0^2}. \quad \text{(A 13)}$$

A 3·2. *A rectangular source viewed by two light detectors with rectangular apertures*

When the source is rectangular in shape with angular dimensions θ_1, θ_2, where $\theta_1\theta_2 = \Omega_0$, it is best to evaluate equation (A 12) by integrating initially over the variables $\mathbf{x}$, $\mathbf{x}'$ to get

$$\Delta(\nu_0)\Gamma^2(\nu_0, d) = \frac{1}{A_1 A_2(\theta_1\theta_2)^2} \iint \frac{\mathrm{d}\eta\,\mathrm{d}\eta'}{R_0^2}\left[\frac{\sin(\pi(\eta-\eta')\,b\nu_0/cR_0)}{\pi(\eta-\eta')\,\nu_0/cR_0}\right] \iint \frac{\mathrm{d}\xi\,\mathrm{d}\xi'}{R_0^2}$$
$$\times \left[\frac{\sin(\pi(\xi-\xi')\,a\nu_0/cR_0)}{\pi(\xi-\xi')\,\nu_0/cR_0}\right]^2 \cos\left[\frac{2\pi\nu_0 d(\xi-\xi')}{cR_0}\right], \quad \text{(A 14)}$$

where
$$A_1 = A_2 = ab. \quad \text{(A 15)}$$

We now introduce new variables defined by,

$$\left.\begin{aligned} \phi &= \frac{\pi a\nu_0}{cR_0}(\xi-\xi'), \quad & \phi' &= \frac{\pi a\nu_0}{cR_0}\left(\frac{\xi+\xi'}{2}\right), \\ \psi &= \frac{\pi b\nu_0}{cR_0}(\eta-\eta'), \quad & \psi' &= \frac{\pi b\nu_0}{cR_0}\left(\frac{\eta+\eta'}{2}\right), \end{aligned}\right\} \quad \text{(A 16)}$$

and integrate over ϕ', ψ' subject to the inequalities

$$\phi' < \left|\frac{\pi a\theta_1\nu_0}{c} - \phi\right|, \quad \psi' < \left|\frac{\pi b\theta_2\nu_0}{c} - \psi\right|. \quad \text{(A 17)}$$

We then get that

$$\Delta(\nu_0)\Gamma^2(\nu_0, d) = \left(\frac{c^2}{\pi^2\nu_0^2\theta_1\theta_2}\right)^2 \frac{1}{A_1 A_2}$$
$$\times \int_0^\Phi \frac{2\sin^2\phi}{\phi^2}(\Phi-\phi)\cos\left(\frac{2d\phi}{a}\right)\mathrm{d}\phi \int_0^\Psi \frac{2\sin^2\psi}{\psi^2}(\Psi-\psi)\,\mathrm{d}\psi, \quad \text{(A 18)}$$

where θ_1, θ_2, the angular dimensions of the source, are defined by

$$\theta_1 = |\xi_1-\xi_2|_{\text{max.}}/R_0, \quad \theta_2 = |\eta_1-\eta_2|_{\text{max.}}/R_0, \quad \text{(A 19)}$$

and
$$\Phi = \frac{\pi a\theta_1\nu_0}{c}, \quad \Psi = \frac{\pi b\theta_2\nu_0}{c}. \quad \text{(A 20)}$$

Equation (A 18) involves integrals of the form

$$\mathfrak{J}(\Phi) = \int_0^\Phi \frac{2\sin^2\phi}{\phi^2}(\Phi-\phi)\,\mathrm{d}\phi \quad \text{(A 21)}$$

and, in the case where the source is not appreciably resolved by the individual photocathode apertures ($\Phi \ll 1$), we may write

$$\mathfrak{J}(\Phi) = \Phi^2 = \frac{\pi^2\theta_1^2 a^2\nu_0^2}{c^2}, \quad \text{(A 22)}$$

so that in this case $\Delta(\nu_0) = 1$, as indeed it must be by definition.

For the opposite extremes, where the source is completely resolved by the individual photocathode apertures ($\Phi \to \infty$), then $\mathfrak{J}(\Phi) \to \pi\Phi$ and hence,

$$\Delta(\nu_0) \to \frac{c^2}{\nu_0^2\theta_1\theta_2}(A_1A_2)^{-\frac{1}{2}}. \quad \text{(A 23)}$$

For intermediate values of Φ, $\mathfrak{J}(\Phi)$ may be expressed conveniently in terms of tabulated functions

$$\mathfrak{J}(\Phi) = 2\Phi\,\mathrm{Si}\,(2\Phi) - (1-\cos 2\Phi) + \ln(\gamma\Phi) - \mathrm{Ci}\,(2\Phi), \quad \text{(A 24)}$$

where γ is Euler's constant given approximately by $\ln(\gamma) = 0{\cdot}5772$, and this result was used in computing the partial coherence factor for the case of the preliminary experiment that we have described elsewhere (Hanbury Brown & Twiss 1956*a*).

A 3·3. *A circular source viewed by two light detectors with rectangular apertures*

When the light source is circular the quantities $\Delta(\nu_0)$ and $\Gamma^2(\nu_0, d)$ can be derived more simply, since the distribution of intensity over the equivalent line source is independent of the direction of the line joining points on the two photocathodes. Thus, the correlation between the fluctuations in the currents emitted from points $(x_1, y_1, 0)$ and $(x_2, y_2, 0)$ on the two photocathodes is proportional to

$$\frac{4J_1^2\{(\pi\theta_0\nu_0/c)\,((x_1-x_2)^2+(y_1-y_2)^2)^{\frac{1}{2}}\}}{(\pi\theta_0\nu_0/c)^2\,((x_1-x_2)^2+(y_1-y_2)^2)},$$

where J_1 is a Bessel function of the first order, and θ_0 is the angular diameter of the source.

Thus from equation (A 12) the partial coherence factor $\Delta(\nu_0)$ is given by

$$\Delta(\nu_0) = \frac{1}{A_1 A_2}\int_{-\frac{1}{2}b}^{\frac{1}{2}b} \mathrm{d}y_1 \int_{-\frac{1}{2}b}^{\frac{1}{2}b} \mathrm{d}y_2 \int_{-\frac{1}{2}a}^{\frac{1}{2}a} \mathrm{d}x_1 \int_{-\frac{1}{2}a}^{\frac{1}{2}a} \mathrm{d}x_2 \, \frac{4J_1^2\{(\pi\theta_0\nu_0/c)\,((x_1-x_2)^2+(y_1-y_2)^2)^{\frac{1}{2}}\}}{(\pi\theta_0\nu_0/c)^2\,((x_1-x_2)^2+(y_1-y_2)^2)} \tag{A 25}$$

and the normalized correlation factor $\Gamma^2(\nu_0, d)$ is given by

$$\Gamma^2(\nu_0, d) = \frac{1}{A_1 A_2 \Delta(\nu_0)}\int_{-\frac{1}{2}b}^{\frac{1}{2}b} \mathrm{d}y_1 \int_{-\frac{1}{2}b}^{\frac{1}{2}b} \mathrm{d}y_2 \int_{-\frac{1}{2}(d+a)}^{-\frac{1}{2}(d-a)} \mathrm{d}x_1 \int_{\frac{1}{2}(d-a)}^{\frac{1}{2}(d+a)} \mathrm{d}x_2 \times \frac{4J_1^2\{(\pi\theta_0\nu_0/c)\,((x_1-x_2)^2+(y_1-y_2)^2)^{\frac{1}{2}}\}}{(\pi\theta_0\nu_0/c)^2\,((x_1-x_2)^2+(y_1-y_2)^2)\}}. \tag{A 26}$$

These last two results have been used to analyze the experiment described in the text.

Appendix B. The correlation between photons measured with a coincidence counter

We shall consider here an extremely simple system consisting of two photomultiplier tubes and a coincidence counter, and shall discuss briefly the idealized case in which a coincidence will be recorded if, and only if, two photoelectrons are emitted from the two photocathodes with a time difference less than the resolving time of the coincidence counter. We shall assume initially that the incident light is a linearly polarized plane wave; the extension to the general case of an arbitrarily polarized and partially coherent beam can then be made along identical lines to those developed in § 2 of the text.

B 1. *Mathematical theory*

If $P(t_n)\,\mathrm{d}t$ is the probability that a photoelectron be emitted from a photocathode in the time interval

$$t_n \leqslant t < t_n + \mathrm{d}t, \tag{B 1}$$

then we showed in equation (3·14) of part I that

$$P(t_n) = \sum_{r=0}^{\infty} \frac{\alpha_r n_r A}{T} + 2\sum_{r>s}\sum_{s=1}^{\infty} \frac{A}{T}(\alpha_r\alpha_s n_r n_s)^{\frac{1}{2}} \cos\left\{\frac{2\pi}{T}(r-s)\,t_n + \phi_r - \phi_s\right\}, \tag{B 2}$$

where T is an arbitrary time interval not less than the observation time of the experiment; α_r is the cathode quantum efficiency at frequency r/T; n_r is the number of linearly polarized quanta of frequency r/T incident in unit time on unit area of the photocathode; A is the area of the photocathode; ϕ_r, ϕ_s are independent random variables distributed with uniform probability over the range $0 \to 2\pi$.

It follows that $P(\tau_c, t_n)$, the probability that *one* electron be emitted in the time interval

$$t_n - \tau_c < t < t_n + \tau_c, \tag{B 3}$$

is given by

$$P(\tau_c, t_n) = \sum_{r=1}^{\infty} \frac{A}{T}\alpha_r n_r 2\tau_c + 2\sum_{r>s}\sum_{s=1}^{\infty} \frac{A}{T}(\alpha_r\alpha_s n_r n_s)^{\frac{1}{2}} \times \frac{2\sin\left(2\pi(r-s)\,\tau_c/T\right)}{2\pi(r-s)/T} \cos\left[\frac{2\pi}{T}(r-s)\,t_n + \phi_r - \phi_s\right], \tag{B 4}$$

provided that τ_c is so small that one can neglect the probability that two or more electrons will be emitted in a time interval of duration $2\tau_c$. This last limitation is obviously essential to the use of a coincidence counter technique.

The ensemble average of the joint probability $P_1(\tau_c, t_n)$ that an election be emitted from one photocathode in the interval given by the inequality (B 1) while an electron is emitted from the other photocathode in the interval given by the inequality (B 3), may be written

$$\overline{P_1(\tau_c, t_n)\, P_2(t_n)\,\mathrm{d}t} = \mathrm{d}t \left\{ \left(\sum_{r=1}^{\infty} \frac{A_1}{T} \alpha_{1r} n_{1r} \right) \left(\sum_{s=1}^{\infty} \frac{A_2}{T} \alpha_{2s} n_{2s} \right) 2\tau_c + 4 \sum_{r>s} \sum_{s=1}^{\infty} \frac{A_1 A_2}{T^2} (\alpha_{1r} \alpha_{2r} n_{1r} n_{2r}) \frac{\sin(2\pi(r-s)\,\tau_c/T)}{2\pi(r-s)/T} \right\}, \tag{B 5}$$

since all the terms which depend explicitly on ϕ_r, ϕ_s average to zero.

In the simple case where $A_1\alpha_1 n_1 = A_2\alpha_2 n_2 = A\alpha(\nu)\, n(\nu)$ the ensemble average $\overline{C(T_0)}$ of the number of coincidences in time T_0, is given by

$$\overline{C(T_0)} = 2\tau_c T_0 \left(\int_0^{\infty} A\alpha(\nu)\, n(\nu)\,\mathrm{d}\nu \right)^2 + 4T_0 \int_{\nu_s}^{\infty} \mathrm{d}\nu_r \int_0^{\infty} \mathrm{d}\nu_s A^2 \alpha(\nu_r)\, \alpha(\nu_s)\, n(\nu_r)\, n(\nu_s) \frac{\sin(2\pi(\nu_r - \nu_s)\,\tau_c)}{2\pi(\nu_r - \nu_s)}, \tag{B 6}$$

where we have let $T \to \infty$.

If the light bandwidth is so large compared with $1/\tau_c$ the reciprocal resolving time of the coincidence counter, that $\alpha(\nu)\, n(\nu)$ does not vary appreciably over the frequency band for which $\sin(2\pi(\nu_r - \nu_s)\,\tau_c)/2\pi(\nu_r - \nu_s)$ differs significantly from zero, we have that

$$\overline{C(T_0)} = 2\tau_c T_0 \left(\int_0^{\infty} A\alpha(\nu)\, n(\nu)\,\mathrm{d}\nu \right)^2 + T_0 \int_0^{\infty} A^2 \alpha^2(\nu)\, n^2(\nu)\,\mathrm{d}\nu, \tag{B 7}$$

which may be written

$$\overline{C(T_0)} = \overline{C_R(T_0)} + \overline{C_c(T_0)} = 2\tau_c T_0\, N_p^2 + \tau_0 T_0 N_p^2, \tag{B 8}$$

where N_p, the average number of photoelectrons emitted by either photocathode in unit time, is

$$N_p = \int_0^{\infty} A\alpha(\nu)\, n_0(\nu)\,\mathrm{d}\nu \tag{B 9}$$

and $c\tau_0$ is the 'coherence length' of the light (Born 1933) which, in the general case where the light bandwidth is of arbitrary shape, may be defined by

$$c\tau_0 = \frac{c\sigma}{B_0}, \tag{B 10}$$

where σ, and B_0 are defined by equations (2·7) and (2·5).

In equation (B 8) $C_R(T_0)$ is the average number of coincidences which would occur by chance if the emission of photoelectrons by the two photocathodes was completely random, and is given by

$$\overline{C_R(T_0)} = 2\tau_c T_0 N_p^2, \tag{B 11}$$

where it is assumed that $\tau_c \gg \tau_0$ and that $N_p \tau_c \ll 1$, while $\overline{C_c(T_0)}$ is the average number of excess coincidences due to the so-called 'bunching' of photons, and is given by

$$\overline{C_c(T_0)} = \tau_0 T_0 N_p^2. \tag{B 12}$$

If the incident light is randomly polarized and if N_0 is now the total average number of photoelectrons emitted in unit time from either photocathode, then the average number of excess coincidences is given by

$$\overline{C_c(T_0)} = \tfrac{1}{2}\tau_0 T_0 N_0^2. \tag{B 13}$$

since there is no correlation between quanta with mutually orthogonal polarizations. The random coincidences, on the other hand, depend only upon the total number of incident quanta so that,

$$\overline{C_R(T_0)} = 2\tau_c T_0 N_0^2 \tag{B 14}$$

as long as

$$\left.\begin{aligned} \tau_c &\gg \tau_0, \\ N_0 \tau_c &\ll 1. \end{aligned}\right\} \tag{B 15}$$

When the incident light is only partially coherent over the individual photocathode apertures and when the centres of the latter are separated by a distance d, the number of excess coincidences will be reduced by a factor $\overline{\Delta \Gamma^2(d)}$, where $\overline{\Delta}$ the *partial coherence factor* and $\overline{\Gamma^2(d)}$ the *normalized correlation factor* are defined in § 2. Thus, in this more general case, the correlation between the photons arriving at the two photocathodes increases the number of coincidences over the random rate by a factor

$$1 + \rho_c = 1 + \overline{\Delta \Gamma^2(d)}\, \frac{\tau_0}{4\tau_c}, \tag{B 16}$$

which is independent of the quantum efficiency and of the intensity of the incident light.

The random coincidences obey a Poisson distribution so that the r.m.s. fluctuation in their number is given by,

$$\{\langle C_R(T_0) - \overline{C_R(T_0)}\rangle^2_{\text{aver.}}\}^{\frac{1}{2}} = \overline{C_R(T_0)}^{\frac{1}{2}} = 2N_0(\tau_c T_0)^{\frac{1}{2}}. \tag{B 17}$$

Therefore the *signal to noise ratio*, defined as the ratio of the average number of excess coincidences to the r.m.s. fluctuation in the random coincidence rate, is

$$\frac{S}{N} = \frac{\overline{C_c(T_0)}}{\overline{C_R(T_0)}^{\frac{1}{2}}} = \frac{\overline{\Delta \Gamma^2(d)}\, N_0 \tau_0}{2} \left(\frac{T_0}{2\tau_c}\right)^{\frac{1}{2}}, \tag{B 18}$$

which from equations (2·3) (2·6) and (B 10) may be written in the equivalent form

$$\frac{S}{N} = \overline{\Delta \Gamma^2(d)}\, A \alpha(\nu_0)\, n_0(\nu_0)\, \sigma \left(\frac{T_0}{4\sqrt{2}\,\tau_c}\right)^{\frac{1}{2}}. \tag{B 19}$$

The results given above have been quoted elsewhere (Hanbury Brown & Twiss 1956*c*) for the special case $\overline{\Gamma^2(d)} = 1$; an exactly similar result has been derived by Purcell (1956) for the case $\overline{\Delta \Gamma^2(d)} = 1$ by an analysis based upon the auto-correlation function for the intensity fluctuations in the incident light. The present analysis, is, in effect, the Fourier transform dual to that of Purcell and it is simple to show that τ_0 as defined in (B 10) is identical with the τ_0 used by Purcell.

B 2. *Comparison with the alternative technique*

If equation (B 19) is compared with equation (2·13) which gives the signal to noise ratio for the linear multiplier technique, we see that the two expressions are the same if we put

$$\frac{b_v}{\eta} = \frac{1}{4\tau_c} \tag{B 20}$$

and assume that both equipments have an ideal performance

$$(\text{i.e.}\quad \epsilon\gamma_0(\mu-1)/\{\mu(1+a)(1+\delta)\} = 1).$$

Now b_v/η is roughly the response time of the amplifiers in the linear multiplier system, or alternatively is roughly the effective bandwidth of the coincidence counter. It is therefore clear that in the ideal case both techniques, coincidence counting of photoelectrons or linear multiplication of intensity fluctuations, give about the same signal to noise ratio provided that the effective bandwidth of their circuits, the spectral distribution of the incident light, and the primary photoemission currents are the same in both cases.

However, in a practical case there are considerable difficulties in meeting the condition that the primary photoemission currents should be the same for both techniques. Thus the need to satisfy the inequalities defined in equation (B 15) severely limits the maximum photoemission on current when the resolving time of coincidence counter is fixed, and in practice it is difficult to let N_0, the average number of photoelectrons emitted per second, rise much above 10^6 when one is using the coincidence-counter technique. As we have shown above the signal to noise ratio, which is by far the most important limitation of an 'intensity' interferometer, is determined by the number of incident quanta per cycle bandwidth rather than by the total incident light flux, and to ensure a workable signal to noise ratio, the average number of photoelectrons produced per second, by quanta in a frequency band of 1 c/s, must be of the order† of 10^{-5} or greater. This latter requirement can only be satisfied by a source of high equivalent temperature, especially if the source is to be negligibly resolved by the apertures of the photocathodes. For example, if $N_0 < 10^6$ this means that the light bandwidth must be less than 10^{11} c/s, which corresponds to a bandwidth of about 0·5 Å at a wavelength of 4000 Å.

In the laboratory these conditions can easily be met by using a low-pressure electrodeless isotope lamp. Thus a source described by Forrester, Gudmundsen & Johnson (1955) had a bandwidth of only $8 \cdot 10^8$ cycles, centred on the 5461 green line of ^{198}Hg, with an output flux of 0·004 W cm^{-2} sterad^{-1} which corresponds to an effective black-body temperature at the centre of the line of 6750° K.

However, in the case of a measurement on a star, where the incident light is spread over the whole visible spectrum, narrow bandwidths can only be obtained by means of interference filters (Ring 1956) which introduce appreciable attenuation and demand well collimated beams of light. For these and other technical reasons the coincidence-counting technique has not been seriously considered for astronomical applications.

† With practical values for the parameters of equation (2·14) and an amplifier bandwidth of 100 Mc/s one would get a signal to noise ratio of 3 to 1 in 1 h with $\sqrt{(A_1 A_2)}\,\alpha(\nu_0)\,n_0(\nu_0) = 10^{-5}$ and this is about the lowest sensitivity with which one could work comfortably.

B3. *The limiting case of arbitrarily small resolving time*

We have, so far, only considered the case in which τ_c, the resolving time of the coincidence counter is very much greater than τ_0, where $c\tau_0$ is the 'coherence length' of the incident light. However, if the light source is, for example, a low-pressure isotope lamp this condition is not necessarily valid and a brief discussion of the more general case is needed.

In the limiting case $\tau_c/\tau_0 \to 0$ it follows immediately from equation (B5) that

$$C(T_0) \to 2\tau_c T_0 \left(\int_0^\infty A\alpha(\nu)\, n(\nu)\, \mathrm{d}\nu \right)^2 + 2\tau_c T_0 \left(\int_0^\infty A\alpha(\nu)\, n(\nu)\, \mathrm{d}\nu \right)^2, \tag{B21}$$

when

$$\overline{C_c(T_0)} \to \overline{C_R(T_0)} = 2\tau_c T_0 N_p^2 \tag{B22}$$

in the idealized case of a linearly polarized plane wave of light. The average number of excess coincidences is therefore equal to the number of purely random coincidences which would arise from completely incoherent light beams, a result which is independent of the spectral distribution of the incident light.

In the general case, when τ_c/τ_0 is finite and non-zero, it can be shown from equation (B5) that

$$C_R(T_0) = \tau_0 T_0 N_p^2 \operatorname{erf}\left(\frac{\sqrt{\pi}\,\tau_c}{\tau_0}\right), \tag{B23}$$

assuming that the spectral distribution of the incident light is Gaussian, as will be approximately the case when the line broadening is due to Doppler effect. The signal to noise ratio is therefore reduced by the factor

$$\operatorname{erf}(\sqrt{\pi}\,\tau_c/\tau_0),$$

which tends to zero as $\tau_c \to 0$, and this result emphasizes that the correlated photons are not in perfect time coincidence. There is a fundamental uncertainty in their arrival time which is of the order of the reciprocal of the bandwidth of the incident light, a result which can be deduced directly from the uncertainty principle.

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